Methods in Philosophy, Politics and Economics: Individual and Group Decision Making

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$u : \mathcal{L} \to \mathbb{R}$ is linear provided for all $L = [L_1 : p_1, \ldots, L_n : p_n] \in \mathcal{L}$,

$$u(L) = \sum_{i=1}^{n} p_i \times u(L_i)$$

von Neumann-Morgenstern Representation Theorem A binary relation $\succeq$ on $\mathcal{L}$ satisfies Preference, Compound Lotteries, Independence and Continuity if, and only if, $\succeq$ is representable by a linear utility function $u : \mathcal{L} \to \mathbb{R}$.

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Moreover, $u' : \mathcal{L} \to \mathbb{R}$ represents $\succeq$ iff there exists real numbers $c > 0$ and d such that $u'(\cdot) = cu(\cdot) + d$. ("$u$ is unique up to linear transformations.")

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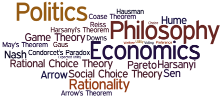

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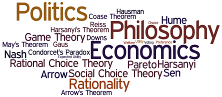

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- Utility is unique only *up to linear transformations*.
- Issue with continuity: \$1 $\succ$ 1 cent $\succ$ death, but who would accept a lottery which is p for \$1 and $(1-p)$ for death??
- Important issues about how to identify correct descriptions of the outcomes and options.

Objections

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- The axioms are too strong. Do rational decisions *have* to obey these axioms?

Allais Paradox

		Red (1)	White (89)	Blue (10)
S_1	A	$1M$	$1M$	$1M$
	B	0	$1M$	$5M$

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		Red (1)	White (89)	Blue (10)
S_2	C	$1M$	0	$1M$
	D	0	0	$5M$

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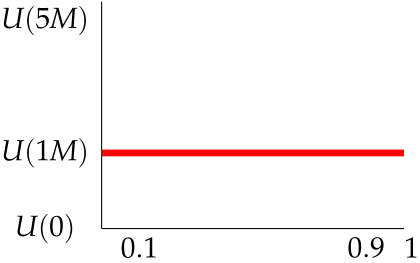

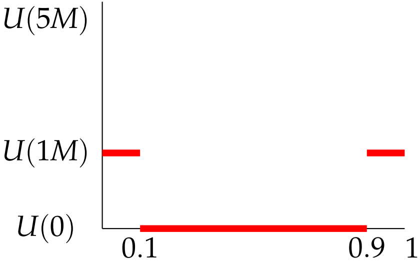

$[\ 1M : 0.01, \quad 1M : 0.89, \quad 1M : 0.01\ ]$

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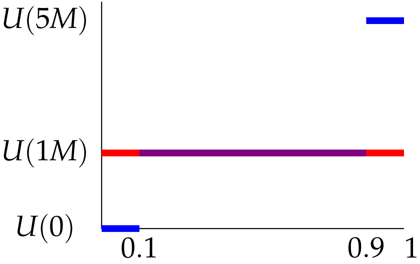

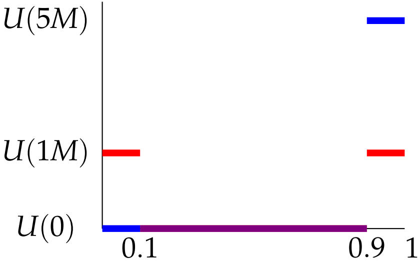

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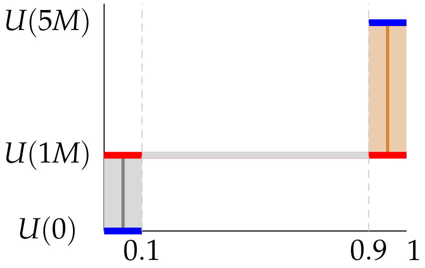

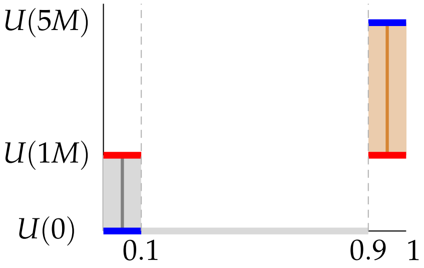

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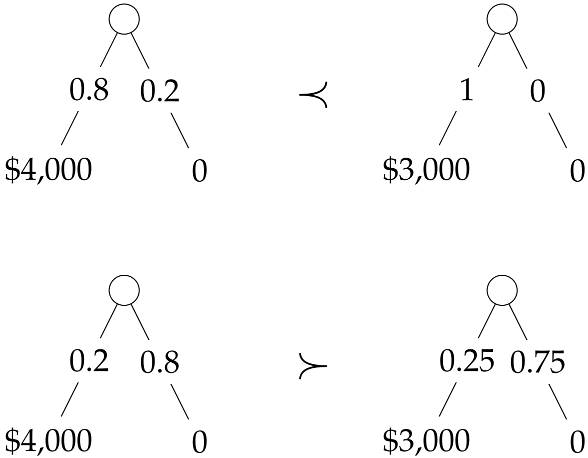
$$A \succeq B \text{ iff } C \succeq D$$

Independence

Independence For all $L_1, L_2, L_3 \in \mathcal{L}$ and $a \in (0, 1]$,

$$L_1 \succ L_2 \text{ if, and only if, } [L_1 : a, L_3 : (1-a)] \succ [L_2 : a, L_3 : (1-a)].$$

$$L_1 \sim L_2 \text{ if, and only if, } [L_1 : a, L_3 : (1-a)] \sim [L_2 : a, L_3 : (1-a)].$$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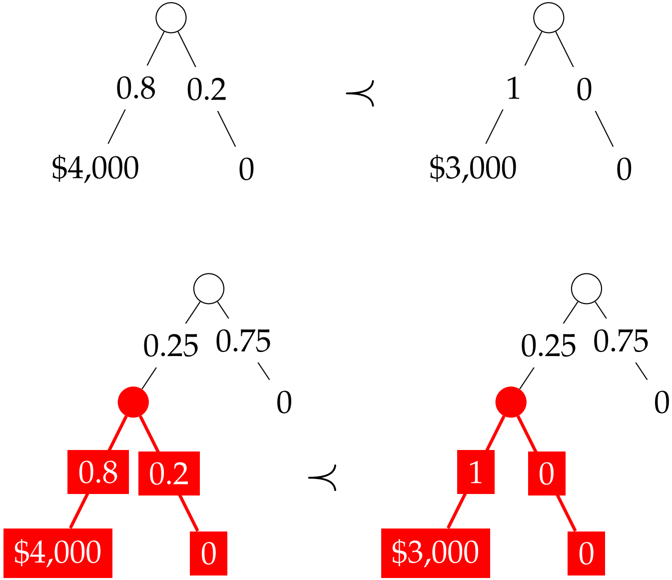
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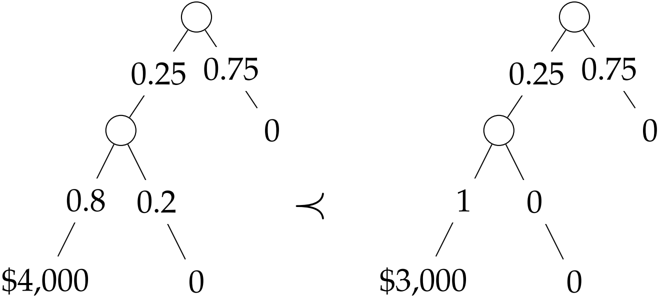
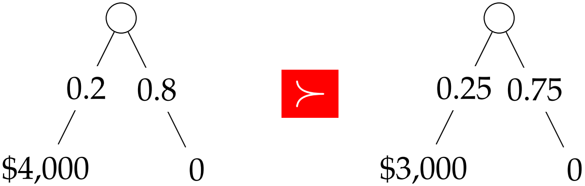
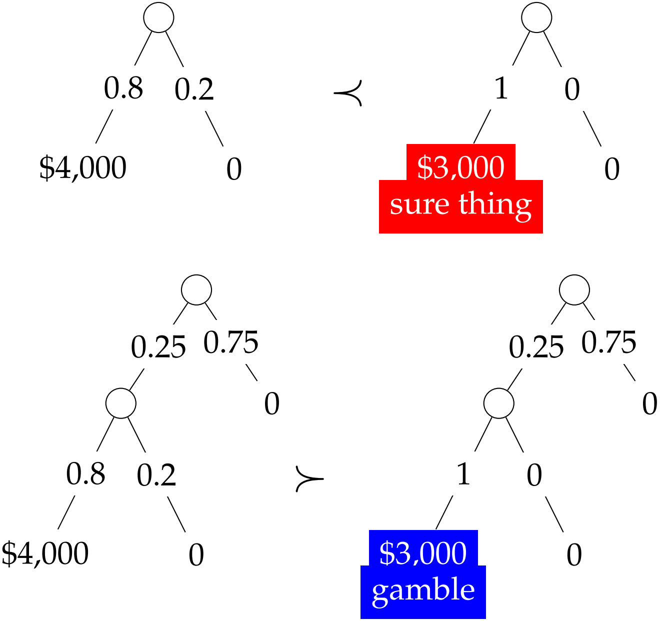




$\mathbf{0.25 * 0.8 = 0.2}$ $\mathbf{0.25 * 1 = 0.25}$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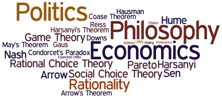
Allais Paradox



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(a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or

(b) those who choose A in S_1 and D is S_2 are irrational.

Rather, people's utility functions (*their rankings over outcomes*) are often far more complicated than the monetary bets would indicate....

L. Buchak. *Risk and Rationality*. Oxford University Press, 2013.

Ellsberg Paradox

	30	60	
Lotteries	Blue	Yellow	Green
L_1	$1M$	0	0
L_2	0	$1M$	0

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$$L_1 \succeq L_2 \text{ iff } L_3 \succeq L_4$$

Ambiguity Aversion

I. Gilboa and M. Marinacci. *Ambiguity and the Bayesian Paradigm*. Advances in Economics and Econometrics: Theory and Applications, Tenth World Congress of the Econometric Society. D. Acemoglu, M. Arellano, and E. Dekel (Eds.). New York: Cambridge University Press, 2013.

Flipping a fair coin vs. flipping a coin of unknown bias: “The probability is 50-50”...

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- Imprecise probabilities
- Non-additive probabilities
- Qualitative probability