## PHIL309P

# Philosophy, Politics and Economics 

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Nash Rational Choice Theory Pareto Harsanyi
ArrowSocial Choice TheorySen
Rationality
Arrow's Theorem

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## Collective decision making







## Voting Situations

 Neshemenerem Economics Nashlouna chice Theory| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| $\boldsymbol{q}_{\text {worst }}$ | D | D | A | A |

- 21 voters and 4 candidates: Ann (A), Bob (B), Charles (C) and Dora (D)


## Voting Situations

 wavs neme thern Economics Nastional Choice Theory ParetoHarsany| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | D | B | C | D |
|  | D | A | A |  |

- 21 voters and 4 candidates: Ann (A), Bob (B), Charles (C) and Dora (D)
- Each voter ranks the candidates from best (at the top of the list) to worst (at the bottom of the list) resulting in the 4 voting blocks given in the above table


## Voting Situations

 Nash conarcets Rational Choice Theory ParetoHarsanyi

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | D | B | C | D |
|  |  | A | A |  |

Who should win the election?

## Which candidate should be chosen?

 Nash Consorcet's Paradot ECO OPM Rational Choice Theory ArrowSocial Choice
Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | D | D | A | A |

## Which candidate should be chosen?

Politicsass numm tume

 ArrowSocial Choice
Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| $\boldsymbol{q}_{\text {worst }}$ | D | D | A | A |

- Candidate $A$ : More people (8) rank $A$ first than any other candidate


## Which candidate should be chosen?

 nsan shime theoryems ArrowSocial Choice
Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\overbrace{\text { worst }}$ | B | C | D | B |
|  | C | B | C | D |
|  |  | A | A |  |

- Candidate $A$ : More people rank $A$ first than any other candidate
- Candidate $A$ should not win: more than half rank $A$ last


## Which candidate should be chosen?

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| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
|  | D | D | A | A |

- Candidate $A$ : More people rank $A$ first than any other candidate
- Candidate $D$ should not win


## Which candidate should be chosen?

 nef shame theornems ArrowSocial Choice
Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
|  | D | D | A | A |

- Candidate $A$ : More people rank $A$ first than any other candidate
- Candidate $D$ should not win: everyone ranks $B$ higher than $D$


## Which candidate should be chosen?

 Nash Consorcets Paradox LCL
Rational Choice Theory ParetoHarsanyi Arrow Sociationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
| D | A | A |  |  |

- Which of $B$ or $C$ should win?


## Which candidate should be chosen?



Marquis de Condorcet (1743-1794)


Jean-Charles de Borda (1733-1799)

## Which candidate should be chosen?

 waveneme weormeconomics Arrowsocial Cholice

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
| D | A | A |  |  |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)


## Which candidate should be chosen?

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| $\uparrow_{\text {worst }}$ | C | B | C | D |
|  | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)


## Which candidate should be chosen?

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| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
|  | C | B | C | D |
| worst | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)


## Which candidate should be chosen?

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| best | A | A | B | C |
|  | B | C | D | B |
|  | C | B | C | D |
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- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)


## Which candidate should be chosen?



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| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
| D | D | A | A |  |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)


## Which candidate should be chosen?

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| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best <br> $\uparrow$ | A | A | B | C |
| worst | D | C | D | B |
|  | C | B | C | D |
|  |  | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- B gets 13 (vs. A)


## Which candidate should be chosen?

 Nash Condorcets Paradox Rational Choice Theory ParetoHarsany

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
|  | B | C | D | B |
|  | C | B | C | D |
| worst | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- B gets 13 (vs. A) + 10 (vs. C)


## Which candidate should be chosen?

 waven same therome Nash Condorcets Paradox Rational Choice Theory ParetoHarsany| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best <br> $\uparrow$ | A | A | B | C |
|  | B | C | D | B |
|  | C | B | C | D |
| worst | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- B gets 13 (vs. $A)+10($ vs. $C)+21$ (vs. $D)=44$ points


## Which candidate should be chosen?


 Nash Consorcet's Paradox ECO
Rational Choice Theory ParetoHarsanyi ArrowSocial Choice
Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\overbrace{\text { worst }}$ | B | C | D | B |
|  | C | B | C | D |
|  | D | A | A |  |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- $C$ get $13($ vs. $A)+11($ vs. $B)+14($ vs. $D)=38$ points


## Which candidate should be chosen?

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| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best <br> $\uparrow$ | A | A | B | C |
|  | B | C | D | B |
| worst | C | B | C | D |
|  | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- C get 13 (vs. $A)+11($ vs. $B)+14($ vs. $D)=38$ points


## Which candidate should be chosen?

 Nash Condorcet's Paradox
Rational Choice Theory Pareto Harsanyi

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
|  | C | B | C | D |
| worst | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- C get 13 (vs. $A)+11($ vs. $B)+14($ vs. $D)=38$ points


## Which candidate should be chosen?

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
|  | D | A | A |  |

- Candidate $A$ should not win: more than half rank $A$ last
- Candidate $D$ should not win: everyone ranks $B$ higher than $D$
- Candidate $C$ : $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ : Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)


## Which candidate should be chosen?

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | D | B | C | D |
|  |  | D | A | A |

Conclusion: there are many ways to answer the above question!
(C is the Condorcet winher)

- Candidate B: Taking into account the entire ordering, $B$ has the most "support" (B is the Borda winner)


## There are many different voting methods

Many different electoral methods: Plurality, Borda Count, Antiplurality/Veto, and k-approval; Plurality with Runoff; Single Transferable Vote (STV)/Hare; Approval Voting; Cup Rule/Voting Trees; Copeland; Banks; Slater Rule; Schwartz Rule; the Condorcet rule; Maximin/Simpson, Kemeny; Ranked Pairs/Tideman; Bucklin Method; Dodgson Method; Young's Method; Majority Judgment; Cumulative Voting; Range/Score Voting; ...

## Choosing how to choose

 Rational Choice Theory ParetoHarsany
ArrowSocial Choice TheorySen $\underset{\substack{\text { Rrows theorem }}}{\substack{\text { Rity } \\ \text { and }}}$

Pragmatic considerations: Is the procedure easy to use? Is it legal? The importance of ease of use should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.

## Choosing how to choose

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Information required from the voters: What type of information do the ballots convey? I.e., Choosing a single alternative, linearly rank all the candidates, report something about the "intensity" of preference.

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Information required from the voters: What type of information do the ballots convey? I.e., Choosing a single alternative, linearly rank all the candidates, report something about the "intensity" of preference.

Axiomatics: Characterize the different voting methods in terms of normative principles of group decision making.

## Voting Methods

Positional Scoring Rules: Given the rankings of the candidates provided by the voters, each candidate is assigned a score. The candidate(s) with the highest score is(are) declared the winner(s).

Examples: Borda, Plurality

Generalized Scoring Rules: Voters assign scores, or "grades", to the candidates. The candidate(s) with the "best" aggregate score is(are) declared the winner(s).

Examples: Approval Voting, Majority Judgement, Range Voting

## Voting Methods

Staged Procedures: The winner(s) is(are) determined in stages. At each stage, one or more candidates are eliminated. The candidate or candidates that are never eliminated are declared the winner(s).

Examples: Plurality with Runoff, Hare, Coombs

Condorcet Consistent Methods: Voting methods that guarantee that the Condorcet winner is elected.

Examples: Copeland, Dodgson, Young
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 Arrowsocial Choice

## Voting Methods Tutorial

# The Condorcet Paradox 

## Recall Condorcet's Idea

 Game Theory Downsmars Theorem Guss
Nash Consorests Paratox ECOMOMICS Nash condorcetsoice Theory ParetoHarsanyi

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | D | B | C | D |
|  | D | A | A |  |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections.


## Recall Condorcet's Idea


 Arrow Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
|  | D | A | A |  |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections.


## Recall Condorcet's Idea

 waven same therams Arrow Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
| w | D | A | A |  |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections.


## Recall Condorcet's Idea


 Arrow Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
|  | D | D | A | A |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections.


## Recall Condorcet's Idea

 nes same theorner Arrow Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| $\overbrace{\text { worst }}$ | D | B | C | D |
|  |  | D | A | A |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections. $B$ is ranked second


## Recall Condorcet's Idea

 nes same theorner Arrow Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
| w | A | A |  |  |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections. $B$ is ranked second


## Recall Condorcet's Idea

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Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst $^{\text {wors }}$ | D | B | C | D |
|  | D | A | A |  |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections. $B$ is ranked second, $D$ is ranked third, and $A$ is ranked last.

$$
C>_{M} B>_{M} D>_{M} A
$$

## The Majority Relation

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Rationality
arrows theocem
Suppose that $X$ and $Y$ are candidates and $P_{i}$ represents voter $i$ 's strict preference.
$\mathbf{N}(X P Y)=\left|\left\{i \mid X P_{i} Y\right\}\right|$
"the number of voters that rank $X$ strictly above $Y$ "

## The Majority Relation

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"the number of voters that rank $X$ strictly above $Y^{\prime \prime}$
$X>_{M} Y$ iff $\mathbf{N}(X P Y)>\mathbf{N}(Y P X)$
"a majority prefers candidate $X$ over candidate $Y$ "

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$X>_{M} Y$ iff $\mathbf{N}(X P Y)>\mathbf{N}(Y P X)$
"a majority prefers candidate $X$ over candidate $Y$ "
$X$ is a Condorcet winner if $X$ beats every other candidate in an head-to-head election: there is no candidate $Y$ such that $Y>_{M} X$

## The Majority Relation

Suppose that $X$ and $Y$ are candidates and $P_{i}$ represents voter $i$ 's strict preference.
$\mathbf{N}(X P Y)=\left|\left\{i \mid X P_{i} Y\right\}\right|$
"the number of voters that rank $X$ strictly above $Y$ "
$X>_{M} Y$ iff $\mathbf{N}(X P Y)>\mathbf{N}(Y P X)$
"a majority prefers candidate $X$ over candidate $Y$ "
$X$ is a Condorcet winner if $X$ beats every other candidate in an head-to-head election: there is no candidate $Y$ such that $Y>_{M} X$
$X$ is a Condorcet loser if $X$ loses to every other candidate in an head-to-head elections: there is no candidate $Y$ such that, $X>_{M} Y$

## The Problem





ArrowSocial Choice
Rationality
Arows theorem
Voter 1 Voter 2 Voter 3
A C B
B
A
C
$\begin{array}{lll}\text { C } & \text { B } & \text { A }\end{array}$

## The Problem

 mavs Theorem Geus Nash Condorcets ParadoxRational Choice Theory
Pareto Harsanyi Arrowsocialionality
Voter 1 Voter 2 Voter 3

| A | C | B |
| :---: | :---: | :---: |
| B | A | C |
| C | B | A |

- Does the group prefer $A$ over $B$ ?


## The Problem

 Mavs Theorem Theory Gery Downs Nash Condorret's Paradox ECO ROMOS ArrowSocial Choice TheorySenVoter 1 Voter 2 Voter 3

| A | C | B |
| :---: | :---: | :---: |
| B | A | C |
| C | B | A |

- Does the group prefer $A$ over $B$ ? Yes


## The Problem

| Voter 1 | Voter 2 | Voter 3 |
| :---: | :---: | :---: |
| A | C | B |
| B | A | C |
| C | B | A |

- Does the group prefer $A$ over $B$ ? Yes
- Does the group prefer $B$ over C? Yes


## The Problem

 Nash Condorcet's Paradox ECO OMOt Mars
Rational Choice Theory ArrowSocial Choice TheorySen $\underset{\substack{\text { Ratrows theosemality }}}{ }$
Voter 1 Voter 2 Voter 3


- Does the group prefer $A$ over $B$ ? Yes
- Does the group prefer $B$ over C? Yes
- Does the group prefer $A$ over $C$ ? No


## The Problem

 Nash Consorcets Paradox ECO
Rational Choice Theory ParetoHarsanyi ArrowSocial Choice TheorySen $\underset{\text { Rrows theorem }}{\text { Ration }}$
Voter 1 Voter 2 Voter 3

| A | C | B |
| :--- | :--- | :--- |
| B | A | C |
| C | B | A |

The majority relation $>_{M}$ is not transitive!
There is a Condorcet cycle: $A>_{M} B>_{M} C>_{M} A$

## How bad is this?

- Final decisions are extremely sensitive to institutional features such as who can set the agenda, arbitrary time limits place on deliberation, who is permitted to make motions, etc.


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- Final decisions are extremely sensitive to institutional features such as who can set the agenda, arbitrary time limits place on deliberation, who is permitted to make motions, etc.
- Is there empirical evidence that Condorcet cycles have shown up in real elections?
W. Riker. Liberalism against Populism. Waveland Press, 1982.
G. Mackie. Democracy Defended. Cambridge University Press, 2003.


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- Final decisions are extremely sensitive to institutional features such as who can set the agenda, arbitrary time limits place on deliberation, who is permitted to make motions, etc.
- Is there empirical evidence that Condorcet cycles have shown up in real elections?
W. Riker. Liberalism against Populism. Waveland Press, 1982.
G. Mackie. Democracy Defended. Cambridge University Press, 2003.
- How likely is a Condorcet cycle?

Should we select a Condorcet winner (when one exists)?

## Condorcet's Other Paradox

 Mars theorem Gews Nash Consorcet's Paradot ECO OPM Rational Choice TheoryArrowSocial Choice
Rationality

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |

## Condorcet's Other Paradox

等| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | A | A | B | B | C | C |
| 1 | B | C | A | C | A | B |
| 0 | C | B | C | A | B | A |

$$
\begin{aligned}
& B S(A)=2 \times 31+1 \times 39+0 \times 11=101 \\
& B S(B)=2 \times 39+1 \times 31+0 \times 11=109 \\
& B S(C)=2 \times 11+1 \times 11+0 \times 59=33
\end{aligned}
$$

$B>_{B C} A>_{B C} C$

## Condorcet's Other Paradox

 Mars theorem Gews tory Nash Consorcet's Paradot ECO OPM Rational Choice TheoryArrowSocial Choice
Rationality

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$

## Condorcet's Other Paradox

Politics
 Nash Condorcet's Paratox ECCO ParetoHarsanyi

Arrowsocial Choice

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
| B | C | A | C | A | B |  |
| C | B | C | A | B | A |  |

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$

## Condorcet's Other Paradox

 Marys theorem tewe cusyNash Condorcels Paratox ECOMOMICS Nash Consorcet's Paradot ECO OPM Rational Choice Theory

ArrowSocial Choice
Rationality

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$

## Condorcet's Other Paradox

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{2}$ | A | A | B | B | C | C |
| $s_{1}$ | B | C | A | C | A | B |
| $s_{0}$ | C | B | C | A | B | A |

Condorcet's Other Paradox: No scoring rule will work...

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$

## Condorcet's Other Paradox

 Nash
Rational Choice Theory ParetoHarsany
ArrowSocial Choice TheorySen

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{2}$ | A | A | B | B | C | C |
| $s_{1}$ | B | C | A | C | A | B |
| $s_{0}$ | C | B | C | A | B | A |

Condorcet's Other Paradox: No scoring rule will work...
Score $(A)=s_{2} \times 31+s_{1} \times 39+s_{0} \times 11$
Score $(B)=s_{2} \times 39+s_{1} \times 31+s_{0} \times 11$
$B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C$

## Condorcet's Other Paradox

 Nash Consorcets Parraot
Rational Choice Theory ParetoHarsany
ArrowSocial Choice TheorySen

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{2}$ | A | A | B | B | C | C |
| $s_{1}$ | B | C | A | C | A | B |
| $s_{0}$ | C | B | C | A | B | A |

Condorcet's Other Paradox: No scoring rule will work...
$\operatorname{Score}(A)=s_{2} \times 31+s_{1} \times 39+s_{0} \times 11$
Score $(B)=s_{2} \times 39+s_{1} \times 31+s_{0} \times 11$
$\operatorname{Score}(A)>\operatorname{Score}(B) \Rightarrow 31 s_{2}+39 s_{1}>39 s_{2}+31 s_{1} \Rightarrow s_{1}>s_{2}$
$B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C$

## Condorcet's Other Paradox

 Mens.ime ween Economics

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{2}$ | A | A | B | B | C | C |
| $s_{1}$ | B | C | A | C | A | B |
| $s_{0}$ | C | B | C | A | B | A |

Theorem (Fishburn 1974). For all $m \geq 3$, there is some voting situation with a Condorcet winner such that every scoring rule will have at least $m-2$ candidates with a greater score than the Condorcet winner.
P. Fishburn. Paradoxes of Voting. The American Political Science Review, 68:2, pgs. 537-546, 1974.




| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A | Whane hrow Economics

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Rationality
Arrows theerem

$$
\begin{array}{ccccccc}
\text { \# voters } & 30 & 1 & 29 & 10 & 10 & 1 \\
\hline 2 & \mathrm{~A} & \mathrm{~A} & \mathrm{~B} & \mathrm{~B} & \mathrm{C} & \mathrm{C} \\
1 & \mathrm{~B} & \mathrm{C} & \mathrm{~A} & \mathrm{C} & \mathrm{~A} & \mathrm{~B} \\
0 & \mathrm{C} & \mathrm{~B} & \mathrm{C} & \mathrm{~A} & \mathrm{~B} & \mathrm{~A} \\
B S(A)=2 \times 31+1 \times 39+0 \times 11=101 \\
B S(B)=2 \times 39+1 \times 31+0 \times 11=109 \\
B S(C)=2 \times 11+1 \times 11+0 \times 59=33
\end{array}
$$

$$
B>_{B C} A>_{B C} C
$$


 ArrowSocial Choice
Rationality
Arrows theorem

$$
\begin{array}{lcccccc}
\text { \# voters } & 30 & 1 & 29 & 10 & 10 & 1 \\
\hline & \text { A } & \text { A } & \text { B } & \text { B } & \text { C } & \text { C } \\
& \text { B } & \text { C } & \text { A } & \text { C } & \text { A } & \text { B } \\
& \text { C } & \text { B } & \text { C } & \text { A } & \text { B } & \text { A }
\end{array}
$$

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$





| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
| B | C | A | C | A | B |  |
| C | B | C | A | B | A |  |

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$


 Arrow Social Chaice
aronsitionemity

$$
\begin{array}{lcccccc}
\text { \# voters } & 30 & 1 & 29 & 10 & 10 & 1 \\
\hline & \mathrm{~A} & \mathrm{~A} & \mathrm{~B} & \mathrm{~B} & \mathrm{C} & \mathrm{C} \\
& \mathrm{~B} & \mathrm{C} & \mathrm{~A} & \mathrm{C} & \mathrm{~A} & \mathrm{~B} \\
& \mathrm{C} & \mathrm{~B} & \mathrm{C} & \mathrm{~A} & \mathrm{~B} & \mathrm{~A}
\end{array}
$$

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$

## Condorcet Triples


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Rationality

| $G_{1}$ | $G_{2}$ | $G_{3}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | C |  |  | $G_{1}$ | $G_{2}$ |
| A | $G_{3}$ |  |  |  |  |  |
| B | C | A | B |  |  |  |
| C | A | B |  |  | C | B |
| A | A | C |  |  |  |  |

If $G_{1}=G_{2}=G_{3}$, then this group of voters "cancel out" each other's votes

## Saari's argument


 ArrowSocial Choice

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |

## Saari's argument



 ArrowSocial Choice
Rationality

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |
| 10 | 10 | 10 |  |  |  |  |
| A | B | C |  |  |  |  |
| B | C | A |  |  |  |  |
| C | A | B |  |  |  |  |
|  |  |  |  |  |  |  |

## Saari's argument

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Rational Choice Theory Pareto Harsanyi ArrowSocial Choice
Rationality

| \# voters | 20 | 1 | 29 | 0 | 0 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |
|  |  |  |  |  |  |  |
| 10 | 10 | 10 |  |  | 1 | 1 |
| A | B | C |  |  | A | C |
| B | C | A |  | C | B | A |
| C | A | B |  | B | A | C |

## Saari's argument

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Rationality

| \# voters | 20 | 0 | 28 | 0 | 0 | 0 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  | B |  |  |  |  |
|  | B |  | A |  |  |  |  |
|  | C |  | C |  |  |  |  |
| 10 | 10 | 10 |  | 1 | 1 | 1 |  |
| A | B | C |  | A | C | B |  |
| B | C | A |  | C | B | A |  |
| C | A | B |  | B | A | C |  |

## Is the Condorcet winner the "best" choice?

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Rationality

| \# voters | 47 | 47 | 3 | 3 |
| :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | C |
|  | C | C | A | B |
|  | B | A | B | A |

$C$ is the Condorcet winner

## Is the Condorcet winner the "best" choice?

| \# voters | 47 | 47 | 3 | 3 |
| :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | C |
|  | C | C | A | B |
|  | B | A | B | A |

$C$ is the Condorcet winner; however, it seems that supporters of the main rivals $A$ and $B$ would rather see $C$ win than their candidate's principal opponent, but this does not mean that there is "positive support" for $C$.

Approval Voting: Each voter selects a subset of candidates. The candidate with the most "approvals" wins the election.
S. Brams and P. Fishburn. Approval Voting. Birkhauser, 1983.
J.-F. Laslier and M. R. Sanver (eds.). Handbook of Approval Voting. Studies in Social Choice and Welfare, 2010. Neshemenem eronomics
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Rationality
Arrows theocem

Under Approval Voting (AV), voters are asked to select the candidates that the voter approves.

Under Approval Voting (AV), voters are asked to select the candidates that the voter approves.

Under ranking voting procedures (such as Borda Count), voters are asked to (linearly) rank the candidates.

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Under ranking voting procedures (such as Borda Count), voters are asked to (linearly) rank the candidates.

The two pieces of information are related, but not derivable from each other

Under Approval Voting (AV), voters are asked to select the candidates that the voter approves.

Under ranking voting procedures (such as Borda Count), voters are asked to (linearly) rank the candidates.

The two pieces of information are related, but not derivable from each other
Approving of a candidate is not (necessarily) the same as simply ranking the candidate first.

## Why Approval Voting?

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www.electology.org/approval-voting
S. Brams and P. Fishburn. Going from Theory to Practice: The Mixed Success of Approval Voting. Handbook of Approval Voting, pgs. 19-37, 2010.

## Approval Voting is more flexible

 Game Theory Downsmars Theorem Gus.
Nash Consorcelts Paratoox ECOMOMICS
 ArrowSocial Choice
Rationality

| \# voters | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | D | D | A |
|  | B | A | B |
|  | C | C | D |

The Condorcet winner is $A$.

## Approval Voting is more flexible

 Nash Condorcets Parasox ECO ParetoHarsanyi Arrowsocial Cholice

There is no fixed rule that always elects a unique Condorcet winner.

| \# voters | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | D | D | A |
|  | B | A | B |
|  | C | C | D |

The Condorcet winner is $A$.
Vote-for-1 elects $\{A, B\}$

## Approval Voting is more flexible

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There is no fixed rule that always elects a unique Condorcet winner.

| \# voters | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | D | D | A |
|  | B | A | B |
|  | C | C | D |

The Condorcet winner is $A$.
Vote-for-1 elects $\{A, B\}$, vote-for-2 elects $\{D\}$

## Approval Voting is more flexible


 Arrowsocial Cholice

There is no fixed rule that always elects a unique Condorcet winner.

| \# voters | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | D | D | A |
|  | B | A | B |
|  | C | C | D |

The Condorcet winner is $A$.
Vote-for-1 elects $\{A, B\}$, vote-for- 2 elects $\{D\}$, vote-for-3 elects $\{A, B\}$.

## Approval Voting is more flexible

 Game Theory Downsmars Theorem Gus.
Nash Consorcelts Paratoox ECOMOMICS Nash Condorcets Paradox ECO
Rational Choice Theory ParetoHarsanyi Arrow Sociaionality

AV may elect the Condorcet winner

| \# voters | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | D | D | A |
|  | B | A | B |
|  | C | C | D |

The Condorcet winner is $A$.
( $\{A\},\{B\},\{C, A\}$ ) elects $A$ under AV.

## Possible Failure of Unanimity


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## Possible Failure of Unanimity

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Rationality

| \# voters | 1 | 1 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | C | D |
|  | B | A | A |
|  | C | B | B |
|  | D | D | C |

Approval Winners: $A, B$

## Indeterminate or Responsive?

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Nash Conem Gexis Nash Consorcet's Paradorec: OMOMCS ArrowSocial Choice TheorySen

| \# voters | 6 | 5 | 4 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | C | C | B |
|  | B | A | A |

Plurality winner: $A$, Borda and Condorcet winner: $C$.

## Indeterminate or Responsive?

 Mas seme temo Nashemanc cheobe Tho $\underset{\text { Rrrows theorem }}{\text { Ratity }}$| \# voters | 6 | 5 | 4 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | C | C | B |
|  | B | A | A |

Plurality winner: $A$, Borda and Condorcet winner: $C$.
Any combination of $A, B$ and $C$ can be an AV winner (or AV winners).

## Generalizing Approval Voting

## Generalizing Approval Voting

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Ask the voters to provide both a linear ranking of the candidates and the candidates that they approve.

## Generalizing Approval Voting

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Ask the voters to provide both a linear ranking of the candidates and the candidates that they approve.

Make the ballots more expressive: Dis\&Approval voting, RangeVoting, Majority Judgement

## Grading

In many group decision situations, people use measures or grades from a common language of evaluation to evaluate candidates or alternatives:

- in figure skating, diving and gymnastics competitions;
- in piano, flute and orchestra competitions;
- in classifying wines at wine competitions;
- in ranking university students;
- in classifying hotels and restaurants, e.g., the Michelin *


## Voting by Grading: Questions

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- What grading language should be used? (e.g., $A-F, 0-10, *-* * * *)$


## Voting by Grading: Questions

 wave same therams Nashemena choce Theary Arrowsocial Cholice- What grading language should be used? (e.g., $A-F, 0-10, *-* * * *$ )
- How should we aggregate the grades? (e.g., Average or Median)


## Voting by Grading: Questions

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- What grading language should be used? (e.g., $A-F, 0-10, *-* * * *$ )
- How should we aggregate the grades? (e.g., Average or Median)
- Should there be a "no opinion" option?


## Voting by Grading: Questions

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- What grading language should be used? (e.g., $A-F, 0-10, *-* * * *$ )
- How should we aggregate the grades? (e.g., Average or Median)
- Should there be a "no opinion" option?


## Voting by Grading: Examples

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Approval Voting: voters can assign a single grade "approve" to the candidates

Dis\&Approval Voting: voters can approve or disapprove of the candidates
Majority Judgement, Score Voting: voters can assign any grade from a fixed set of grades to the candidates

## Strong Paradox of Grading Systems

Grades: $\{0,1,2,3\}$
Candidates: $\{A, B, C\}$ 3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :--- | :--- | :--- | :--- |
| $A$ | 3 | 2 | 0 |  |
| $B$ | 0 | 3 | 1 |  |
| $C$ | 0 | 3 | 1 |  |

Grades: $\{0,1,2,3\}$

 Arrow Rationality
Candidates: $\{A, B, C\}$ 3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 3 | 2 | 0 | $5 / 3$ |
| $B$ | 0 | 3 | $1 \mid$ | $4 / 3$ |
| $C$ | 0 | 3 | 1 | $4 / 3$ |

Average Grade Winner: $A$

Grades: $\{0,1,2,3\}$ Mens shemen wem Economics
 ArrowSocial Choice
Candidates: $\{A, B, C\}$ $\underset{\text { Arows theorem }}{\text { Rationality }}$ 3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 3 | 2 | 0 |  |
| $B$ | 0 | 3 | 1 |  |
| $C$ | 0 | 3 | 1 |  |

Average Grade Winner: $A$

$$
B>A
$$

Grades: $\{0,1,2,3\}$
Candidates: $\{A, B, C\}$

 ArrowSocial Choice
Rationality 3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 3 | 2 | 0 |  |
| $B$ | 0 | 3 | 1 |  |
| $C$ | 0 | 3 | 1 |  |

Average Grade Winner: $A$

$$
C \sim B>A
$$

Grades: $\{0,1,2,3\}$ Mens shemen wem Economics
 ArrowSocial Choice
Candidates: $\{A, B, C\}$ $\underset{\text { Arows theorem }}{\text { Rationality }}$

3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 3 | 2 | 0 |  |
| $B$ | 0 | 3 | 1 |  |
| $C$ | 0 | 3 | 1 |  |

Average Grade Winner: $A$

$$
C \sim B>A
$$

Grades: $\{0,1,2,3\}$
Candidates: $\{A, B, C\}$

## 3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 3 | 2 | 0 |  |
| $B$ | 0 | 3 | 1 |  |
| $C$ | 0 | 3 | 1 |  |

Average Grade Winner: $A$
Superior Grade Winners: $C, B$

Grades: $\{0,1,2,3,4,5\}$
Candidates: $\{A, B, C\}$ 5 Voters

| \# voters | 1 | 4 | Avg |
| :---: | :---: | :---: | :---: |
| $A$ | 5 | 0 | $5 / 5$ |
| $B$ | 0 | 1 | $4 / 5$ |
| $C$ | 0 | 1 | $4 / 5$ |

Average Grade Winner: $A$
Superior Grade Winner: B, C

To conclude, we have identified a paradox of grading systems, which is not just a mirror of the well-known differences that crop up in aggregating votes under ranking systems. Unlike these systems, for which there is no accepted way of reconciling which candidate to choose when, for example, the Hare, Borda and Condorcet winners differ, AV provides a solution when the AG and SG winners differ.

Theorem (Brams and Potthoff). When there are two grades, the AG and SG winners are identical.

## Further Investigation

 Nash Condorcets Paradox
Rational Choice
Theory Arrow Rationality

- W. Poundstone, Gaming the Vote: Why Elections Aren't Fair (and What We Can Do About It), Hill and Wang, 2009
- EP, Voting Methods (Stanford Encyclopedia of Philosophy)
- C. List, Social Choice Theory (Stanford Encyclopedia of Philosophy)
- M. Morreau, Arrow's Theorem (Stanford Encyclopedia of Philosophy)


## Further Investigation

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- https://www.electology.org
- http://www.fairvote.org
- http://rangevoting.org
- https://www.opavote.com
- http://www.preflib.org


## There are many different voting methods

Many different electoral methods: Plurality, Borda Count, Antiplurality/Veto, and k-approval; Plurality with Runoff; Single Transferable Vote (STV)/Hare; Approval Voting; Cup Rule/Voting Trees; Copeland; Banks; Slater Rule; Schwartz Rule; the Condorcet rule; Maximin/Simpson, Kemeny; Ranked Pairs/Tideman; Bucklin Method; Dodgson Method; Young's Method; Majority Judgment; Cumulative Voting; Range/Score Voting; ...

## Choosing how to choose

Pragmatic considerations: Is the procedure easy to use? Is it legal? The importance of ease of use should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.

Behavioral considerations: Do the different procedures really lead to different outcomes in practice?

Information required from the voters: What type of information do the ballots convey? I.e., Choosing a single alternative, linearly rank all the candidates, report something about the "intensity" of preference.

Axiomatics: Characterize the different voting methods in terms of normative principles of group decision making.

## Principles of group decision making

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## Principles of group decision making


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- Condorcet Condition: Always choose the candidate that beats every other candidate in head-to-head elections.


## Principles of group decision making


 Arrow Rationality

- Condorcet Condition: Always choose the candidate that beats every other candidate in head-to-head elections.
- Unanimity (Pareto): If everyone ranks $A$ above $B$, then $B$ should not win the election.


## Principles of group decision making

- Condorcet Condition: Always choose the candidate that beats every other candidate in head-to-head elections.
- Unanimity (Pareto): If everyone ranks $A$ above $B$, then $B$ should not win the election.
- Anonymity: The names of the voters do not matter (if two voters swap votes, then the outcome is unaffected).


## Monotonicity

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Arous

A candidate receiving more "support" shouldn't maker her worse off.

## Monotonicity

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A candidate receiving more "support" shouldn't maker her worse off.

More-is-Less Paradox: If a candidate $C$ is elected under a given a profile of rankings of the competing candidates, it is possible that, ceteris paribus, C may not be elected if some voter(s) raise $C$ in their rankings.
P. Fishburn and S. Brams. Paradoxes of Preferential Voting. Mathematics Magazine (1983).

## More-is-Less Paradox: Plurality with Runoff

 uns came terer Economics| \# voters | 6 | 5 | 4 | 2 | \# voters | 6 | 5 | 4 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | C | B | B |  | A | C | B | A |
|  | B | A | C | A |  | B | A | C | B |
|  | C | B | A | C |  | C | B | A | C |

## More-is-Less Paradox: Plurality with Runoff

 uns came terer Economics| \# voters | 6 | 5 | 4 | 2 |  |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | C | B | B |  |  |  |  |  |
| B | A voters | C | A | 5 | 4 | 2 |  |  |  |
|  |  |  | A | C | B | A |  |  |  |
|  | C | B | A | C |  | B | A | C | B |
|  |  |  | C | B | A | C |  |  |  |

## More-is-Less Paradox: Plurality with Runoff

 uns same temo Economics| \# voters | 6 | 5 | 4 | 2 | \# voters | 6 | 5 | 4 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | C | B | B |  | A | C | B | A |
|  | B | A | C | A |  | B | A | C | B |
|  | C | B | A | C |  | C | B | A | C |

# More-is-Less Paradox: Plurality with Runoff 

 uns came terer Economics| \# voters | 6 | 5 | 4 | 2 |
| :--- | :--- | :--- | :--- | :--- |
|  | A | C | B | B |
|  | B | A | C | A |
|  | C | B | A | C |


| \# voters | 6 | 5 | 4 | 2 |
| ---: | :--- | :--- | :--- | :--- |
|  | A | C | B | A |
|  | B | A | C | B |
|  | C | B | A | C |

## More-is-Less Paradox: Plurality with Runoff

 uns same weon Economics| \# voters | 6 | 5 | 4 | 2 |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | A | C | B | B |$\quad$| \# voters | 6 | 5 | 4 | 2 |  |
| :--- | :--- | :--- | :--- | :--- | :--- |
|  | B | A | C | A |  |
| C | B | A | C |  | A |
| C | B | A |  |  |  |
|  |  |  | C | A | C |
| B | B | A | C |  |  |

Winner: $A$

## More-is-Less Paradox: Plurality with Runoff

| \# voters | 6 | 5 | 4 | 2 |
| :--- | :--- | :--- | :--- | :--- |
|  | A | C | B | B |
|  | B | A | C | A |
|  | C | B | A | C |


| \# voters | 6 | 5 | 4 | 2 |
| :--- | :--- | :--- | :--- | :--- |
|  | A | C | B | A |
|  | B | A | C | B |
|  | C | B | A | C |

Winner: $A$

| \# voters | 6 | 5 | 4 | 2 |
| :--- | :--- | :--- | :--- | :--- |
|  | A | C | B | B |
| B | A | C | A |  |
|  | C | B | A | C |

Winner: $A$

| \# voters | 6 | 5 | 4 | 2 |
| :--- | :---: | :---: | :---: | :---: |
|  | A | C | B | A |
|  | B | A | C | B |
|  | C | B | A | C |

Winner: C

## More-is-Less Paradox: Plurality with Runoff

 wes shemencer misconomics| \# voters | 6 | 5 | 4 | 2 | \# voters | 6 | 5 | 4 | 2 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | C | B | B |  | A | C | B | A |
|  | B | A | C | A |  | B | A | C | B |
|  | C | B | A | C |  | C | B | A | C |
|  | Winner: $A$ |  |  |  |  | Winner: C |  |  |  | wns nememem ECONOMICS Arowsocial Coice theerrsen

Monotonicity: A candidate receiving more "support" shouldn't maker her worse off.

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No-Show Paradox: A voter may obtain a more preferable outcome if he decides not to participate in an election than, ceteris paribus, if he decides to participate in the election.

Monotonicity: A candidate receiving more "support" shouldn't maker her worse off.

No-Show Paradox: A voter may obtain a more preferable outcome if he decides not to participate in an election than, ceteris paribus, if he decides to participate in the election.

- Twin Paradox: A voter may obtain a less preferable outcome if his "twin" (a voter with the exact same ranking) decides to participate in the election.

Monotonicity: A candidate receiving more support shouldn't make her worse off

No-Show Paradox: A voter may obtain a more preferable outcome if he decides not to participate in an election than, ceteris paribus, if he decides to participate in the election.

- Twin Paradox: A voter may obtain a less preferable outcome if his "twin" (a voter with the exact same ranking) decides to participate in the election.
- Truncation Paradox: A voter may obtain a more preferable outcome if, ceteris paribus, he only reveals part of his ranking of the candidates.


## No-Show Paradox: Plurality with Runoff

 Nash Condorcet's Paradox ECO
Rational Choice Theory Pareto Harsanyi

Arrow Rationality

\# voters | 4 | 3 | 1 | 3 |  |
| ---: | :--- | :--- | :--- | :--- |
|  | A | B | C | C |
|  | B | C | A | B |
|  | C | A | B | A |

## No-Show Paradox: Plurality with Runoff

 Game Theory Downsmars Theorem Guss
Nash Consorests Paratox ECOMOMICS Nash condorcets Paradox ECO ParetoHarsanyi
Rational Choice Theory
ArrowSocial Choice TheorySen Arrowsocial Rnalice

| \# voters | 4 | 3 | 1 | 3 |
| ---: | :---: | :---: | :---: | :---: |
|  | A | B | C | C |
| B | C | A | B |  |
| C | A | B | A |  |

## No-Show Paradox: Plurality with Runoff

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 ArrowSocial Choice
Rationality

| \# voters | 4 | 3 | 1 | 3 |
| :--- | :---: | :---: | :---: | :---: |
|  | A | B | C | C |
|  | $B$ | C | A | B |
|  | C | A | B | A |

Winner: C

## No-Show Paradox: Plurality with Runoff

 Arrowsocial Cholice

| \# voters | 4 | 3 | 1 | 3 | \# voters | 2 | 3 | 1 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | C |  | A | B | C | C |
|  | B | C | A | B |  | B | C | A | B |
|  | C | A | B | A |  | C | A | B | A |

## No-Show Paradox: Plurality with Runoff


 Arrowsocial Cholice

| \# voters | 4 | 3 | 1 | 3 |  |  |  |  |
| ---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  | A | B | C | C |  |  |  |  |
| B voters | 2 | 3 | 1 | 3 |  |  |  |  |
|  | C | A | B |  | A | B | C | C |
| C | A | B | A |  | B | C | A | B |
|  |  | C | A | B | A |  |  |  |

Winner: C

## No-Show Paradox: Plurality with Runoff

| \# voters | 4 | 3 | 1 | 3 |
| :--- | :--- | :--- | :--- | :--- |
|  | A | B | C | C |
|  | B | C | A | B |
|  | C | A | B | A |

Winner: C

| \# voters | 2 | 3 | 1 | 3 |
| :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | C |
|  | B | C | A | B |
| C | A | B | A |  |

Winner: $B$

## Twin Paradox: Plurality with Runoff

 Nash Consorcets Paradox ECCO ParetoHarsanyi
Rational Choice Theory Arrowsocial Rnalice

| \# voters | 4 | 3 | 1 | 3 | \# voters | 2 | 3 | 1 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | C |  | A | B | C | C |
|  | B | C | A | B |  | B | C | A | B |
|  | C | A | B | A |  | C | A | B | A |
|  | Winner: C |  |  |  |  | Winner: $B$ |  |  |  |

## Failures of Monotonicity

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Example: Burlington, VT 2009 Mayoral Race (rangevoting.org/Burlington.html)

## Failures of Monotonicity

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Example: Burlington, VT 2009 Mayoral Race (rangevoting.org/Burlington.html)
D. Felsenthal and N. Tideman. Varieties of Failure of Monotonicity and Participation under Five Voting Methods. Theory and Decision, 75, pgs. 59-77, 2013.

## Failures of Monotonicity

Example: Burlington, VT 2009 Mayoral Race (rangevoting.org/Burlington.html)
D. Felsenthal and N. Tideman. Varieties of Failure of Monotonicity and Participation under Five Voting Methods. Theory and Decision, 75, pgs. 59-77, 2013.

Theorem (Moulin). If there are four or more candidates, then every Condorcet consistent voting methods is susceptible to the No-Show paradox.
H. Moulin. Condorcet's Principle Implies the No Show Paradox. Journal of Economic Theory, 45, pgs. 53-64, 1988.

## Spoiler Candidates: Plurality Rule

 Nash Condorcet's Paradox ECO OPM PaticS ArrowSocial Choice
Rationality

\# voters | 49 | 48 | 3 |  |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | B | A | B |
|  | C | C | A |

Winner: $A$

## Spoiler Candidates: Plurality Rule

 Nash Consorcet's Paradot ECO OPM Rational Choice Theory ArrowSocial Choice
Rationality

| \# voters | 49 | 48 | 3 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | B | A | B |
|  | C | C | A |

Winner: $B$

Independence of Irrelevant Alternatives: If the voters in two different electorates rank $A$ and $B$ in exactly the same way, then $A$ and $B$ should be ranked the same way in both elections.

## Failure of IIA: Borda Count

 Nash condorcets Paradox ECO ParetoHarsanyi
Rational Choice Theory
ArrowSocial Choice TheorySen Arrowsocia Choice

| \# voters | 3 | 2 | 2 |
| :---: | :---: | :---: | :---: |
| 3 | A | B | C |
| 2 | B | C | A |
| 1 | C | A | B |
| 0 | X | X | X |

## Failure of IIA: Borda Count

 un ane fow Economics Nash Condorcets Paradox ECO ParetoHarsanyiRational Choice Theory
ArrowSocial Choice Theory Sen Arrowsocial Cholice

| \# voters | 3 | 2 | 2 |
| :---: | :---: | :---: | :---: |
| 3 | A | B | C |
| 2 | B | C | A |
| 1 | C | A | B |
| 0 | X | X | X |

$A(15)>_{B C} B(14)>_{B C} C(13)>_{B C} X(0)$

## Failure of IIA: Borda Count

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| \# voters | 3 | 2 | 2 |
| :---: | :---: | :---: | :---: |
| 3 | A | B | C |
| 2 | B | C | X |
| 1 | C | X | A |
| 0 | X | A | B |

$$
A(15)>_{B C} B(14)>_{B C} C(13)>_{B C} X(0)
$$

## Failure of IIA: Borda Count

| \# voters | 3 | 2 | 2 |
| :---: | :---: | :---: | :---: |
| 3 | A | B | C |
| 2 | B | C | A |
| 1 | C | A | B |
| 0 | X | X | X |

$A(15)>_{B C} B(14)>_{B C} C(13)>_{B C} X(0)$

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 ArrowSocial Choice
Rationality

| \# voters | 3 | 2 | 2 |
| :---: | :---: | :---: | :---: |
| 3 | A | B | C |
| 2 | B | C | X |
| 1 | C | X | A |
| 0 | X | A | B |

$C(13)>_{B C} B(12)>_{B C} A(11)>_{B C} X(6)$

## Principles

Condorcet: Elect the Condorcet winner whenever it exists.
Monotonicity: More support should never hurt a candidate.
Participation: It should never be in a voter's best interests not to vote.
Multiple-Districts: If a candidate wins in each district, then that candidate should also win when the districts are merged.

Independence: The group's ranking of $A$ and $B$ should only depend on the voter's rankings of $A$ and $B$.

## More Principles

 Mas seme temy conomics Nash Consorcets saraodRational Choice 'Theory ParetoHarsany $\underset{\text { Rrows theorem }}{\text { Ration }}$

Pareto: Never elect a candidate if another candidate is strictly preferred by all voters.

Anonymity: The outcome does not depend on the names of the voters.
Neutrality: The outcome does not depend on the names of the candidates.
Universal Domain: The voters are free to rank the candidates (or grade the candidates) in any way they want.

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Rationality
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What are the relationships between these principles? Is there a procedure that satisfies all of them?

What are the relationships between these principles? Is there a procedure that satisfies all of them?

A few observations:

- Condorcet winners may not exist.
- No positional scoring method satisfies the Condorcet Principle.
- The Condorcet and Participation principles cannot be jointly satisfied.


## Axiomatics

"When a set of axioms regarding social choice can all be simultaneously satisfied, there may be several possible procedures that work, among which we have to choose.
A. Sen. The Possibility of Social Choice. The American Economic Review, 89:3, pgs. 349-378, 1999 (reprint of his Nobel lecture).

## Axiomatics

"When a set of axioms regarding social choice can all be simultaneously satisfied, there may be several possible procedures that work, among which we have to choose. In order to choose between different possibilities through the use of discriminating axioms, we have to introduce further axioms, until only and only one possible procedure remains.
A. Sen. The Possibility of Social Choice. The American Economic Review, 89:3, pgs. 349-378, 1999 (reprint of his Nobel lecture).

## Axiomatics

"When a set of axioms regarding social choice can all be simultaneously satisfied, there may be several possible procedures that work, among which we have to choose. In order to choose between different possibilities through the use of discriminating axioms, we have to introduce further axioms, until only and only one possible procedure remains. This is something of an exercise in brinkmanship. We have to go on and on cutting alternative possibilities, moving-implicitly-towards an impossibility, but then stop just before all possibilities are eliminated, to wit, when one and only one options remains."
(pg. 354)
A. Sen. The Possibility of Social Choice. The American Economic Review, 89:3, pgs. 349-378, 1999 (reprint of his Nobel lecture).

The Social Choice Model

## Notation

 was semme whorn Nonomics Nash condores Choice' Theory ParetoHarsany Arrowsocial Cholice- $N$ is a finite set of voters (assume that $N=\{1,2,3, \ldots, n\}$ )
- X is a (typically finite) set of alternatives, or candidates
- A relation on $X$ is a linear order if it is transitive, irreflexive, and complete (hence, acyclic)
- $L(X)$ is the set of all linear orders over the set $X$
- $O(X)$ is the set of all reflexive and transitive relations over the set $X$


## Notation

 Nash Nastonal Choice' Theory ParetoHarsanyi ArrowSocial Choice TheorySen- A profile for the set of voters $N$ is a sequence of (linear) orders over $X$, denoted $\mathbf{R}=\left(R_{1}, \ldots, R_{n}\right)$.
- $L(X)^{n}$ is the set of all profiles for $n$ voters (similarly for $\left.O(X)^{n}\right)$
- For a profile $\mathbf{R}=\left(R_{1}, \ldots, R_{n}\right) \in O(X)^{n}$, let $\mathbf{N}_{\mathbf{R}}(A P B)=\left\{i \mid A P_{i} B\right\}$ be the set of voters that rank $A$ above $B$ (similarly for $\mathbf{N}_{\mathbf{R}}(A$ I $B)$ and $\mathbf{N}_{\mathbf{R}}(B P A)$ )


## Preference Aggregation Methods


 Arrow Rationality

Social Welfare Function: $F: \mathcal{D} \rightarrow L(X)$, where $\mathcal{D} \subseteq L(X)^{n}$

## Preference Aggregation Methods

Social Welfare Function: $F: \mathcal{D} \rightarrow L(X)$, where $\mathcal{D} \subseteq L(X)^{n}$
Comments

- $\mathcal{D}$ is the domain of the function: it is the set of all possible profiles
- Aggregation methods are decisive: every profile $\mathbf{R}$ in the domain is associated with exactly one ordering over the candidates
- The range of the function is $L(X)$ : the social ordering is assumed to be a linear order
- Tie-breaking rules are built into the definition of a preference aggregation function


## Preference Aggregation Methods

 ArrowSocial Choice TheorySen $\underset{\text { Rrows theorem }}{\text { Ratity }}$

Social Welfare Function: $F: \mathcal{D} \rightarrow L(X)$, where $\mathcal{D} \subseteq L(X)^{n}$

## Variants

- Social Choice Function: $F: \mathcal{D} \rightarrow \wp(X)$ - $\emptyset$, where $\mathcal{D} \subseteq L(X)^{n}$ and $\wp(X)$ is the set of all subsets of $X$.
- Allow Ties: $F: \mathcal{D} \rightarrow O(X)$ where $O(X)$ is the set of orderings (reflexive and transitive) over $X$
- Allow Indifference and Ties: $F: \mathcal{D} \rightarrow O(X)$ where $O(X)$ is the set of orderings (reflexive and transitive) over $X$ and $\mathcal{D} \subseteq O(X)^{n}$


## Examples

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$\operatorname{Maj}(\mathbf{R})=>_{M}$ where $A>_{M} B$ iff $\left|\mathbf{N}_{\mathbf{R}}(A P B)\right|>\left|\mathbf{N}_{\mathbf{R}}(B P A)\right|$
(the problem is that $>_{M}$ may not be transitive (or complete))

## Examples

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$\operatorname{Maj}(\mathbf{R})=>_{M}$ where $A>_{M} B$ iff $\left|\mathbf{N}_{\mathbf{R}}(A P B)\right|>\left|\mathbf{N}_{\mathbf{R}}(B P A)\right|$
(the problem is that $>_{M}$ may not be transitive (or complete))
$\operatorname{Borda}(\mathbf{R})=\geq_{B C}$ where $A \geq_{B C} B$ iff the Borda score of $A$ is greater than the Borda score for $B$.
(the problem is that $\geq_{B C}$ may not be a linear order)

## Characterizing Majority Rule

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## Characterizing Majority Rule

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When there are only two candidates $A$ and $B$, then all voting methods give the same results

Majority Rule: $A$ is ranked above (below) $B$ if more (fewer) voters rank $A$ above $B$ than $B$ above $A$, otherwise $A$ and $B$ are tied.

## Characterizing Majority Rule

When there are only two candidates $A$ and $B$, then all voting methods give the same results

Majority Rule: $A$ is ranked above (below) $B$ if more (fewer) voters rank $A$ above $B$ than $B$ above $A$, otherwise $A$ and $B$ are tied.

When there are only two options, can we argue that majority rule is the "best" procedure?
K. May. A Set of Independent Necessary and Sufficient Conditions for Simple Majority Decision. Econometrica, Vol. 20 (1952).

## May's Theorem: Details

Let $N=\{1,2,3, \ldots, n\}$ be the set of $n$ voters and $X=\{A, B\}$ the set of candidates.

Social Welfare Function: $F: O(X)^{n} \rightarrow O(X)$, where $O(X)$ is the set of orderings over X (there are only three possibilities: A P B, A I B, or B P A)

$$
F_{M a j}(\mathbf{R})=\left\{\begin{array}{ll}
A P & P
\end{array} \quad \text { if }\left|\mathbf{N}_{\mathbf{R}}\left(\begin{array}{ll}
A & P
\end{array}\right)\right|>\left|\mathbf{N}_{\mathbf{R}}\left(\begin{array}{lll}
B & P & A
\end{array}\right)\right|\right.
$$

## May's Theorem: Details

Let $N=\{1,2,3, \ldots, n\}$ be the set of $n$ voters and $X=\{A, B\}$ the set of candidates.

Social Welfare Function: $F:\{1,0,-1\}^{n} \rightarrow\{1,0,-1\}$,
where 1 means $A$ P $B, 0$ means $A$ I $B$, and -1 means $B P A$

$$
F_{M a j}(\mathbf{v})= \begin{cases}1 & \text { if }\left|\mathbf{N}_{\mathbf{v}}(1)\right|>\left|\mathbf{N}_{\mathbf{v}}(-1)\right| \\ 0 & \text { if }\left|\mathbf{N}_{\mathbf{v}}(1)\right|=\left|\mathbf{N}_{\mathbf{v}}(-1)\right| \\ -1 & \text { if }\left|\mathbf{N}_{\mathbf{v}}(-1)\right|>\left|\mathbf{N}_{\mathbf{v}}(1)\right|\end{cases}
$$

## Warm-up Exercise

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## Warm-up Exercise

Suppose that there are two voters and two candidates. How many social choice functions are there? 19,683

- There are three possible rankings for 2 candidates.
- When there are two voters there are $3^{2}=9$ possible profiles:

$$
\{(1,1),(1,0),(1,-1),(0,1),(0,0),(0,-1),(-1,1),(-1,0),(-1,-1)\}
$$

- Since there are 9 profiles and 3 rankings, there are $3^{9}=19,683$ possible preference aggregation functions.


## May's Theorem: Details

- Unanimity: unanimously supported alternatives must be the social outcome.
- Anonymity: all voters should be treated equally.
- Neutrality: all candidates should be treated equally.


## May's Theorem: Details

 Nash Condorcet's Paradox ECO ParetoHarsany Arrow Rationality

- Unanimity: unanimously supported alternatives must be the social outcome.
If $\mathbf{v}=\left(v_{1}, \ldots, v_{n}\right)$ with for all $i \in N, v_{i}=x$ then $F(\mathbf{v})=x$ (for $x \in\{1,0,-1\}$ ).
- Anonymity: all voters should be treated equally.
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## May's Theorem: Details

- Unanimity: unanimously supported alternatives must be the social outcome.
If $\mathbf{v}=\left(v_{1}, \ldots, v_{n}\right)$ with for all $i \in N, v_{i}=x$ then $F(\mathbf{v})=x$ (for $x \in\{1,0,-1\}$ ).
- Anonymity: all voters should be treated equally.
$F\left(v_{1}, \ldots, v_{n}\right)=F\left(v_{\pi(1)}, v_{\pi(2)}, \ldots, v_{\pi(n)}\right)$ where $v_{i} \in\{1,0,-1\}$ and $\pi$ is a permutation of the voters.
- Neutrality: all candidates should be treated equally.


## May's Theorem: Details

- Unanimity: unanimously supported alternatives must be the social outcome.
If $\mathbf{v}=\left(v_{1}, \ldots, v_{n}\right)$ with for all $i \in N, v_{i}=x$ then $F(\mathbf{v})=x$ (for $x \in\{1,0,-1\}$ ).
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- Neutrality: all candidates should be treated equally.

$$
F(-\mathbf{v})=-F(\mathbf{v}) \text { where }-\mathbf{v}=\left(-v_{1}, \ldots,-v_{n}\right) .
$$

## May's Theorem: Details


 Arrow Rationality

- Positive Responsiveness (Monotonicity): unidirectional shift in the voters' opinions should help the alternative toward which this shift occurs

If $F(\mathbf{v})=0$ or $F(\mathbf{v})=1$ and $\mathbf{v}<\mathbf{v}^{\prime}$, then $F\left(\mathbf{v}^{\prime}\right)=1$ where $\mathbf{v}<\mathbf{v}^{\prime}$ means for all $i \in N v_{i} \leq v_{i}^{\prime}$ and there is some $i \in N$ with $v_{i}<v_{i}^{\prime}$.

## Warm-up Exercise

Suppose that there are two voters and two candidates. How many social choice functions are there that satisfy anonymity?

Anonymity: all voters should be treated equally.
$F\left(v_{1}, v_{2}, \ldots, v_{n}\right)=F\left(v_{\pi(1)}, v_{\pi(2)}, \ldots, v_{\pi(n)}\right)$ where $\pi$ is a permutation of the voters.

## Warm-up Exercise

Suppose that there are two voters and two candidates. How many social choice functions are there that satisfy anonymity?

Anonymity: all voters should be treated equally.
$F\left(v_{1}, v_{2}, \ldots, v_{n}\right)=F\left(v_{\pi(1)}, v_{\pi(2)}, \ldots, v_{\pi(n)}\right)$ where $\pi$ is a permutation of the voters.

- Imposing anonymity reduces the number of preference aggregation functions.
- If $F$ satisfies anonymity, then $F(1,0)=F(0,1), F(1,-1)=F(-1,1)$ and $F(-1,0)=F(0,-1)$.
- This means that there are essentially 6 elements of the domain. So, there are $3^{6}=729$ preference aggregation functions.


## May's Theorem: Details

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May's Theorem (1952) A social decision method $F$ satisfies unanimity, neutrality, anonymity and positive responsiveness iff $F$ is majority rule.

## Proof Idea

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If $(1,0,-1)$ is assigned 1 or -1 then

## Proof Idea


 Arrow Socialionality

If $(1,0,-1)$ is assigned 1 or -1 then
$\checkmark$ Anonymity implies $(-1,0,1)$ is assigned 1 or -1

## Proof Idea

 Nashtional Choice Theory Peratetorassny Arrowsocial Cholice

If $(1,0,-1)$ is assigned 1 or -1 then
$\checkmark$ Anonymity implies $(-1,0,1)$ is assigned 1 or -1
$\checkmark$ Neutrality implies $(1,0,-1)$ is assigned -1 or 1 Contradiction.

## Proof Idea

 nes nemene wemeconomics Arrow Sociationality

If $(1,1,-1)$ is assigned 0 or -1 then

## Proof Idea

 mass Game theor) wouns Nash tonarects eise thery Peretorarsany Arrow RationalityIf $(1,1,-1)$ is assigned 0 or -1 then
$\checkmark$ Neutrality implies $(-1,-1,1)$ is assigned 0 or 1

## Proof Idea

 mavs sheorem Geus Nash Condorceets Paradox ECO\OMOMS Nash Consorcets Parabox ECO Parn ArrowSocial Choice TheorySenIf $(1,1,-1)$ is assigned 0 or -1 then
$\checkmark$ Neutrality implies $(-1,-1,1)$ is assigned 0 or 1
$\checkmark$ Anonymity implies $(1,-1,-1)$ is assigned 0 or 1

## Proof Idea

If $(1,1,-1)$ is assigned 0 or -1 then
$\checkmark$ Neutrality implies $(-1,-1,1)$ is assigned 0 or 1
$\checkmark$ Anonymity implies $(1,-1,-1)$ is assigned 0 or 1
$\checkmark$ Positive Responsiveness implies $(1,0,-1)$ is assigned 1

## Proof Idea

 Nash condor
Rational Choice
Theory ParetoHarsany Arrow Rationality

If $(1,1,-1)$ is assigned 0 or -1 then
$\checkmark$ Neutrality implies $(-1,-1,1)$ is assigned 0 or 1
$\checkmark$ Anonymity implies $(1,-1,-1)$ is assigned 0 or 1
$\checkmark$ Positive Responsiveness implies $(1,0,-1)$ is assigned 1
$\checkmark$ Positive Responsiveness implies $(1,1,-1)$ is assigned 1 Contradiction.

## Other characterizations


 ArrowSocial Choice TheorySen $\underset{\text { Rrows theorem }}{\text { Rationality }}$
G. Asan and R. Sanver. Another Characterization of the Majority Rule. Economics Letters, 75 (3), 409-413, 2002.
E. Maskin. Majority rule, social welfare functions and game forms. in Choice, Welfare and Development, The Clarendon Press, pgs. 100-109, 1995.
G. Woeginger. A new characterization of the majority rule. Economic Letters, 81, pgs. 89-94, 2003.

Can May's Theorem be generalized to more than 2 candidates?

Can May's Theorem be generalized to more than 2 candidates? No!

