PHIL309P Philosophy, Politics and Economics

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Politics coase theorem Harsanyis Theorem Gaus Nash Condorcet's Paradox Rational Choice Theory Pareto Harsanyi Arrow Social Choice Theory Sen Rational Choice Theory Sen Rational Choice Theory Martin Harsanyi Arrow Social Choice Theory Sen Rational Choice Theory Sen Rational Choice Theory Martin Harsanyi Arrow Social Choice Theory Sen

The Social Choice Model



- *N* is a finite set of voters (assume that $N = \{1, 2, 3, ..., n\}$)
- *X* is a (typically finite) set of alternatives, or candidates
- A relation on X is a linear order if it is transitive, irreflexive, and complete (hence, acyclic)
- ► *L*(*X*) is the set of all linear orders over the set *X*
- O(X) is the set of all reflexive and transitive relations over the set X

Notation



- ► A profile for the set of voters N is a sequence of (linear) orders over X, denoted **R** = (R₁,..., R_n).
- $L(X)^n$ is the set of all **profiles** for *n* voters (similarly for $O(X)^n$)

► For a profile $\mathbf{R} = (R_1, ..., R_n) \in O(X)^n$, let $\mathbf{N}_{\mathbf{R}}(A \ P \ B) = \{i \mid A \ P_i \ B\}$ be the set of voters that rank *A* above *B* (similarly for $\mathbf{N}_{\mathbf{R}}(A \ I \ B)$ and $\mathbf{N}_{\mathbf{R}}(B \ P \ A)$)

Preference Aggregation Methods



Social Welfare Function: $F : \mathcal{D} \to L(X)$, where $\mathcal{D} \subseteq L(X)^n$

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Comments

- *D* is the *domain* of the function: it is the set of all possible profiles
- Aggregation methods are *decisive*: every profile **R** in the domain is associated with exactly one ordering over the candidates
- The range of the function is *L*(*X*): the social ordering is assumed to be a linear order
- Tie-breaking rules are built into the definition of a preference aggregation function

Preference Aggregation Methods



Social Welfare Function: $F : \mathcal{D} \to L(X)$, where $\mathcal{D} \subseteq L(X)^n$

Variants

- Social Choice Function: $F : \mathcal{D} \to \wp(X) \emptyset$, where $\mathcal{D} \subseteq L(X)^n$ and $\wp(X)$ is the set of all subsets of *X*.
- Allow Ties: $F : \mathcal{D} \to O(X)$ where O(X) is the set of orderings (reflexive and transitive) over *X*
- Allow Indifference and Ties: $F : \mathcal{D} \to O(X)$ where O(X) is the set of orderings (reflexive and transitive) over X and $\mathcal{D} \subseteq O(X)^n$

Examples



$Maj(\mathbf{R}) = >_M$ where $A >_M B$ iff $|\mathbf{N}_{\mathbf{R}}(A P B)| > |\mathbf{N}_{\mathbf{R}}(B P A)|$ (the problem is that $>_M$ may not be transitive (or complete))

Examples



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 $Borda(\mathbf{R}) = \geq_{BC}$ where $A \geq_{BC} B$ iff the Borda score of A is greater than the Borda score for B.

(the problem is that \geq_{BC} may not be a linear order)

Axiomatic characterizations



- 1. Fix a set \mathfrak{F} of possible aggregation methods.
- 2. Identify a set of properties that discriminate between the different methods in \mathfrak{F} .
- 3. Characterize the subset of \mathfrak{F} consisting of the methods that satisfy the principles identified in the second step.

Competing desiderata



- 1. The voters' inputs (rankings, judgements) should *completely determine* the group decision.
- 2. The group decision should depend *in the right way* on the voters' inputs.
- 3. The voters' inputs are not constrained in any way (unless there is good reason to think otherwise).

Characterizing Majority Rule



May's Theorem (1952) A social decision method *F* satisfies unanimity, neutrality, anonymity and positive responsiveness iff *F* is majority rule.

Can May's Theorem be generalized to more than 2 candidates?

Can May's Theorem be generalized to more than 2 candidates? No!

Spoiler Candidates: Plurality Rule



# voters	49	48	3	
	А	В	С	
	В	А	В	
	С	С	А	



Spoiler Candidates: Plurality Rule





IIA



Independence of Irrelevant Alternatives: If the voters in two different electorates rank *A* and *B* in exactly the same way, then *A* and *B* should be ranked the same way in both elections.



# voters	3	2	2
3	Α	В	С
2	В	С	Α
1	С	Α	В
0	Х	Х	Х



# voters	3	2	2
3	Α	В	С
2	В	С	Α
1	С	Α	В
0	Х	Х	Х

 $A (15) >_{BC} B (14) >_{BC} C (13) >_{BC} X (0)$



# voters	3	2	2	# voters	3	2	2
3	А	В	С	3	Α	В	С
2	В	С	Α	2	В	С	X
1	С	Α	В	1	С	Х	Α
0	Х	Х	Х	0	Х	А	В

 $A (15) >_{BC} B (14) >_{BC} C (13) >_{BC} X (0)$





Arrow's Theorem



Let *X* be a finite set with *at least three elements* and *N* a finite set of *n* voters.

Social Welfare Function: $F : \mathcal{D} \to O(X)$ where $\mathcal{D} \subseteq O(X)^n$

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Reminders:

- O(X) is the set of transitive and complete relations on X
- ► For $R \in O(X)$, let P_R denote the strict subrelation and I_R the indifference subrelation:
 - $A P_R B$ iff A R B and not B R A
 - $A I_R B$ iff A R B and B R A

Unanimity



 $F:\mathcal{D}\to O(X)$

If each agent ranks *A* above *B*, then so does the social ranking.

Unanimity



 $F:\mathcal{D}\to O(X)$

If each agent ranks *A* above *B*, then so does the social ranking.

For all profiles $\mathbf{R} = (R_1, \ldots, R_n) \in \mathcal{D}$:

If for each $i \in N$, $A P_i B$ then $A P_{F(\mathbf{R})} B$

Universal Domain



 $F:\mathcal{D}\to O(X)$

Voter's are free to choose any ranking, and the voters' choices are independent.

Universal Domain



 $F:\mathcal{D}\to O(X)$

Voter's are free to choose any ranking, and the voters' choices are independent.

The domain of *F* is the set of *all* profiles, i.e., $\mathcal{D} = O(X)^n$.

Independence of Irrelevant Alternatives



 $F: \mathcal{D} \to O(X)$

The social ranking (higher, lower, or indifferent) of two alternatives *A* and *B* depends only the relative rankings of *A* and *B* for each voter.

Independence of Irrelevant Alternatives



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The social ranking (higher, lower, or indifferent) of two alternatives *A* and *B* depends only the relative rankings of *A* and *B* for each voter.

For all profiles **R** =
$$(R_1, ..., R_n)$$
 and **R**' = $(R'_1, ..., R'_n)$:

If $R_{i\{A,B\}} = R'_{i\{A,B\}}$ for all $i \in N$, then $F(\mathbf{R})_{\{A,B\}} = F(\mathbf{R}')_{\{A,B\}}$. where $R_{\{X,Y\}} = R \cap \{X,Y\} \times \{X,Y\}$

Voter 1	Voter 2	Group
A B C	CBA	BAC
A C B	BCA	C B A
BAC	C A B	A B C
BCA	A C B	C A B
СВА	A B C	BCA
C A B	BAC	A B C

Voter 1	Voter 2	Group
A B	BA	BA
A B	B A	BA
B A	A B	A B
B A	A B	A B
B A	A B	B A
A B	ΒA	A B

Voter 1	Voter 2	Group
A B	BA	BA
A B	B A	BA
BA	A B	A B
B A	A B	A B
B A	A B	A B
A B	BA	BA








Voter 1	Voter 2	Group
A B C	CBA	BAC
A C B	BCA	A B C
BAC	C A B	A B C
BCA	A C B	BAC
CBA	A B C	A B C
C A B	BAC	BAC

Dictatorship



 $F:\mathcal{D}\to O(X)$

A voter $d \in N$ is a **dictator** if society strictly prefers A over B whenever d strictly prefers A over B.

Dictatorship



 $F:\mathcal{D}\to O(X)$

A voter $d \in N$ is a **dictator** if society strictly prefers *A* over *B* whenever *d* strictly prefers *A* over *B*.

There is a $d \in N$ such that for each profile $\mathbf{R} = (R_1, \dots, R_d, \dots, R_n)$, if $A P_d B$, then $A P_{F(\mathbf{R})} B$

M. Morreau. Arrow's Theorem. Stanford Encyclopedia of Philosophy, 2014.



Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.



D. Campbell and J. Kelly. *Impossibility Theorems in the Arrovian Framework*. Handbook of Social Choice and Welfare Volume 1, pgs. 35 - 94, 2002.

W. Gaertner. A Primer in Social Choice Theory. Oxford University Press, 2006.

J. Geanakoplos. *Three Brief Proofs of Arrow's Impossibility Theorem*. Economic Theory, **26**, 2005.

P. Suppes. *The pre-history of Kenneth Arrow's social choice and individual values*. Social Choice and Welfare, 25, pgs. 319 - 326, 2005.



Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.

1	2	Society
A	С	
В	В	
С	A	

1	2	Society
A	С	В
В	В	A C
С	A	

1	2	Society	_	1	2	Society
A	С	В		A	С	
В	В	A C		С	В	
С	A			В	Α	



IIA





IIA





IIA

1	2	Society	1	2	Society
A	С	В	A	С	$A \sim C$
В	В	A C	С	В	B > A
С	A		В	A	B > C

Transitivity

1	2	Society	1	2	Society
A	С	В	A	С	В
В	В	A C	С	В	A C
С	A		В	A	

1	2	Society	1	2	Society
A	С	В	A	С	В
В	В	A C	С	В	A C
С	A		В	Α	

Pareto!

1	2	Society	
A	С		
В	В		
С	A		

1	2	Society	
Α	С	A C	
В	В	В	
С	A		

1	2	Society	1	2	Society
A	С	A C	A	В	
В	В	В	В	С	
С	Α		С	A	



Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.



 $F:\mathcal{D}\to O(X)$

Dictatorial: there is a $d \in N$ such that for all $A, B \in X$ and all profiles **R**: if $A P_d B$, then $A P_{F(\mathbf{R})} B$

Inversely Dictatorial: there is a $d \in N$ such that for all $A, B \in X$ and all profiles **R**: if $A P_d B$, then $B P_{F(\mathbf{R})} A$



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Null: For all $A, B \in X$ and for all $\mathbf{R} \in \mathcal{D}$: $A I_{F(\mathbf{R})} B$



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Null: For all $A, B \in X$ and for all $\mathbf{R} \in \mathcal{D}$: $A I_{F(\mathbf{R})} B$

Non-Imposition: For all $A, B \in X$, there is a $\mathbf{R} \in \mathcal{D}$ such that $A F(\mathbf{R}) B$



Theorem (Wilson) Suppose that *N* is a finite set. If a social welfare function satisfies universal domain, independence of irrelevant alternatives and non-imposition, then it is either null, dictatorial or inversely dictatorial.

R. Wilson. *Social Choice Theory without the Pareto principle*. Journal of Economic Theory, 5, pgs. 478 - 486, 1972.

Y. Murakami. Logic and Social Choice. Routledge, 1968.

S. Cato. *Social choice without the Pareto principle: A comprehensive analysis*. Social Choice and Welfare, 39, pgs. 869 - 889, 2012.



Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.

Social Choice Functions



 $F: \mathcal{D} \to \wp(X) - \emptyset$

Resolute: For all profiles $\mathbf{R} \in \mathcal{D}$, $|F(\mathbf{R})| = 1$

Non-Imposed: For all candidates $A \in X$, there is a $\mathbf{R} \in \mathcal{D}$ such that $F(\mathbf{R}) = \{A\}$.

Monotonicity: For all profiles **R** and **R'**, if $A \in F(\mathbf{R})$ and for all $i \in N$, $\mathbf{N}_{\mathbf{R}}(A \ P_i \ B) \subseteq \mathbf{N}_{\mathbf{R}'}(A \ P'_i \ B)$ for all $B \in X - \{A\}$, then $A \in F(\mathbf{R'})$.

Dictator: A voter *d* is a dictator if for all $\mathbf{R} \in \mathcal{D}$, $F(\mathbf{R}) = \{A\}$, where *A* is *d*'s top choice.

Social Choice Functions



Muller-Satterthwaite Theorem. Suppose that there are more than three alternatives and finitely many voters. Every resolute social choice function $F : L(X)^n \to X$ that is monotonic and non-imposed is a dictatorship.

E. Muller and M.A. Satterthwaite. *The Equivalence of Strong Positive Association and Strategy-Proofness.* Journal of Economic Theory, 14(2), pgs. 412 - 418, 1977.



Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.

- Infinitely many voters.
- Domain restrictions.
- Richer ballots.

Universal Domain



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Universal Domain



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Epistemic Rationale: "If we do not wish to require any prior knowledge of the tastes of individuals before specifying our social welfare function, that function will have to be defined for every logically possible set of individual orderings." (Arrow, 1963, pg. 24)

Domain Restrictions



- Single-Peaked preferences
- Sen's Value Restriction
- Assumptions about the distribution of preferences

W. Gaertner. Domain Conditions in Social Choice Theory. Cambridge University Press, 2001.



1	1	1
A	В	С
В	С	A
С	A	В






























































D. Black. *On the rationale of group decision-making*. Journal of Political Economy, 56:1, pgs. 23 - 34, 1948.



Single-Peakedness: the preferences of group members are said to be single-peaked if the alternatives under consideration can be represented as points on a line and each of the utility functions representing preferences over these alternatives has a maximum at some point on the line and slopes away from this maximum on either side.



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Theorem. If there is an odd number of voters that display single-peaked preferences, then a Condorcet winner exists.



D. Miller. Deliberative Democracy and Social Choice. Political Studies, 40, pgs. 54 - 67, 1992.

C. List, R. Luskin, J. Fishkin and I. McLean. *Deliberation, Single-Peakedness, and the Possibility of Meaningful Democracy: Evidence from Deliberative Polls.* Journal of Politics, 75(1), pgs. 80 - 95, 2013.

Sen's Value Restriction



A. Sen. A Possibility Theorem on Majority Decisions. Econometrica 34, 1966, pgs. 491 - 499.

Sen's Theorem



Assume *n* voters (*n* is odd).

Sen's Theorem



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Triplewise value-restriction: For every triple of distinct candidates *A*, *B*, *C* there exists an $x_i \in \{A, B, C\}$ and $r \in \{1, 2, 3\}$ such that no voter ranks x_i has her *r*th preference among *A*, *B*, *C*.

Sen's Theorem



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Theorem (Sen, 1966). For every profile satisfying triplewise value-restriction, pairwise majority voting generates a transitive group preference ordering.

Restrict the distribution of preferences

M. Regenwetter, B. Grofman, A.A.J. Marley and I. Tsetlin. *Behavioral Social Choice*. Cambridge University Press, 2006.

Proceduralist Justifications



"identifies a set of ideals with which any collective decision-making procedure ought to comply. [A] process of collective decision making would be more or less justifiable depending on the extent to which it satisfies them...

Proceduralist Justifications



"identifies a set of ideals with which any collective decision-making procedure ought to comply. [A] process of collective decision making would be more or less justifiable depending on the extent to which it satisfies them...What justifies a [collective] decision-making procedure is strictly a necessary property of the procedure—one entailed by the definition of the procedure alone."

J. Coleman and J. Ferejohn. Democracy and social choice. Ethics, 97(1): 6-25, 1986..

Epistemic Justifications



Epistemic Justifications



"An epistemic interpretation of voting has three main elements: (1) an independent standard of correct decisions that is, an account of justice or of the common good that is independent of current consensus and the outcome of votes; (2) a cognitive account of voting that is, the view that voting expresses beliefs about what the correct policies are according to the independent standard, not personal preferences for policies; and (3) an account of decision making as a process of the adjustment of beliefs. adjustments that are undertaken in part in light of the evidence about the correct answer that is provided by the beliefs of others. (p. 34) "

J. Cohen. An epistemic conception of democracy. Ethics, 97(1): 26-38, 1986.

"Condorcet begins with the premise that the object of government is to make decisions that are in the best interest of society. This leads naturally to the question: what voting rules are most likely to yield good outcomes?.... "Condorcet begins with the premise that the object of government is to make decisions that are in the best interest of society. This leads naturally to the question: what voting rules are most likely to yield good outcomes?....

Why should we buy the idea, though, that there really is such a thing as an objectively "best" choice? Aren't values relative, and isn't the point of voting to strike a balance between conflicting opinions, not to determine a correct one?"

H. P. Young. *Optimal Voting Rules*. The Journal of Economic Perspectives, 9:1, pgs. 51 - 64, 1995.



 Group decision problems often exhibit a *combinatorial structure*. For example, selecting a committee from a set of candidates or voting on a number of yes/no issues in a referendum.



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- The different issues under consideration may be *interconnected*.



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Topics



- Voting in Combinatorial Domains: Anscombe's Paradox, Multiple Elections Paradox
- Epistemic Voting: The Condorcet Jury Theorem
- Judgement Aggregation