PHIL309P
Philosophy, Politics and Economics

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Group of voters

Assume that there are an odd number of experts

Candidates:

Two candidates A and B

Preferences:

Rank A above B

Rank B above A

Indifferent between A and B

Aggregation method

Majority rule: A wins if more voters rank A above B than B above A; B wins if more voters rank B above A than A above B;

Group of experts

- Assume that there are an odd number of experts

Agenda:

- A single proposition P

Judgements:

- Accept P/Judge that P is true
- Reject P/Judge that P is false
- Suspend judgement about P

Aggregation method

- Majority rule: Accept P if more people accept P than reject P; Reject P if more people reject P than accept P

Condorcet Jury Theorem

Suppose that the P takes values 0 and 1

R_i is the event that voter i reports correctly.

Condorcet Jury Theorem

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R_i is the event that voter i reports correctly.

Independence The reports of the voters are independent conditional on the state of the world: $R_1, R_2, \ldots$ are *independent conditional* on P

Competence: For each voter, the probability that the reports correctly is greater than 1/2: for each $x \in \{0, 1\}$, $p(R_i \mid P = x) > \frac{1}{2}$ and

Condorcet Jury Theorem

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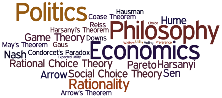
Condorcet Jury Theorem. Suppose Independence and Competence. As the group size increases, the probability that majority opinion is correct (i) increases and (ii) converges to one.

Literature



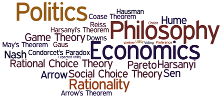
D. Austen-Smith and J. Banks. *Aggregation, Rationality and the Condorcet Jury Theorem*. The American Political Science Review, 90:1, pgs. 34 - 45, 1996.

D. Estlund. *Opinion Leaders, Independence and Condorcet's Jury Theorem*. Theory and Decision, 36, pgs. 131 - 162, 1994.

F. Dietrich. *The premises of Condorcet's Jury Theorem are not simultaneously justified*. Episteme,5:1, pgs. 56 - 73, 2008.

S. Nitzan. *Collective Preference and Choice (Part III)*. Cambridge University Press, 2010.

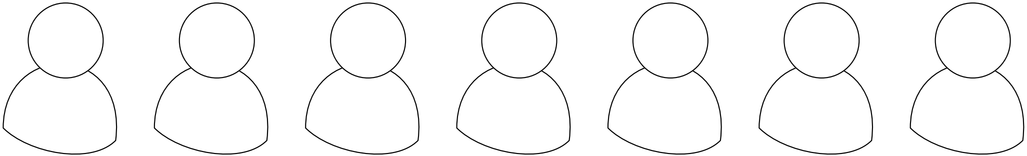
Judgement aggregation model



- Group of experts
- Agenda
- Judgement
- Aggregation method

Group of experts

- Evidence: shared or independent
- Communication: Allow communication/sharing of opinions
- Opinionated
- Coherent: logically and/or probabilistically



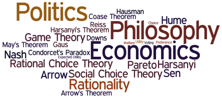
Agenda

- Single issue/proposition
- Set of independent issues/propositions
- Set of logically connected issues/propositions

Is P true?

Do you accept P_1?
Do you accept P_2?
$\vdots$
Do you accept P_n?

Do you accept P?
Do you accept $P \rightarrow Q$?
Do you accept Q?



Agenda

- Single issue/proposition
- Set of independent issues/propositions
- Set of logically connected issues/propositions
- Value from some range (quantity/chance)
- Causal relationships between variables

What is the chance that E will happen?

What is the value of x?

Which intervention will be most effective?

Judgements

- Expressions of judgement vs. expressions of preference
- Qualitative: Accept/Reject; Orderings; Grades

Accept P

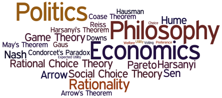
P_1	P_2	$\cdots$	P_n
Y	N	$\cdots$	Y

$P \geq Q \geq R \geq \cdots$

Reject P

P	$P \rightarrow Q$	Q
Y	N	N

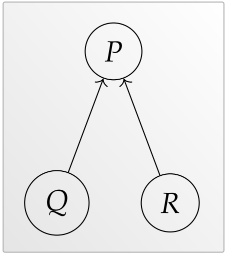
P is very likely
Q is very likely
R is very unlikely
$\vdots$



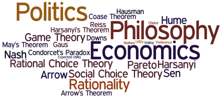
Judgements

- Expressions of judgement vs. expressions of preference
- Qualitative: Accept/Reject; Orderings; Grades
- Quantitative: Probabilities; Imprecise probabilities
- Causal models
- Do the experts provide their reasons/arguments/confidence?

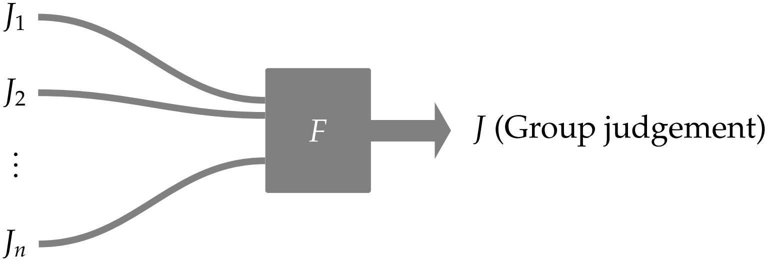
$\boxed{Pr(P) = p}$ $\boxed{Pr(P) = [l, h]}$



Aggregation method

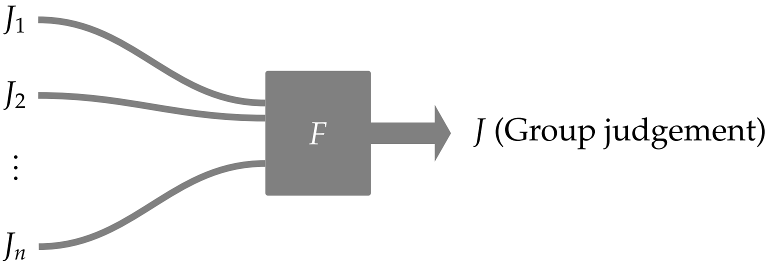


- Functions from *profiles* of judgements to judgements.
- Is the group judgement the same type as the individual judgements?
- Hides *disagreement* among the experts.

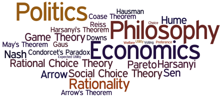


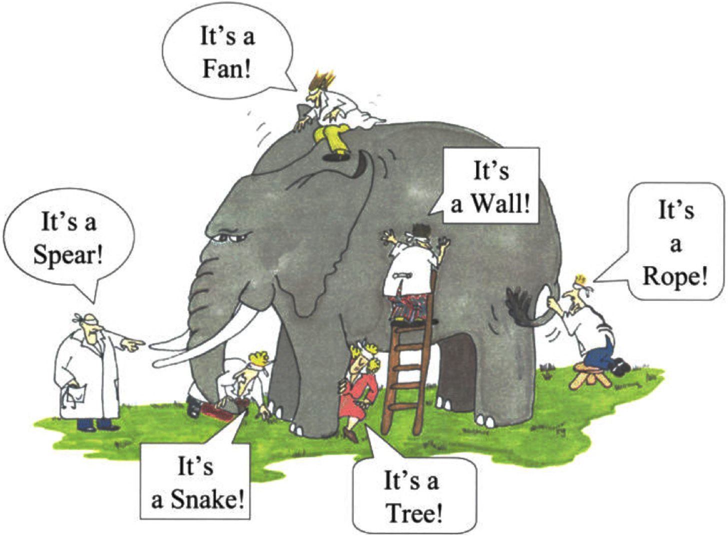
Aggregation method

- Epistemic considerations: How likely is it that the group judgement is *correct*?
- Procedural/fairness considerations: Does the group judgement *reflect* the individual judgements?

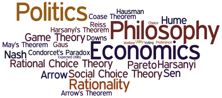


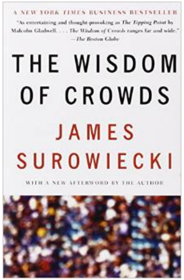
Wisdom of the Crowds

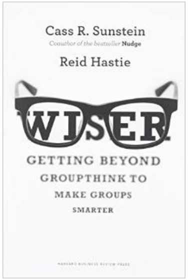


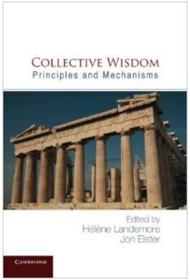


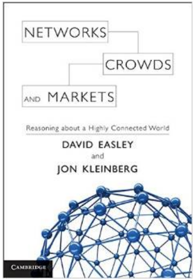
Collective Intelligence

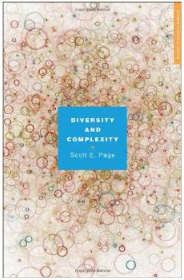












Collective wisdom

A. Lyon and EP. *The wisdom of crowds: Methods of human judgement aggregation*. Handbook of Human Computation, pp. 599 - 614, 2013.

C. Sunstein. *Deliberating groups versus prediction markets (or Hayek's challenge to Habermas)*. Episteme, 3:3, pgs. 192 - 213, 2006.

A. B. Kao and I. D. Couzin. *Decision accuracy in complex environments is often maximized by small group sizes*. Proceedings of the Royal Society: Biological Sciences, 281(1784), 2014.

- In many group decision making problems, one of the alternatives is the *correct* one. Which aggregation method is best for finding the "correct" alternative?

- Group decision problems often exhibit a *combinatorial structure*. For example, selecting a committee from a set of candidates or voting on a number of yes/no issues in a referendum. Furthermore, the propositions in the agenda may be *interconnected*.

S. Brams, D. M. Kilgour, and W. Zwicker. *The paradox of multiple elections*. Social Choice and Welfare, 15(2), pgs. 211 - 236, 1998.

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

Multiple Elections Paradox

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Outcome by majority vote

Proposition 1: N (7 - 6)

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Outcome by majority vote

Proposition 1: N (7 - 6)
Proposition 2: N (7 - 6)

Multiple Elections Paradox

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YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: N (7 - 6)
Proposition 2: N (7 - 6)
Proposition 3: N (7 - 6)

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	*YYN*	*YNY*	*YNN*	*NYY*	*NYN*	*NNY*	*NNN*
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: *N* (7 - 6)
Proposition 2: *N* (7 - 6)
Proposition 3: *N* (7 - 6)

But there is no support for NNN!

Complete Reversal

YYYN	*YYNY*	*YNYY*	*NYYY*	*NNNN*
2	2	2	2	3

Outcome by majority vote

Proposition 1: *Y* (6 - 5)
Proposition 2: *Y* (6 - 5)
Proposition 3: *Y* (6 - 5)
Proposition 4: *Y* (6 - 5)

YYYY wins proposition-wise voting, but the "opposite" outcome *NNN* has the *most* overall support!

S. Brams, M. Kilgour and W. Zwicker. *Voting on referenda: the separability problem and possible solutions*. Electoral Studies, 16(3), pp. 359 - 377, 1997.

D. Lacy and E. Niou. *A problem with referenda*. Journal of Theoretical Politics 12(1), pp. 5 - 31, 2000.

J. Lang and L. Xia. *Sequential composition of voting rules in multi-issue domains*. Mathematical Social Sciences 57(3), pp. 304 - 324, 2009.

L. Xia, V. Conitzer and J. Lang. *Strategic Sequential Voting in Multi-Issue Domains and Multiple-Election Paradoxes*. In Proceedings of the Twelfth ACM Conference on Electronic Commerce (EC-11), pp. 179-188, 2010.

A decision has to be made about whether or not to build a new swimming pool (S or $\overline{S}$) and a new tennis court (T or $\overline{T}$). Consider 5 voters with rankings over $\{S\,T, \overline{S}\,T, S\,\overline{T}, \overline{S}\,\overline{T}\}$:

rank	2 voters	2 voters	1 voter
1	$S\,\overline{T}$	$\overline{S}\,T$	$S\,T$
2	$\overline{S}\,T$	$S\,\overline{T}$	$S\,\overline{T}$
3	$\overline{S}\,\overline{T}$	$\overline{S}\,\overline{T}$	$\overline{S}\,T$
4	$S\,T$	$S\,T$	$\overline{S}\,\overline{T}$

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3	$\overline{S}\,\overline{T}$	$\overline{S}\,\overline{T}$	$\overline{S}\,T$
4	$S\,T$	$S\,T$	$\overline{S}\,\overline{T}$

The preferences of voters 1-4 are not *separable*. So, they will have a hard time voting on S vs. $\overline{S}$ and T vs. $\overline{T}$.

A decision has to be made about whether or not to build a new swimming pool (S or $\overline{S}$) and a new tennis court (T or $\overline{T}$). Consider 5 voters with rankings over $\{S\,T, \overline{S}\,T, S\,\overline{T}, \overline{S}\,\overline{T}\}$:

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3	$\overline{S}\,\overline{T}$	$\overline{S}\,\overline{T}$	$\overline{S}\,T$
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Assume that the voters are *optimistic*: They vote for the options that are top on their list.

A decision has to be made about whether or not to build a new swimming pool (S or $\overline{S}$) and a new tennis court (T or $\overline{T}$). Consider 5 voters with rankings over $\{S\,T, \overline{S}\,T, S\,\overline{T}, \overline{S}\,\overline{T}\}$:

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3	$\overline{S}\,\overline{T}$	$\overline{S}\,\overline{T}$	$\overline{S}\,T$
4	$S\,T$	$S\,T$	$\overline{S}\,\overline{T}$

When voting on the individual issues, S wins (3-2) and T wins (3-2), but the outcome $S\,T$ is a *Condorcet loser*.

“Is a conflict between the proposition and combination winners necessarily bad?

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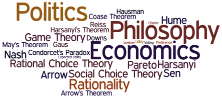
“Is a conflict between the proposition and combination winners necessarily bad? ... The paradox does not just highlight problems of aggregation and packaging, however, but strikes at the core of social choice—both what it means and how to uncover it. In our view, the paradox shows there may be a clash between two different meanings of social choice, leaving unsettled the best way to uncover what this elusive quantity is.” (pg. 234).

S. Brams, D. M. Kilgour, and W. Zwicker. *The paradox of multiple elections*. Social Choice and Welfare, 15(2), pgs. 211 - 236, 1998.

The Doctrinal Paradox/Discursive Dilemma



Kornhauser and Sager. *Unpacking the court*. Yale Law Journal, 1986.

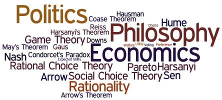
P. Mongin. *The doctrinal paradox, the discursive dilemma, and logical aggregation theory*. Theory and Decision, 73(3), pp 315 - 355, 2012.

C. List and P. Pettit. *Aggregating sets of judgments: An impossibility result*. Economics and Philosophy 18, pp. 89 - 110, 2002.

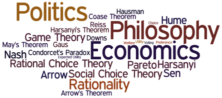
Suppose that three experts *independently* form opinions about three propositions. For instance,

1. c: “Carbon dioxide emissions are above the threshold x.”
2. $c \rightarrow g$: “If carbon dioxide emissions are above the threshold x, then there will be global warming.”
3. g: “There will be global warming.”

	c	$c \to g$	g
Expert 1			
Expert 2			
Expert 3			



	c	$c \to g$	g
Expert 1	True	True	
Expert 2			
Expert 3			



	c	$c \to g$	g
Expert 1	True	True	True
Expert 2			
Expert 3			

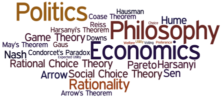
	c	$c \to g$	g
Expert 1	True	True	True
Expert 2	True		False
Expert 3			

	c	$c \to g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3			

	c	$c \rightarrow g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False

	c	$c \to g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group			

	c	$c \to g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True		



	c	$c \to g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True	True	

	c	$c \to g$	g
Expert 1	True	True	True
Expert 2	True	False	False
Expert 3	False	True	False
Group	True	True	False

	c	$c \to g$	g
Expert 1	True	True	True
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Group	True	True	False

Should we hire (h) candidate C?
Is C good at research (r)? Is C good at teaching (t)?

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Is C good at research (r)? Is C good at teaching (t)?

	r	t		h
Voter 1				
Voter 2				
Voter 3				
Group				

Should we hire (h) candidate C?
Is C good at research (r)? Is C good at teaching (t)?
We should hire (h) if and only if $r \wedge t$.

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1			Yes	
Voter 2			Yes	
Voter 3			Yes	
Group			Yes	

Should we hire (h) candidate C?
Is C good at research (r)? Is C good at teaching (t)?
We should hire (h) if and only if $r \wedge t$.

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes	Yes	Yes
Voter 2	Yes	No	Yes	No
Voter 3	No	Yes	Yes	No
Group			Yes	

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Is C good at research (r)? Is C good at teaching (t)?
We should hire (h) if and only if $r \wedge t$.

	r	t	$(r \wedge t) \leftrightarrow h$	h
Voter 1	Yes	Yes	Yes	Yes
Voter 2	Yes	No	Yes	No
Voter 3	No	Yes	Yes	No
Group	Yes	Yes	Yes	No

Suppose that there are five experts $\{1, 2, 3, 4, 5\}$ that are asked about five atomic sentences $\{p_1, p_2, p_3, p_4, p_5\}$ and the disjunction $p_1 \vee p_2 \vee p_3 \vee p_4 \vee p_5$.

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Suppose that each expert i, believes p_i, disbelieves each of the other atomic propositions and believes the disjunction.

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Suppose that each expert i, believes p_i, disbelieves each of the other atomic propositions and believes the disjunction.

There is unanimous support for the disjunction $p_1 \vee p_2 \vee p_3 \vee p_4 \vee p_5$, but 0.8 support *against* each disjunct (i.e., for the negation of each disjunct).

F. Cariani. *Local Supermajorities*. Erkenntnis, 81(2), pp. 391 - 406, 2016.

Philosophy department: Should we hire a logician, epistemologist or a metaphysician?

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	Epistemologist?	Logician?	Metaphysician?
1	Yes	Yes	No
2	No	Yes	Yes
3	Yes	No	Yes
4	Yes	No	No
5	No	Yes	Yes
Majority	Yes	Yes	Yes

Philosophy department: Should we hire a logician, epistemologist or a metaphysician?

	Epistemologist?	Logician?	Metaphysician?
1	Yes	Yes	No
2	No	Yes	Yes
3	Yes	No	Yes
4	Yes	No	No
5	No	Yes	Yes
Majority	Yes	Yes	Yes

University: You can't hire three people.

U. Endriss. *Judgment Aggregation with Rationality and Feasibility Constraints*. In Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS-2018).

Philosophy department: Should we hire a logician, epistemologist or a metaphysician? (**Rationality constraint**: $e \vee l \vee m$)

	Epistemologist?	Logician?	Metaphysician?
1	Yes	Yes	No
2	No	Yes	Yes
3	Yes	No	Yes
4	Yes	No	No
5	No	Yes	Yes
Majority	Yes	Yes	Yes

University: You can't hire three people. (**Feasibility constraint**: $\neg(e \wedge l \wedge m)$)

U. Endriss. *Judgment Aggregation with Rationality and Feasibility Constraints*. In Proceedings of the 17th International Conference on Autonomous Agents and Multiagent Systems (AAMAS-2018).

Electing Diverse Committees

Choose a committee that consists of members from different parts of the university *and* is diverse.

Social Sciences	Natural Sciences	Humanities
Ann	Carol	Ellen
Bob	David	Fred

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Choose a committee that consists of members from different parts of the university *and* is diverse.

Social Sciences	Natural Sciences	Humanities
Ann	Carol	Ellen
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Voter 1	Voter 2	Voter 3
Ann, David, Fred	Bob, Carol, Fred	Bob, David, Ellen

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Choose a committee that consists of members from different parts of the university *and* is diverse.

Social Sciences	Natural Sciences	Humanities
Ann	Carol	Ellen
Bob	David	Fred

Voter 1	Voter 2	Voter 3
Ann, David, Fred	Bob, Carol, Fred	Bob, David, Ellen

Winners: Bob, David, Fred

Electing Diverse Committees

T. Ratliff. *Selecting committees*. Public Choice, 126, pp. 242 - 255, 2006.

T. Ratliff. *Some startling inconsistencies when electing committees*. Social Choice and Welfare, 21(3), pp. 433- 454, 2003.

T. Ratliff and D. Saari. *Complexities of electing diverse committees*. Social Choice and Welfare, 43(1), pp. 55 - 71, 2014.

Taking stock

- Aggregating judgements: single event, multiple issues, logically connected issues, probabilistic opinions, imprecise probabilities, causal models, ...

- May's Theorem: axiomatic characterization of majority rule

- Condorcet Jury Theorem: epistemic analysis of majority rule

- Aggregation paradoxes: multiple election paradox, doctrinal paradox, discursive dilemma, the problem with conjunction, the corroboration paradox

Judgement Aggregation

U. Endriss. *Judgment Aggregation*. In F. Brandt, V. Conitzer, U. Endriss, J. Lang, and A. D. Procaccia, editors, Handbook of Computational Social Choice, Cambridge University Press, 2016.

C. List. *The theory of judgment aggregation: An introductory review*. Synthese 187(1): 179-207, 2012.

D. Grossi and G. Pigozzi. *Judgement Aggregation: A Primer*. Morgan & Claypool Publishers, 2014.

Vote by Grading

Approval Voting: Each voter selects a subset of candidates. The candidate with the most "approvals" wins the election.

S. Brams and P. Fishburn. *Approval Voting*. Birkhauser, 1983.

J.-F. Laslier and M. R. Sanver (eds.). *Handbook of Approval Voting*. Studies in Social Choice and Welfare, 2010.

Under Approval Voting (AV), voters are asked to select the candidates that the voter *approves*.

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Approving of a candidate is not (necessarily) the same as simply ranking the candidate first.

Why Approval Voting?

www.electology.org/approval-voting

S. Brams and P. Fishburn. *Going from Theory to Practice: The Mixed Success of Approval Voting*. Handbook of Approval Voting, pgs. 19-37, 2010.

Approval Voting is more flexible

# voters	2	2	1
	A	B	C
	D	D	A
	B	A	B
	C	C	D

The Condorcet winner is A.

Approval Voting is more flexible

There is no fixed rule that always elects a unique Condorcet winner.

# voters	2	2	1
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	D	D	A
	B	A	B
	C	C	D

The Condorcet winner is A.
Vote-for-1 elects $\{A, B\}$

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# voters	2	2	1
	A	B	C
	D	D	A
	B	A	B
	C	C	D

The Condorcet winner is A.
Vote-for-1 elects $\{A, B\}$, vote-for-2 elects $\{D\}$

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There is no fixed rule that always elects a unique Condorcet winner.

# voters	2	2	1
	A	B	C
	D	D	A
	B	A	B
	C	C	D

The Condorcet winner is A.
Vote-for-1 elects $\{A, B\}$, vote-for-2 elects $\{D\}$, vote-for-3 elects $\{A, B\}$.

Approval Voting is more flexible

AV may elect the Condorcet winner

# voters	2	2	1
	A	B	C
	D	D	A
	B	A	B
	C	C	D

The Condorcet winner is A.
$(\{A\}, \{B\}, \{C, A\})$ elects A under AV.

Possible Failure of Unanimity

# voters	1	1	1
	A	C	D
	B	A	A
	C	B	B
	D	D	C

Possible Failure of Unanimity

# voters	1	1	1
	A	C	D
	B	A	A
	C	B	B
	D	D	C

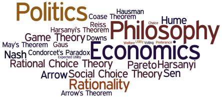
Approval Winners: A, B

Indeterminate or Responsive?

# voters	6	5	4
	A	B	C
	C	C	B
	B	A	A

Plurality winner: *A*, Borda and Condorcet winner: *C*.

Indeterminate or Responsive?



# voters	6	5	4
	A	B	C
	C	C	B
	B	A	A

Plurality winner: A, Borda and Condorcet winner: C.
Any combination of A, B and C can be an AV winner (or AV winners).

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Ask the voters to provide both a linear ranking of the candidates and the candidates that they approve.

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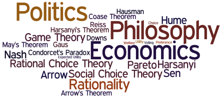
Make the ballots more expressive: Dis&Approval voting, RangeVoting, Majority Judgement

Grading



In many group decision situations, people use measures or grades from a **common language of evaluation** to evaluate candidates or alternatives:

- in figure skating, diving and gymnastics competitions;
- in piano, flute and orchestra competitions;
- in classifying wines at wine competitions;
- in ranking university students;
- in classifying hotels and restaurants, e.g., the Michelin $*$

Voting by Grading: Questions

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- How should we *aggregate* the grades? (e.g., Average or Median)
- Should there be a “no opinion” option?

Voting by Grading: Examples

Approval Voting: voters can assign a single grade "approve" to the candidates

Dis&Approval Voting: voters can approve or disapprove of the candidates

Majority Judgement, Score Voting: voters can assign any grade from a fixed set of grades to the candidates

Strong Paradox of Grading Systems

Grades: $\{0, 1, 2, 3\}$

Candidates: $\{A, B, C\}$

3 Voters

# voters	1	1	1	Avg
A	3	2	0	
B	0	3	1	
C	0	3	1	

Grades: $\{0, 1, 2, 3\}$

Candidates: $\{A, B, C\}$

3 Voters

# voters	1	1	1	Avg
A	3	2	0	5/3
B	0	3	1	4/3
C	0	3	1	4/3

Average Grade Winner: A

Grades: $\{0, 1, 2, 3\}$

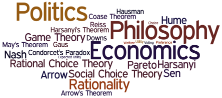
Candidates: $\{A, B, C\}$

3 Voters

# voters	1	1	1	Avg
A	3	2	0	
B	0	3	1	
C	0	3	1	

Average Grade Winner: A

$$B \succ A$$



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3 Voters

# voters	1	1	1	Avg
A	3	2	0	
B	0	3	1	
C	0	3	1	

Average Grade Winner: A

$$C \sim B \succ A$$

Grades: $\{0, 1, 2, 3\}$

Candidates: $\{A, B, C\}$

3 Voters

# voters	1	1	1	Avg
A	3	2	0	
B	0	3	1	
C	0	3	1	

Average Grade Winner: A

$$C \sim B \succ A$$

Grades: $\{0, 1, 2, 3\}$

Candidates: $\{A, B, C\}$

3 Voters

# voters	1	1	1	Avg
A	3	2	0	
B	0	3	1	
C	0	3	1	

Average Grade Winner: A

Superior Grade Winners: C, B

Grades: $\{0, 1, 2, 3, 4, 5\}$

Candidates: $\{A, B, C\}$

5 Voters

# voters	1	4	Avg
A	5	0	5/5
B	0	1	4/5
C	0	1	4/5

Average Grade Winner: A

Superior Grade Winner: B, C

To conclude, we have identified a paradox of grading systems, which is not just a mirror of the well-known differences that crop up in aggregating votes under ranking systems. Unlike these systems, for which there is no accepted way of reconciling which candidate to choose when, for example, the Hare, Borda and Condorcet winners differ, AV provides a solution when the AG and SG winners differ.

Theorem (Brams and Potthoff). When there are two grades, the AG and SG winners are identical.