# Methods in Philosophy, Politics and Economics: Individual and Group Decision Making 

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# Independence Axiom 

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\left[B_{2}: p_{2}, B_{3}: p_{3}\right] \succ / \sim / \prec\left[B_{2}: p_{2}, B_{3}: p_{3}\right]
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\left[A_{1}: p_{1}, B_{2}: p_{2}, B_{3}: p_{3}\right]
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\begin{aligned}
& {\left[A_{1}: p_{1}, B_{2}: p_{2}, B_{3}: p_{3}\right] \succ / \sim / \prec\left[A_{1}: p_{1}, B_{2}: p_{2}, B_{3}: p_{3}\right] } \\
& \text { iff } \\
& {\left[B_{2}: p_{2}, B_{3}: p_{3}\right] } \sim / \prec\left[B_{2}: p_{2}, B_{3}: p_{3}\right]
\end{aligned}
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## Independence

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For all $L_{1}, L_{2}, L_{3} \in \mathcal{L}$ and $a \in(0,1]$,

$$
L_{1} \succ L_{2} \text { if, and only if, }\left[L_{1}: a, L_{3}:(1-a)\right] \succ\left[L_{2}: a, L_{3}:(1-a)\right] .
$$

$$
L_{1} \sim L_{2} \text { if, and only if, }\left[L_{1}: a, L_{3}:(1-a)\right] \sim\left[L_{2}: a, L_{3}:(1-a)\right] .
$$

## Better Prizes

Better prizes: When two lotteries are the same except for one outcome, then the decision maker prefers the lottery with the better outcome.


## Better Chances

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Better Chances: A decision maker prefers a better chance for a better prize


## Better Chances

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a \succ b
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$$
p>q, \text { so } p=q+r \text { and }(1-q)=(1-p)+r \text { for some } r
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## Better Chances

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Better Chances: A decision maker prefers a better chance for a better prize


If $L_{1} \succeq L_{2}$, then for all $p,\left[L_{1}: 1\right] \succeq\left[L_{1}: p, L_{2}:(1-p)\right]$

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## Describing the Outcomes

Suppose you have a kitten, which you plan to give away to either Ann or Bob. Ann and Bob both want the kitten very much. Both are deserving, and both would care for the kitten. You are sure that giving the kitten to Ann $(x)$ is at least as good as giving the kitten to $\operatorname{Bob}(y)$ (so $x \succeq y$ ). But you think that would be unfair to Bob. You decide to flip a fair coin: if the coin lands heads, you will give the kitten to Bob, and if it lands tails, you will give the kitten to Ann.
(J. Drier, "Morality and Decision Theory" in Handbook of Rationality)


- $x$ is the outcome "Ann gets the kitten"
- $y$ is the outcome "Bob gets the kitten"

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- $x$ is the outcome "Ann gets the kitten, in a fair way"
- $y$ is the outcome "Bob gets the kitten"

- $x$ is the outcome "Ann gets the kitten"
- $z$ is the outcome "Ann gets the outcome, fairly
- $y$ is the outcome "Bob gets the kitten, fairly"


If all the agent cares about is who gets the kitten, then $L_{1} \succeq L_{2}$
If all the agent cares about is being fair, then $L_{1} \preceq L_{2}$

# Continuity Axiom 

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\begin{aligned}
& U\left(L_{1}\right)=r_{1} \\
& U\left(L_{2}\right)=r_{2} \\
& U\left(L_{3}\right)=r_{3}
\end{aligned}
$$


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\begin{aligned}
& U\left(L_{1}\right)=r_{1} U\left(\left[L_{1}: 1\right]\right) \\
& U\left(L_{2}\right)=r_{2} \\
& U\left(L_{3}\right)=\left.r_{3}\right|_{0} U\left(\left[L_{3}: 1\right]\right)
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$u: \mathcal{L} \rightarrow \mathbb{R}$ is linear provided for all $L=\left[L_{1}: p_{1}, \ldots, L_{n}: p_{n}\right] \in \mathcal{L}$,

$$
u(L)=\sum_{i=1}^{n} p_{i} \times u\left(L_{i}\right)
$$

von Neumann-Morgenstern Representation Theorem A binary relation $\succeq$ on $\mathcal{L}$ satisfies Preference, Compound Lotteries, Independence and Continuity if, and only if, $\succeq$ is representable by a linear utility function $u: \mathcal{L} \rightarrow \mathbb{R}$.
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Moreover, $u^{\prime}: \mathcal{L} \rightarrow \mathbb{R}$ represents $\succeq$ iff there exists real numbers $c>0$ and $d$ such that $u^{\prime}(\cdot)=c u(\cdot)+d$. (" $u$ is unique up to linear transformations.") Nas shemen mo Rational choote Theory peratethisisn Arrow Social Choice
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Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agent's ordinal utility function can be turned into cardinal utility function.

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- Issue with continuity: $\$ 1 \succ 1$ cent $\succ$ death, but who would accept a lottery which is $p$ for $\$ 1$ and $(1-p)$ for death??
- Important issues about how to identify correct descriptions of the outcomes and options.


## The Two Envelop Paradox

Suppose that you have a choice between two envelops, each containing some money. A trustworthy informant tells you that one of the envelops contains exactly twice as much as the other, but not which is which. Since this is all you know, you pick an envelop at random. Just before you open the envelop, you are given the opportunity to switch envelops. Should you swap?

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Yes: Suppose the chosen envelop has $\$ x$. The other envelop has either $\frac{1}{2} \cdot x$ dollars or $2 \cdot x$ dollars. Each is equally likely, so the expected utility of switching is

$$
\frac{1}{2} \cdot \frac{1}{2} \cdot x+\frac{1}{2} \cdot 2 \cdot x=1.25 \cdot x
$$

## Objections

- No action guidance. Rational decision makers do not prefer an act because its expected utility is favorable, but can only be described as if they were acting from this principle.


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- No action guidance. Rational decision makers do not prefer an act because its expected utility is favorable, but can only be described as if they were acting from this principle.
- Utility without chance. It seems rather odd from a linguistic point of view to say that the meaning of utility has something to do with preferences over lotteries.
- The axioms are too strong. Do rational decisions have to obey these axioms?

