Methods in Philosophy, Politics and Economics: Individual and Group Decision Making

Eric Pacuit
University of Maryland
pacuit.org

Independence Axiom

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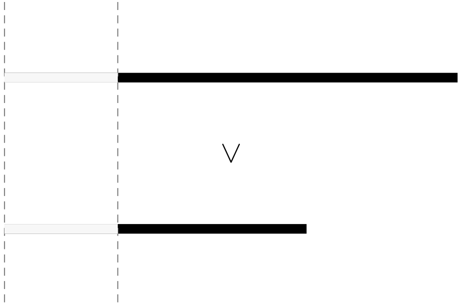

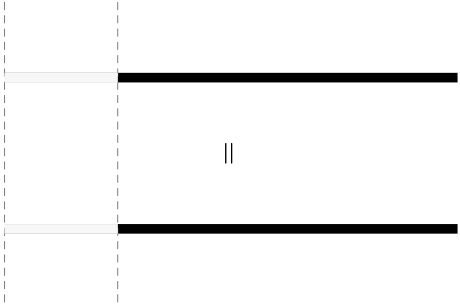

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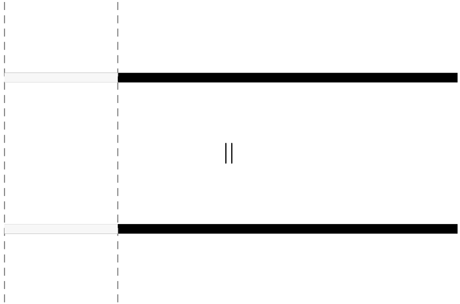

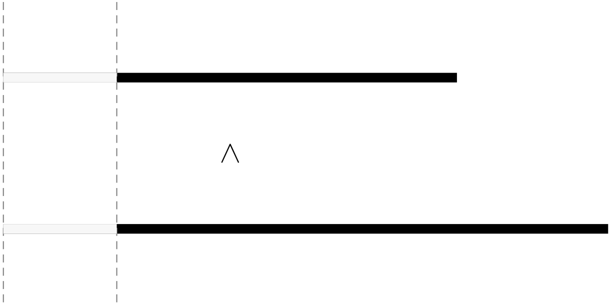

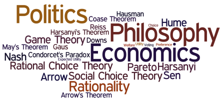

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$$L = [A_1 : p_1, A_2 : p_2, A_3 : p_3]$$

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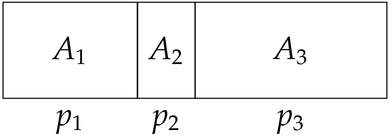

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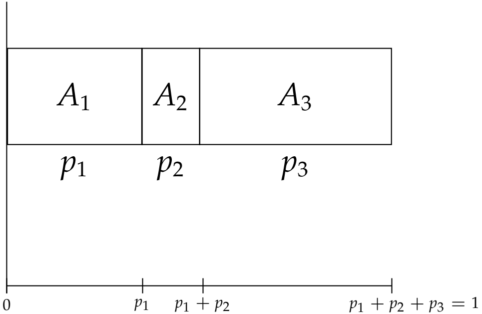

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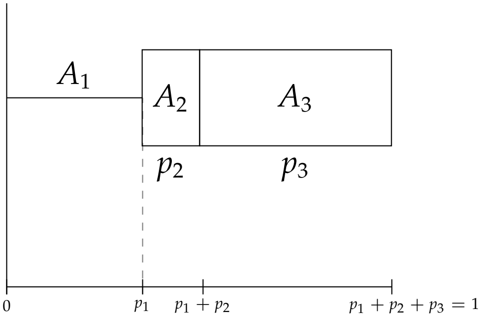

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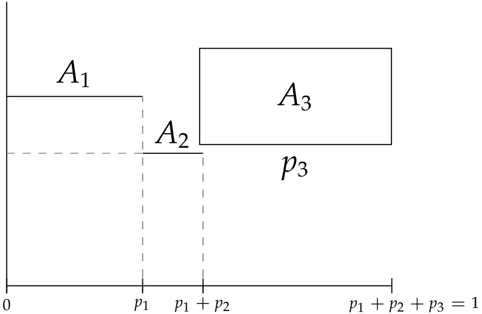

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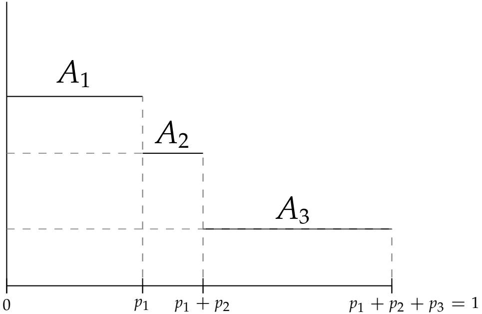

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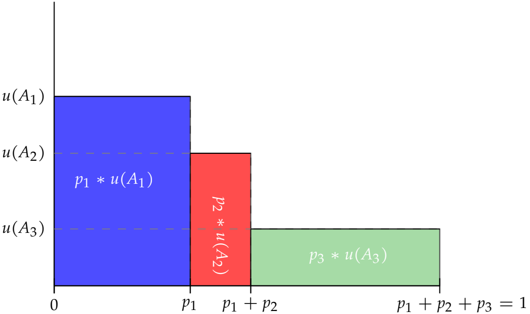

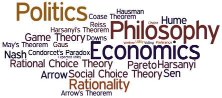

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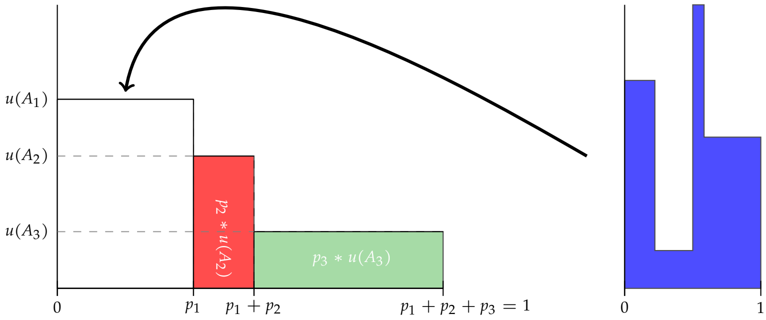

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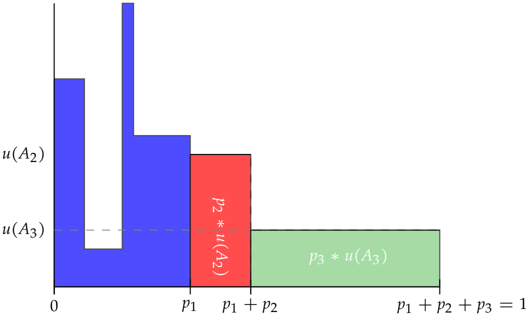
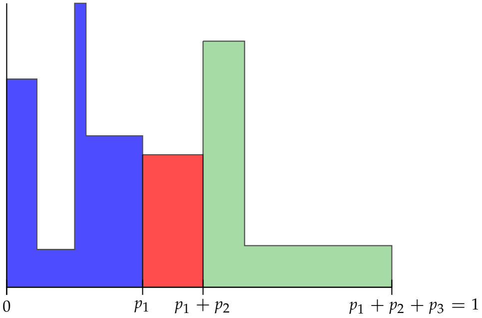
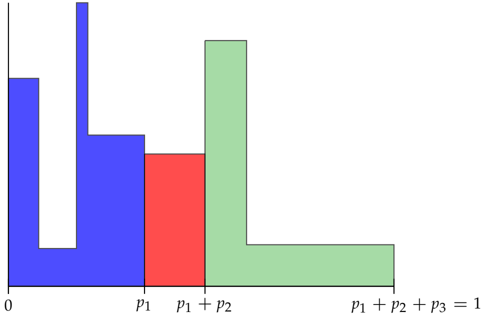


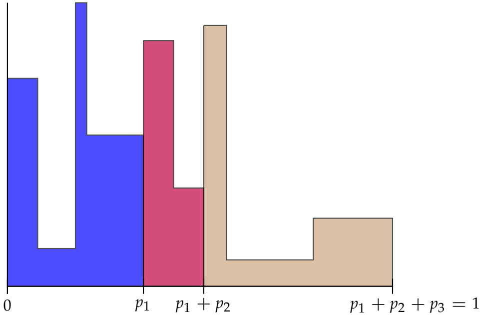
$$L = [A_1 : p_1, A_2 : p_2, A_3 : p_3]$$



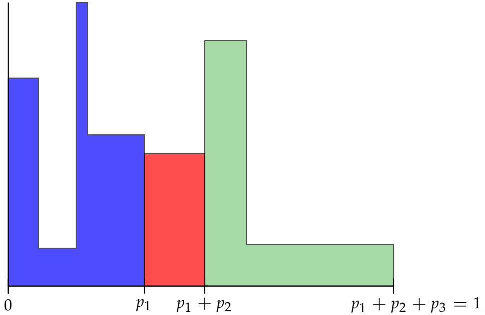
$[A_1 : p_1, A_2 : p_2, A_3 : p_3]$

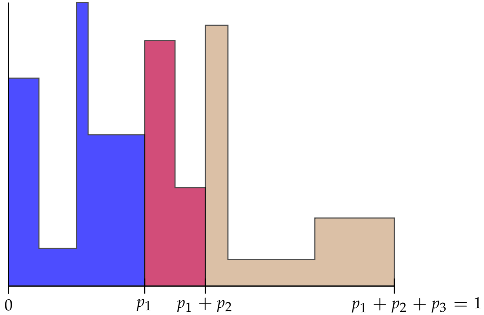


$[A_1 : p_1, B_2 : p_2, B_3 : p_3]$



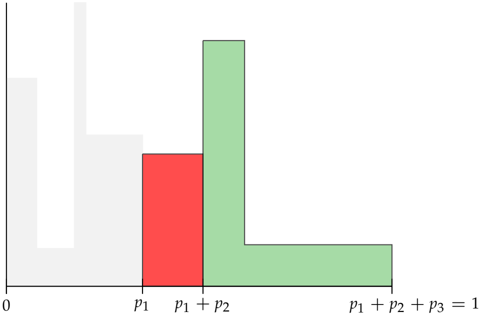
$[A_1 : p_1, A_2 : p_2, A_3 : p_3]$ $[A_1 : p_1, B_2 : p_2, B_3 : p_3]$



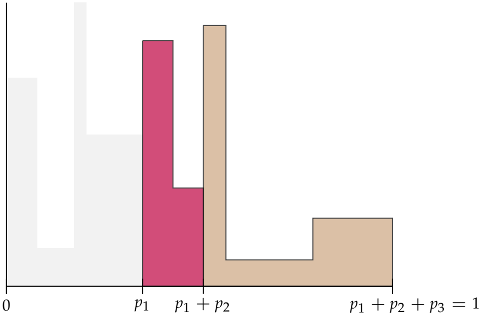


$$[A_1 : p_1, B_2 : p_2, B_3 : p_3] \succ [A_1 : p_1, B_2 : p_2, B_3 : p_3]$$

$[A_1 : p_1, A_2 : p_2, A_3 : p_3]$



$[A_1 : p_1, B_2 : p_2, B_3 : p_3]$

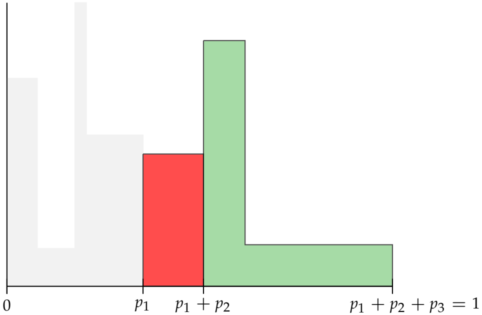


$$[A_1 : p_1, B_2 : p_2, B_3 : p_3] \succ [A_1 : p_1, B_2 : p_2, B_3 : p_3]$$

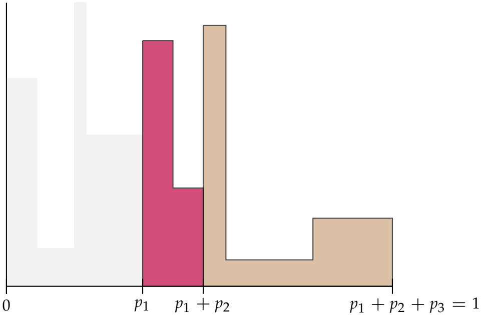
iff

$$[B_2 : p_2, B_3 : p_3] \succ [B_2 : p_2, B_3 : p_3]$$

$$[A_1 : p_1, A_2 : p_2, A_3 : p_3]$$



$$[A_1 : p_1, B_2 : p_2, B_3 : p_3]$$

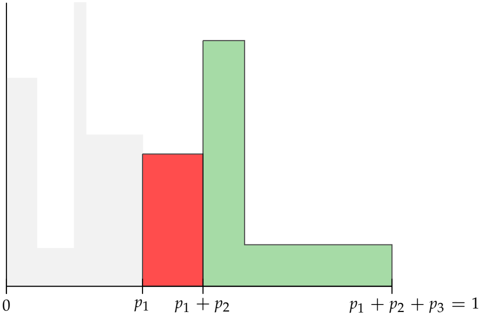


$$[A_1 : p_1, B_2 : p_2, B_3 : p_3] \sim [A_1 : p_1, B_2 : p_2, B_3 : p_3]$$

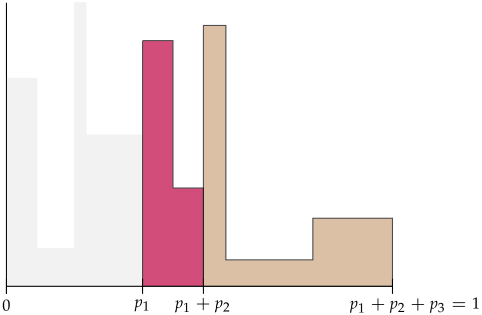
iff

$$[B_2 : p_2, B_3 : p_3] \sim [B_2 : p_2, B_3 : p_3]$$

$[A_1 : p_1, A_2 : p_2, A_3 : p_3]$



$[A_1 : p_1, B_2 : p_2, B_3 : p_3]$

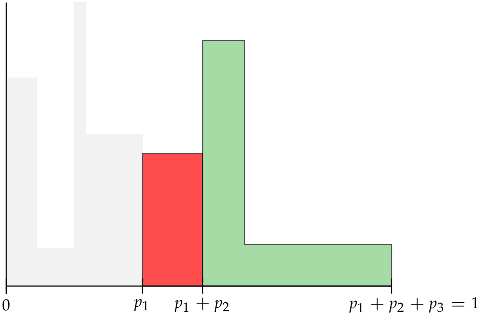


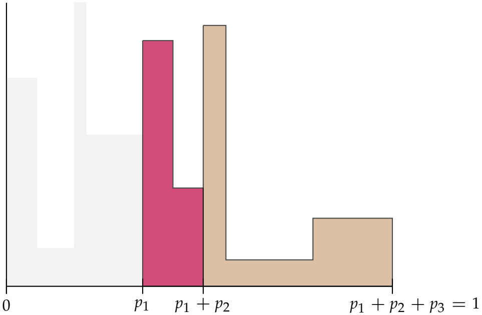
$$[A_1 : p_1, B_2 : p_2, B_3 : p_3] \prec [A_1 : p_1, B_2 : p_2, B_3 : p_3]$$

iff

$$[B_2 : p_2, B_3 : p_3] \prec [B_2 : p_2, B_3 : p_3]$$

$[A_1 : p_1, A_2 : p_2, A_3 : p_3]$ $[A_1 : p_1, B_2 : p_2, B_3 : p_3]$



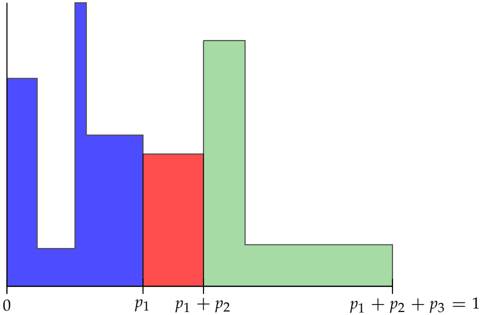


$$[A_1 : p_1, B_2 : p_2, B_3 : p_3] \;\succ / \sim / \prec\; [A_1 : p_1, B_2 : p_2, B_3 : p_3]$$

iff

$$[B_2 : p_2, B_3 : p_3] \;\succ / \sim / \prec\; [B_2 : p_2, B_3 : p_3]$$

$[A_1 : p_1, A_2 : p_2, A_3 : p_3]$



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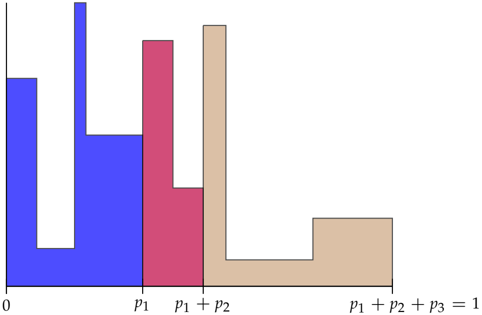


$$[A_1 : p_1, B_2 : p_2, B_3 : p_3] \ \succ / \sim / \prec \ [A_1 : p_1, B_2 : p_2, B_3 : p_3]$$

iff

$$[B_2 : p_2, B_3 : p_3] \ \succ / \sim / \prec \ [B_2 : p_2, B_3 : p_3]$$

Independence

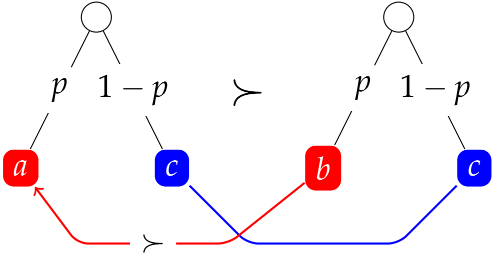
For all $L_1, L_2, L_3 \in \mathcal{L}$ and $a \in (0,1]$,

$$L_1 \succ L_2 \text{ if, and only if, } [L_1 : a,\ L_3 : (1-a)] \succ [L_2 : a,\ L_3 : (1-a)].$$

$$L_1 \sim L_2 \text{ if, and only if, } [L_1 : a,\ L_3 : (1-a)] \sim [L_2 : a,\ L_3 : (1-a)].$$

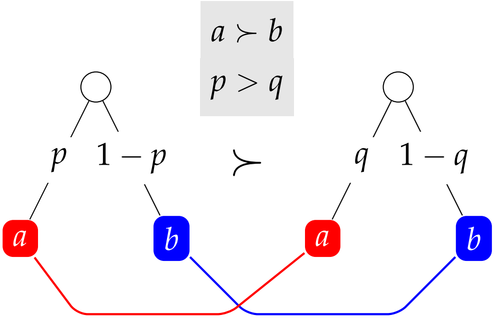
Better Prizes

Better prizes: When two lotteries are the same except for one outcome, then the decision maker prefers the lottery with the better outcome.



Better Chances

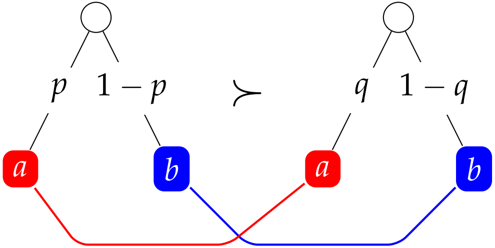
Better Chances: A decision maker prefers a better chance for a better prize



Better Chances

$$a \succ b$$

$$p > q, \text{ so } p = q + r \text{ and } (1 - q) = (1 - p) + r \text{ for some } r$$



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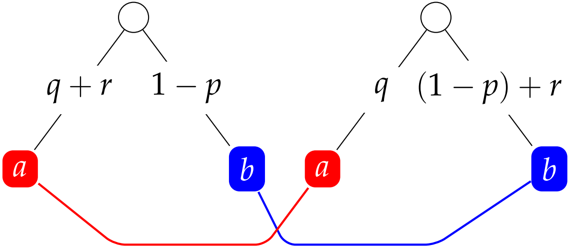

Better Chances

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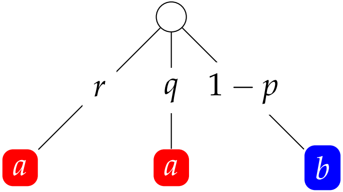

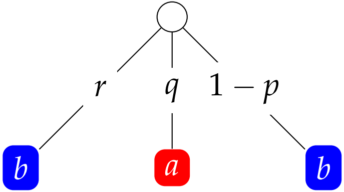

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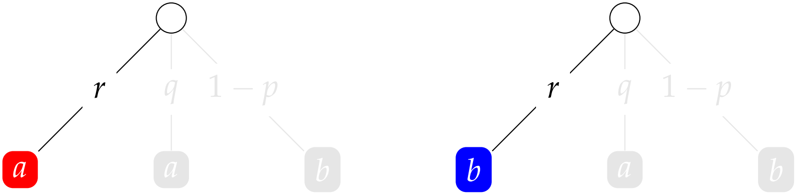
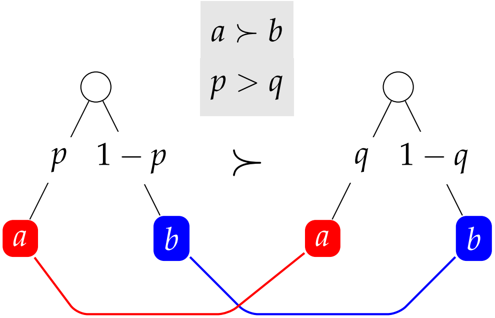


Better Chances

Better Chances: A decision maker prefers a better chance for a better prize



If $L_1 \succeq L_2$, then for all p, $[L_1 : 1] \succeq [L_1 : p, L_2 : (1-p)]$

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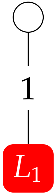
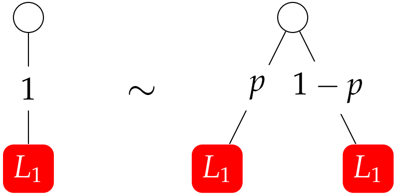


If $L_1 \succeq L_2$, then for all p, $[L_1 : 1] \succeq [L_1 : p, L_2 : (1-p)]$



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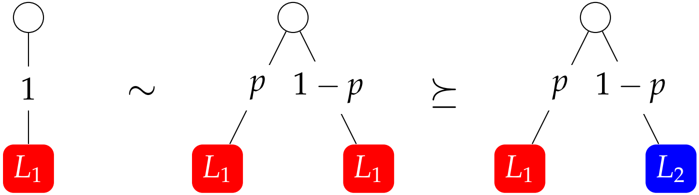
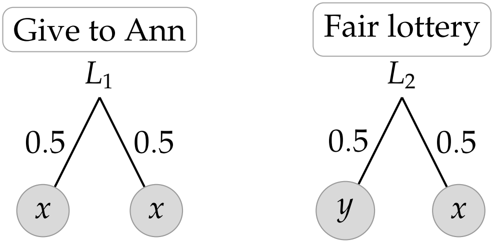


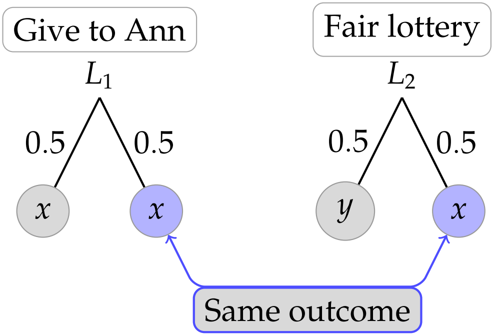
Describing the Outcomes

Suppose you have a kitten, which you plan to give away to either Ann or Bob. Ann and Bob both want the kitten very much. Both are deserving, and both would care for the kitten. You are sure that giving the kitten to Ann (x) is at least as good as giving the kitten to Bob (y) (so $x \succeq y$). But you think that would be unfair to Bob. You decide to flip a fair coin: if the coin lands heads, you will give the kitten to Bob, and if it lands tails, you will give the kitten to Ann.

(J. Drier, "Morality and Decision Theory" in *Handbook of Rationality*)



- x is the outcome “Ann gets the kitten”
- y is the outcome “Bob gets the kitten”



- x is the outcome “Ann gets the kitten”
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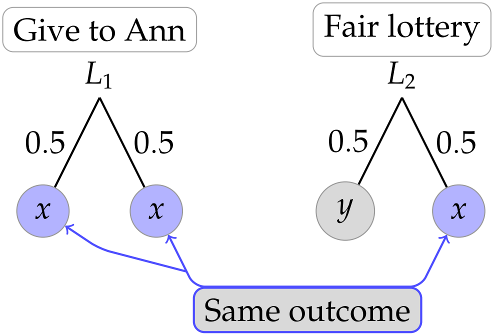
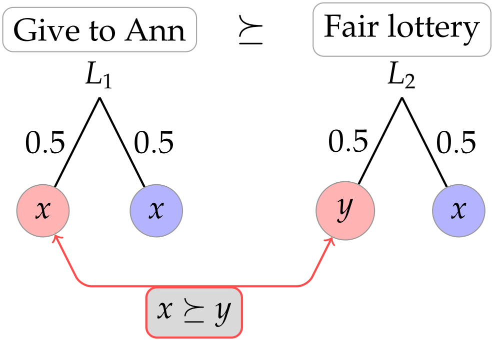
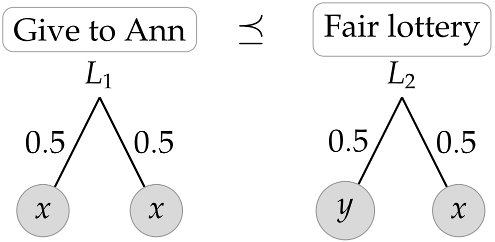


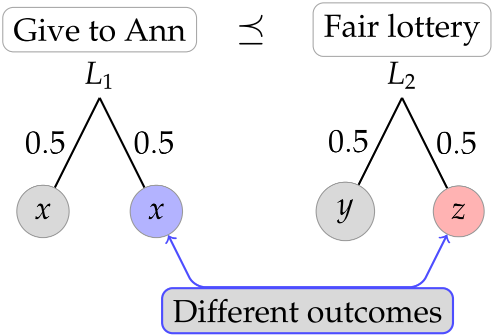
- x is the outcome "Ann gets the kitten"
- y is the outcome "Bob gets the kitten"



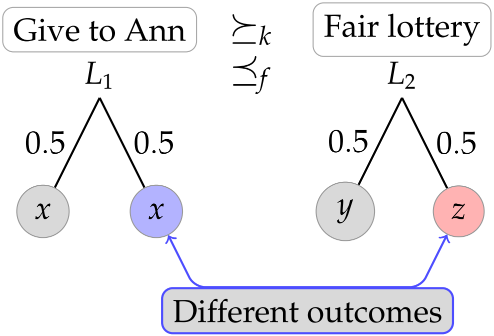
- x is the outcome "Ann gets the kitten"
- y is the outcome "Bob gets the kitten"



- x is the outcome “Ann gets the kitten, *in a fair way*”
- y is the outcome “Bob gets the kitten”



- x is the outcome “Ann gets the kitten”
- z is the outcome “Ann gets the outcome, *fairly*
- y is the outcome “Bob gets the kitten, *fairly*”



If all the agent cares about is who gets the kitten, then $L_1 \succeq L_2$

If all the agent cares about is being fair, then $L_1 \preceq L_2$

Continuity Axiom

$$L_1 \succ L_2 \succ L_3$$

L_1

L_2

L_3

$$U(L_1) = r_1$$

$$U(L_2) = r_2$$

$$U(L_3) = r_3$$

0

$U(L_1) = r_1$ | $U([L_1 : 1])$

$U(L_2) = r_2$

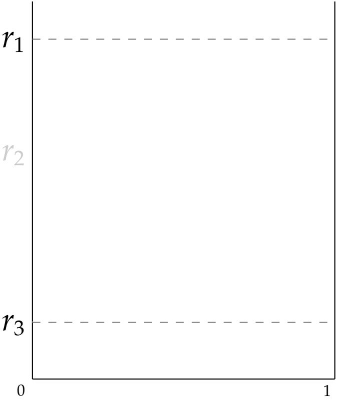
$U(L_3) = r_3$ | $U([L_3 : 1])$

0

$$U(L_1) = r_1 \qquad U([L_1 : 1]) = U([L_1 : 1, L_3 : 0])$$

$$U(L_2) = r_2$$

$$U(L_3) = r_3 \qquad U([L_3 : 1]) = U([L_1 : 0, L_3 : 1])$$

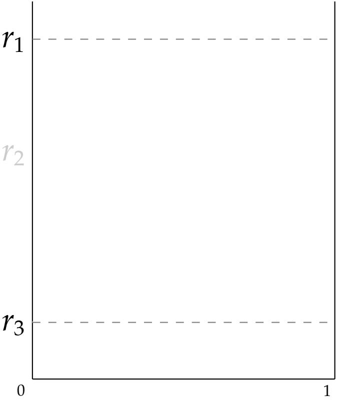


$$U(L_1) = r_1 \qquad U([L_1 : 1]) = U([L_1 : 1, L_3 : 0])$$

$$U(L_2) = r_2$$

$$U([L_1 : p, L_3 : (1-p)])$$

$$U(L_3) = r_3 \qquad U([L_3 : 1]) = U([L_1 : 0, L_3 : 1])$$

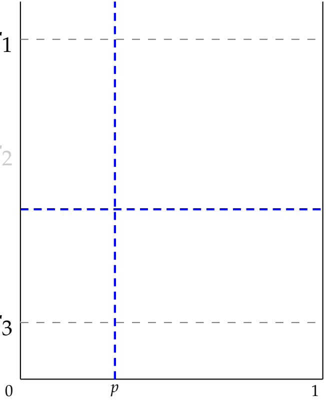


$U(L_1) = r_1$ $\qquad U([L_1 : 1]) = U([L_1 : 1, L_3 : 0])$

$U(L_2) = r_2$

$U([L_1 : p, L_3 : (1 - p)])$

$U(L_3) = r_3$ $\qquad U([L_3 : 1]) = U([L_1 : 0, L_3 : 1])$



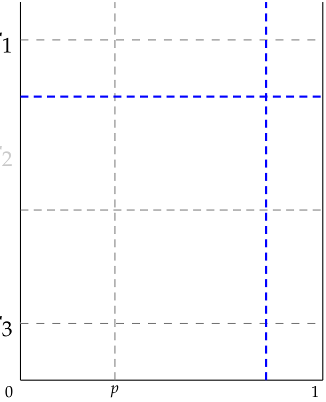
$$U(L_1) = r_1 \qquad U([L_1 : 1]) = U([L_1 : 1, L_3 : 0])$$

$$U([L_1 : q, L_3 : (1-q)])$$

$$U(L_2) = r_2$$

$$U([L_1 : p, L_3 : (1-p)])$$

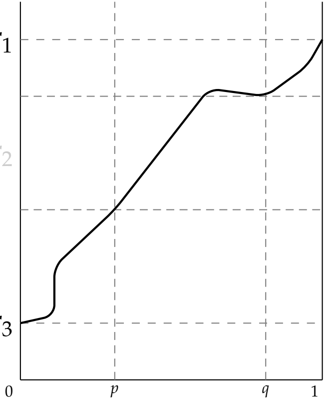
$$U(L_3) = r_3 \qquad U([L_3 : 1]) = U([L_1 : 0, L_3 : 1])$$



$U(L_1) = r_1$

$U(L_2) = r_2$

$U(L_3) = r_3$



$U([L_1 : 1]) = U([L_1 : 1, L_3 : 0])$

$U([L_1 : q, L_3 : (1-q)])$

$U([L_1 : p, L_3 : (1-p)])$

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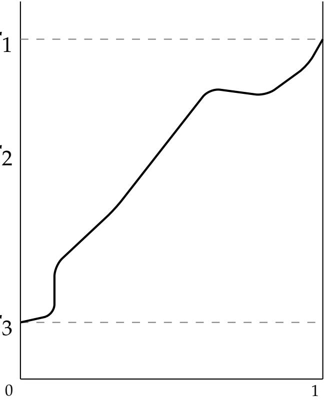

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$$U(L_2) = r_2 \qquad U([L_1 : a, L_3 : (1-a)])$$

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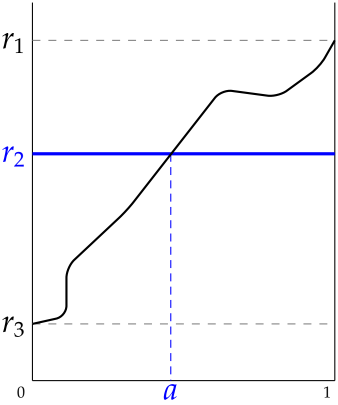

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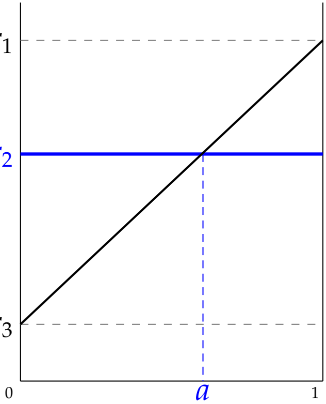
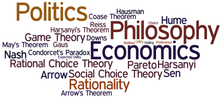


$u : \mathcal{L} \to \mathbb{R}$ is linear provided for all $L = [L_1 : p_1, \ldots, L_n : p_n] \in \mathcal{L}$,

$$u(L) = \sum_{i=1}^{n} p_i \times u(L_i)$$

von Neumann-Morgenstern Representation Theorem A binary relation $\succeq$ on $\mathcal{L}$ satisfies Preference, Compound Lotteries, Independence and Continuity if, and only if, $\succeq$ is representable by a linear utility function $u : \mathcal{L} \to \mathbb{R}$.



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Moreover, $u' : \mathcal{L} \to \mathbb{R}$ represents $\succeq$ iff there exists real numbers $c > 0$ and d such that $u'(\cdot) = cu(\cdot) + d$. ("$u$ is unique up to linear transformations.")

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- Important issues about how to identify correct descriptions of the outcomes and options.

The Two Envelop Paradox

Suppose that you have a choice between two envelops, each containing some money. A trustworthy informant tells you that one of the envelops contains exactly twice as much as the other, but not which is which. Since this is all you know, you pick an envelop at random. Just before you open the envelop, you are given the opportunity to switch envelops. Should you swap?

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Yes: Suppose the chosen envelop has \$$x$. The other envelop has either $\frac{1}{2} \cdot x$ dollars or $2 \cdot x$ dollars. Each is equally likely, so the expected utility of switching is

$$\frac{1}{2} \cdot \frac{1}{2} \cdot x + \frac{1}{2} \cdot 2 \cdot x = 1.25 \cdot x$$

Objections

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- Utility without chance. It seems rather odd from a linguistic point of view to say that the *meaning* of utility has something to do with preferences over lotteries.
- The axioms are too strong. Do rational decisions *have* to obey these axioms?