## PHIL309P

# Methods in Philosophy, Politics and Economics 

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- Beliefs: How should we represent the decision makers beliefs about the decision problems (e.g., the available outcomes, menu items, consequences of actions, etc.). What makes a belief rational or reasonable?
- Preferences: How should we represent the decision maker's preferences about the available choices? What makes a preference rational or reasonable?


## Preferences

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Preferring or choosing $x$ is different that "liking" $x$ or "having a taste for $x$ ": one can prefer $x$ to $y$ but dislike both options

Preferences are always understood as comparative: "preference" is more like "bigger" than "big"

## Concepts of preference

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## Partial/Total/Overal Comparisons

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## Partial/Total/Overal Comparisons

1. Lauren drank water rather than wine with dinner, despite preferring to drink wine, because she promised her husband she would stay sober.
2. Lauren drank water with dinner because she preferred to do so. But for the promise she made her husband to stay sober, she would have preferred to drink wine rather than water with dinner.

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Preferences will be understood as mental rankings of alternatives "all things considered".

# Mathematically describing preferences 

## Mathematical background: Relations

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E.g., $X=\{a, b, c, d\}, R=\{(a, a),(b, a),(c, d),(a, c),(d, d)\}$

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\begin{aligned}
& a R a \\
& b R a \\
& c R d \\
& a R c \\
& d R d
\end{aligned}
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## Mathematical background: Relations

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Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Reflexive relation: for all $x \in X, x R x$

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| :--- |
| Rational Choice Theory ParetoHarsany | Arrowsocial Cholice

Suppose that $X$ is a set and $R \subseteq X \times X$ is a relation.
Complete relation: for all $x, y \in X$, either $x R y$ or $y R x$

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## Representing Preferences


 Arrow Social Choice
Rationality
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Let $X$ be a set of options/outcomes. A decision maker's preference over $X$ is represented by a relation $\succeq \subseteq X \times X$.

## Representing Preferences


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Given $x, y \in X$, there are four possibilities:

## Representing Preferences

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Given $x, y \in X$, there are four possibilities:

1. $x \succeq y$ and $y \nsucceq x$ : The decision maker ranks $x$ above $y$ (the decision maker strictly prefers $x$ to $y$ ).

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3. $x \succeq y$ and $y \succeq x$ : The agent is indifferent between $x$ and $y$.

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## Representing Preferences

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Suppose that $\succeq$ is a relation on $X$ (called the weak preference). Then, define the following:

- Strict preference: $x \succ y$ iff $x \succeq y$ and $y \nsucceq x$
- Indifference: $x \sim y$ iff $x \succeq y$ and $y \succeq x$
- Non-comparability $x N y$ iff $x \nsucceq y$ and $y \nsucceq x$


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What properties should weak/strict preference, indifference, non-comparability satisfy?

## Rational preferences

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A relation $\succeq \subseteq X \times X$ is a rational preference relation (for a decision maker) provided that

1. $\succeq$ is complete (and hence reflexive)
2. $\succeq$ is transitive

- What is the relationship between choice and preference?
- What makes a preference rational?
- Should a decision maker's preference be complete and transitive?
- Are people's preferences complete and transitive?


## Choices

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It is important to distinguish between mere behavior on the one hand and "action" or "choice" on the other.

## Choices

 Mas seme temo M Nonomics Nash Consorcets Paradox LCO Pareto Harsany Arrowsocia ChoiceIt is important to distinguish between mere behavior on the one hand and "action" or "choice" on the other.

Decisions are between beliefs and desires on the one hand and actions on the other. Whateme wisem ECOMOMICS ArowSocil chice theor owain
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Should preferences be identified with choices? wave neme thern Economics Nash benate feyme ArrowSocial Choice TheorySen ${ }_{\text {Rrows }}$ Rationality

Should preferences be identified with choices?

The verb "to prefer" can either mean "to choose" or "to like better," and these two senses are frequently confused in economic literature. That fact that an individual chooses $A$ rather than $B$ is far from conclusive evidence that he likes $A$ better. But whether he likes $A$ better or not should be completely irrelevant to the theory of price.
(Little, 1949).

## Preferences and Choices


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Preferences are closely related to choices: preferences may cause and to help to explain choices; preferences may be invoked to justify choices, in fortuitous circumstances, we can use preference data to make predictions about choice. But to identify the two would be a mistake.

## Preferences and Choices

 wens nemen wem Economics Nash condores Choice Theory ParetoHarsanyi Arrow Rationality- We have preferences over vastly more states of affairs than we can ever hope (or dread) to be in the position to choose.


## Preferences and Choices

Can't we stipulate a concept of preference that is only loosely based on our ordinary concept?

## Preferences and Choices

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Can't we stipulate a concept of preference that is only loosely based on our ordinary concept?
-What about counter-preferential choice?

## Preferences and Choices

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- What about counter-preferential choice?
- Preferences must be stable over a reasonable amount of time in a way that (observed) choices aren't (needed to predict and explain choices).


## Preferences and Choices

Can't we stipulate a concept of preference that is only loosely based on our ordinary concept?

- What about counter-preferential choice?
- Preferences must be stable over a reasonable amount of time in a way that (observed) choices aren't (needed to predict and explain choices).
- Beliefs and expectations over future states of affairs are needed in addition to preferences in order to explain choices. To banish preferences understood as mental rankings because they are unobservable or subjective would mean that beliefs and expectations would have to be banished as well.


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Rationality


## Revealed Preference Theory

 Rational Choice Theory ParetoHarsany Rationality

Standard economics focuses on revealed preference because economic data comes in this form. Economic data can-at best-reveal what the agent wants (or has chosen) in a particular situation. Such data do not enable the economist to distinguish between what the agent intended to choose and what he ended up choosing; what he chose and what he ought to have chosen.
(Gul and Pesendorfer, 2008)

Given some choices of a decision maker, in what circumtances can we understand those choices as being made by a rational decision maker?

## Sen's $\alpha$ Condition




ArrowSocial Choice
Rationality

## $R$ : red wine

$W$ : white wine
L: lemonade

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## Sen's $\alpha$ Condition

$R$ : red wine
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$R$ : red wine
$W$ : white wine

If the world champion is American, then she must be a US champion too.

Observations of actual choices will only partially constrain preference attribution. That someone chooses red wine when white wine is available does not allow one to conclude that the choice of an white wine was ruled out by her preferences, only that her preferences ruled the red wine in.

## Sen's $\beta$ Condition

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## $R$ : red wine

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## $R$ : red wine

$W$ : white wine

## Sen's $\beta$ Condition


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Rationality

## $R$ : red wine

W: white wine
L: lemonade

## Sen's $\beta$ Condition

## $R$ : red wine

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L: lemonade

## Sen's $\beta$ Condition



## $R$ : red wine

## $W$ : white wine

L: lemonade

If some American is a world champion, then all champions of America must be world champions.

## Revealed Preference Theory

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Arrow Social Choice Theory Sen Arrow Sacia Choice

A decision maker's choices over a set of alternatives $X$ are rationalizable iff there is a (rational) preference relation on $X$ such that the decision maker's choices maximize the preference relation.

## Revealed Preference Theory

A decision maker's choices over a set of alternatives $X$ are rationalizable iff there is a (rational) preference relation on $X$ such that the decision maker's choices maximize the preference relation.

Revelation Theorem. A decision maker's choices satisfy Sen's $\alpha$ and $\beta$ if and only if the decision maker's choices are rationalizable.

## Choice Functions

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Suppose $X$ is a set of options. And consider $B \subseteq X$ as a choice problem. A choice function is any function where $C(B) \subseteq B$. $B$ is sometimes called a menu and $C(B)$ the set of "rational" or "desired" choices.

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Sen's $\alpha$ : If $x \in C(A)$ and $B \subseteq A$ and $x \in B$ then $x \in C(B)$

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Sen's $\alpha$ : If $x \in C(A)$ and $B \subseteq A$ and $x \in B$ then $x \in C(B)$
Sen's $\beta$ : If $x, y \in C(A), A \subseteq B$ and $y \in C(B)$ then $x \in C(B)$. waven weme teon Economics Nash Condorcets Parasox Rational Choice' Theory ParetoHarsany ArrowSocial Choice
Rationality

Invoking someone's preferences will suffice to explain why some choices were not made (i.e. in terms of rational impermissibility) but not typically why some particular choice was made. To take up the slack, explanations must draw on factors other than preference: psychological one such as the framing of the choice problem or the saliency of particular options, or sociological ones such as the existence of norms or conventions governing choices of the relevant kind.

## Ordinal Utility Theory

## Utility Function

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A preference ordering is represented by a utility function iff $x$ is (weakly) preferred to $y$ provided $u(x) \geq u(y)$

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A preference ordering is represented by a utility function iff $x$ is (weakly) preferred to $y$ provided $u(x) \geq u(y)$

What properties does such a preference ordering have?

## Ordinal Utility Theory

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Fact. Suppose that $X$ is finite and $\succeq$ is a complete and transitive ordering over $X$, then there is a utility function $u: X \rightarrow \mathfrak{R}$ that represents $\succeq$ (i.e., $x \succeq y$ iff $u(x) \geq u(y)$ )

## Ordinal Utility Theory

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Utility is defined in terms of preference (so it is an error to say that the agent prefers $x$ to $y$ because she assigns a higher utility to $x$ than to $y$ ).

## Important

All three of the utility functions represent the preference $x \succ y \succ z$

| Item | $u_{1}$ | $u_{2}$ | $u_{3}$ |
| :---: | :---: | :---: | :---: |
| $x$ | 3 | 10 | 1000 |
| $y$ | 2 | 5 | 99 |
| $z$ | 1 | 0 | 1 |

$x \succ y \succ z$ is represented by both $(3,2,1)$ and $(1000,999,1)$, so one cannot say that $y$ is "closer" to $x$ than to $z$.

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X=\{M, C, P, L\}
$$

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$$

## $M \subset P L$

$M P L$

C PL
$M \subset P$
$M \subset L$


M P

M L

C $P$

C L

$L$

$$
X=\{M, C, P, L\}
$$

## $M \subset P L$


$\boldsymbol{C}^{P L}$

M C P
$M C L$


$$
X=\{M, C, P, L\}
$$



$$
\begin{gathered}
\succeq=\{(M, C),(C, M),(M, P),(M, L),(C, P),(C, L),(P, L), \\
(M, M),(P, P),(C, C),(L, L)\}
\end{gathered}
$$

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\succeq=\{(M, C),(C, M),(M, P),(M, L),(C, P),(C, L),(P, L), \\
(M, M),(P, P),(C, C),(L, L)\}
\end{gathered}
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$M C P L$

(C) $P L$

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## Maximizing

 Mas semen wey Arrow Sacia Choice

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