Methods in Philosophy, Politics and Economics: Individual and Group Decision Making

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From Lotteries to Decision Problems

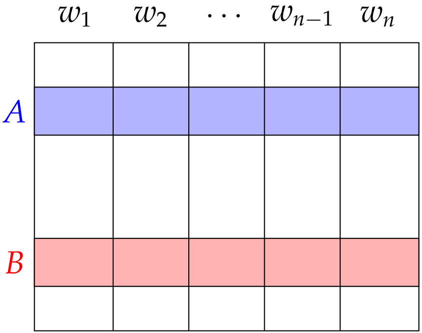
A $x_1 : p_1,\ x_2 : p_2,\ \cdots\ x_n : p_n$

B $x_1 : q_1,\ x_2 : q_2,\ \cdots\ x_n : q_n$

From Lotteries to Decision Problems

	w_1	w_2	$\cdots$	w_{n-1}	w_n
A					
B					

From Lotteries to Decision Problems



An **act** is a function $F : W \to O$

Making an omelet

States: {the sixth egg is good, the sixth egg is rotten}

Consequences: { six-egg omelet, no omelet and five good eggs destroyed, six-egg omelet and a cup to wash....}

Acts: { break egg into bowl, break egg into a cup, throw egg away}

Making an omelet

	Good egg (s_1)	Bad egg (s_2)
Break into a bowl (A_1)	six egg omelet (o_1)	no omelet and five good eggs destroyed (o_2)
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3)	five egg omelet and a cup to wash (o_4)
Throw away (A_3)	five egg omelet and one good egg destroyed (o_5)	five egg omelet (o_6)

Making an omelet

	Good egg (s_1)	Bad egg (s_2)
Break into a bowl (A_1)	six egg omelet (o_1)	no omelet and five good eggs destroyed (o_2)
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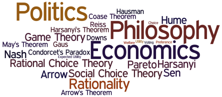
$A_1(s_1) = o_1$ $\qquad$ $A_1(s_2) = o_2$

Making an omelet



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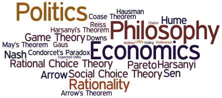
$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2$$

Making an omelet



	Good egg (s_1)	Bad egg (s_2)
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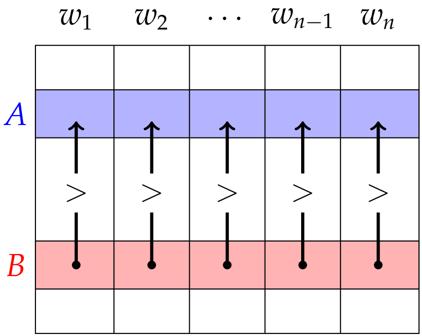
$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2$

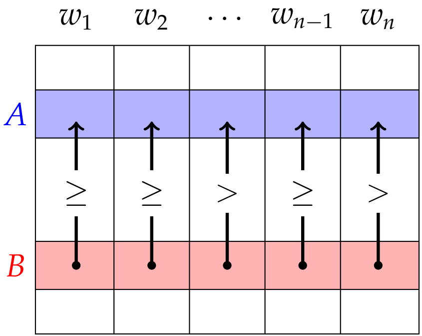
How should A_1, A_2 and A_3 be ranked?

Strict Dominance



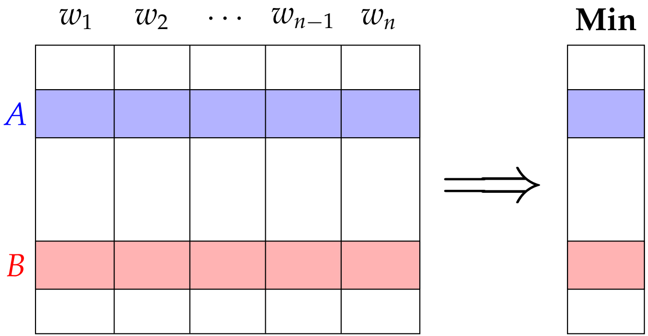
$$\forall\, w \in W, u(A(w)) > u(B(w))$$

Weak Dominance



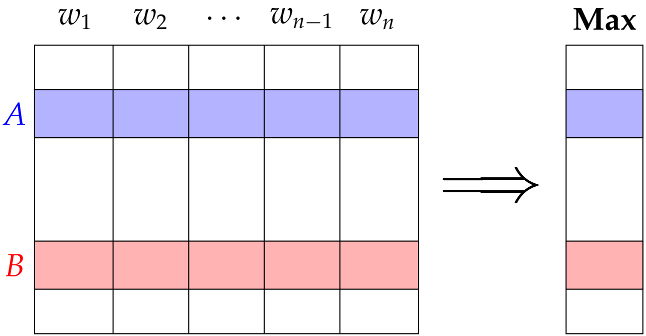
$$\forall\, w \in W, u(A(w)) \geq u(B(w)) \text{ and } \exists\, w \in W, u(A(w)) > u(B(w))$$

MaxMin (Security)



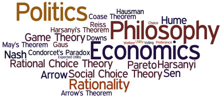
$$\min(\{u(A(w)) \mid w \in W\}) > \min(\{u(B(w)) \mid w \in W\})$$

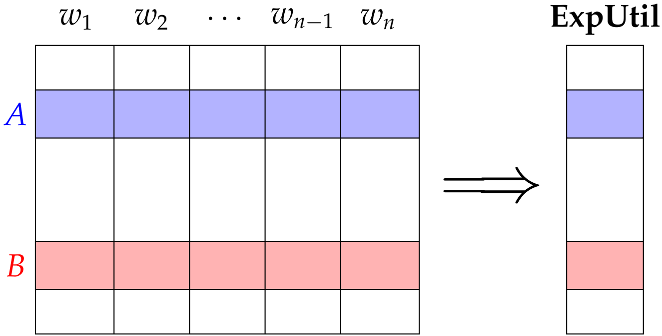
MaxMax



$$\max(\{u(A(w)) \mid w \in W\}) > \max(\{u(B(w)) \mid w \in W\})$$

Maximize (Subjective) Expected Utility





$$\sum_{w \in W} P_A(w) * u(A(w)) > \sum_{w \in W} P_A(w) * u(B(w))$$

Subjective Expected Utility

Probability: Suppose that $W = \{w_1, \ldots, w_n\}$ is a finite set of states. A probability function on W is a function $P : W \to [0,1]$ where $\sum_{w \in W} P(w) = 1$ (i.e., $P(w_1) + P(w_2) + \cdots + P(w_n) = 1$).

Suppose that A is an act for a set of outcomes O (i.e., $A : W \to O$). The **expected utility** of A is:

$$\sum_{w \in W} P(w) * u(A(w))$$

Two issues

- Utility is unique up to linear transformations

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- Probabilities depends, in part, on the description of the problem

Making an omelet

	Good egg (s_1)	Bad egg (s_2)
Break into a bowl (A_1)	six egg omelet (o_1)	no omelet and five good eggs destroyed (o_2)
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3)	five egg omelet and a cup to wash (o_4)
Throw away (A_3)	five egg omelet and a good egg destroyed (o_5)	five egg omelet (o_6)

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **6**	no omelet and five good eggs destroyed (o_2) **1**
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3) **4**	five egg omelet and a cup to wash (o_4) **3**
Throw away (A_3)	five egg omelet and a good egg destroyed (o_5) **2**	five egg omelet (o_6) **5**

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$u(o_1) = 6, u(o_6) = 5, u(o_3) = 4, u(o_4) = 3, u(o_5) = 2, u(o_2) = 1$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **6**	no omelet and five good eggs destroyed (o_2) **1**
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$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$EU(A_1) = P(s_1) * u(A_1(s_1)) + P(s_2) * u(A_1(s_2)) = 0.8 * 6 + 0.2 * 1 = 5.0$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **6**	no omelet and five good eggs destroyed (o_2) **1**
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3) **4**	five egg omelet and a cup to wash (o_4) **3**
Throw away (A_3)	five egg omelet and a good egg destroyed (o_5) **2**	five egg omelet (o_6) **5**

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$EU(A_2) = P(s_1) * u(A_2(s_1)) + P(s_2) * u(A_2(s_2)) = 0.8 * 4 + 0.2 * 3 = 3.8$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **6**	no omelet and five good eggs destroyed (o_2) **1**
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3) **4**	five egg omelet and a cup to wash (o_4) **3**
Throw away (A_3)	five egg omelet and a good egg destroyed (o_5) **2**	five egg omelet (o_6) **5**

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$EU(A_3) = P(s_1) * u(A_3(s_1)) + P(s_2) * u(A_3(s_2)) = 0.8 * 2 + 0.2 * 5 = 2.6$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **6**	no omelet and five good eggs destroyed (o_2) **1**
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3) **4**	five egg omelet and a cup to wash (o_4) **3**
Throw away (A_3)	five egg omelet and a good egg destroyed (o_5) **2**	five egg omelet (o_6) **5**

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$EU(A_1) = 5 > EU(A_2) = 3.8 > EU(A_3) = 2.6$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **6**	no omelet and five good eggs destroyed (o_2) **0**
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3) **5.5**	five egg omelet and a cup to wash (o_4) **5**
Throw away (A_3)	five egg omelet and a good egg destroyed (o_5) **4.75**	five egg omelet (o_6) **5.75**

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$u(o_1) = 6, u(o_6) = 5.75, u(o_3) = 5.5, u(o_4) = 5, u(o_5) = 4.75, u(o_2) = 0$$

Making an omelet

	Good egg (s_1) **0.8**	Bad egg (s_2) **0.2**
Break into a bowl (A_1)	six egg omelet (o_1) **6**	no omelet and five good eggs destroyed (o_2) **0**
Break into a cup (A_2)	six egg omelet and a cup to wash (o_3) **5.5**	five egg omelet and a cup to wash (o_4) **5**
Throw away (A_3)	five egg omelet and a good egg destroyed (o_5) **4.75**	five egg omelet (o_6) **5.75**

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2 \qquad P(s_1) = 0.8, P(s_2) = 0.2$$

$$EU(A_2) = 5.4 > EU(A_3) = 4.95 > EU(A_1) = 4.8$$

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2$$

$$\begin{aligned} EU(A_1) &= 0.2 * u(o_1) + 0.8 * u(o_2) \\ EU(A_2) &= 0.2 * u(o_3) + 0.8 * u(o_4) \\ EU(A_3) &= 0.2 * u(o_5) + 0.8 * u(o_6) \end{aligned}$$

	o_1	o_6	o_3	o_4	o_5	o_2	Expected Utility Ranking
u_1	6	5	4	3	2	1	$EU(A_1) > EU(A_2) > EU(A_3)$
u_2	6	5.75	5.5	5	4.75	0	$EU(A_2) > EU(A_3) > EU(A_1)$

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2$$

$$\begin{aligned} EU(A_1) &= 0.2 * u(o_1) + 0.8 * u(o_2) \\ EU(A_2) &= 0.2 * u(o_3) + 0.8 * u(o_4) \\ EU(A_3) &= 0.2 * u(o_5) + 0.8 * u(o_6) \end{aligned}$$

	o_1	o_6	o_3	o_4	o_5	o_2	Expected Utility Ranking
u_1	6	5	4	3	2	1	$EU(A_1) > EU(A_2) > EU(A_3)$
u_2	6	5.75	5.5	5	4.75	0	$EU(A_2) > EU(A_3) > EU(A_1)$
u_3	7.5	6.5	5.5	4.5	3.5	2.5	

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2$$

$$\begin{aligned} EU(A_1) &= 0.2 * u(o_1) + 0.8 * u(o_2) \\ EU(A_2) &= 0.2 * u(o_3) + 0.8 * u(o_4) \\ EU(A_3) &= 0.2 * u(o_5) + 0.8 * u(o_6) \end{aligned}$$

	o_1	o_6	o_3	o_4	o_5	o_2	Expected Utility Ranking
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	o_1	o_6	o_3	o_4	o_5	o_2	Expected Utility Ranking
u_1	6	5	4	3	2	1	$EU(A_1) > EU(A_2) > EU(A_3)$
u_2	6	5.75	5.5	5	4.75	0	$EU(A_2) > EU(A_3) > EU(A_1)$
u_3	7.5	6.5	5.5	4.5	3.5	2.5	$EU(A_1) > EU(A_2) > EU(A_3)$
u_4	12	10	8	6	4	2	

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2$$

$$\begin{aligned} EU(A_1) &= 0.2 * u(o_1) + 0.8 * u(o_2) \\ EU(A_2) &= 0.2 * u(o_3) + 0.8 * u(o_4) \\ EU(A_3) &= 0.2 * u(o_5) + 0.8 * u(o_6) \end{aligned}$$

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u_1	6	5	4	3	2	1	$EU(A_1) > EU(A_2) > EU(A_3)$
u_2	6	5.75	5.5	5	4.75	0	$EU(A_2) > EU(A_3) > EU(A_1)$
u_3	7.5	6.5	5.5	4.5	3.5	2.5	$EU(A_1) > EU(A_2) > EU(A_3)$
u_4	12	10	8	6	4	2	$EU(A_1) > EU(A_2) > EU(A_3)$

$$o_1 \succ o_6 \succ o_3 \succ o_4 \succ o_5 \succ o_2$$

$$\begin{aligned} EU(A_1) &= 0.2 * u(o_1) + 0.8 * u(o_2) \\ EU(A_2) &= 0.2 * u(o_3) + 0.8 * u(o_4) \\ EU(A_3) &= 0.2 * u(o_5) + 0.8 * u(o_6) \end{aligned}$$

	o_1	o_6	o_3	o_4	o_5	o_2	Expected Utility Ranking
u_1	6	5	4	3	2	1	$EU(A_1) > EU(A_2) > EU(A_3)$
u_2	6	5.75	5.5	5	4.75	0	$EU(A_2) > EU(A_3) > EU(A_1)$
u_3	7.5	6.5	5.5	4.5	3.5	2.5	$EU(A_1) > EU(A_2) > EU(A_3)$
u_4	12	10	8	6	4	2	$EU(A_1) > EU(A_2) > EU(A_3)$

Two issues

- ✓ Utility is unique up to linear transformations
- ▸ Probabilities depends, in part, on the description of the problem

Imagine that you are a paparazzi photographer and that rumor has it that actress Julia Roberts will show up in either New York (NY), Los Angeles (LA) or Paris (P). Nothing is known about the probability of these states of the world. You have to decide if you should stay in America or catch a plane to Paris. If you stay and actress Julia Roberts shows up in Paris you get $0; otherwise you get your photos, which you will be able to sell for $10,000. If you catch a plane to Paris and Julia Roberts shows up in Paris your net gain after having paid for the ticket is $5,000, and if she shows up in America you for some reason, never mind why, get $6,000.

	s_1	s_2	s_3
A	0	10	10
B	5	6	6

	s_1	s_2	s_3
A	0	10	10
B	5	6	6

$$EU(A) = \frac{1}{3} \times 0 + \frac{1}{3} \times 10 + \frac{1}{3} \times 10 = 6.667$$

$$EU(B) = \frac{1}{3} \times 5 + \frac{1}{3} \times 6 + \frac{1}{3} \times 6 = 5.667$$

	s_1	s_2	s_3
A	0	10	10
B	5	6	6

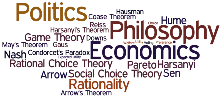
	s_1	t
A	0	10
B	5	6

	s_1	t
A	0	10
B	5	6

$$EU(A) = \frac{1}{2} \times 0 + \frac{1}{2} \times 10 = 5$$

$$EU(B) = \frac{1}{2} \times 5 + \frac{1}{2} \times 6 = 5.5$$

Two issues



- ✓ Utility is unique up to linear transformations
- ✓ Probabilities depends, in part, on the description of the problem

Three issues

- ✓ Utility is unique up to linear transformations
- ✓ Probabilities depends, in part, on the description of the problem
- ▸ The probability of states are *independent* of the chosen act