PHIL309P
Methods in Philosophy, Politics and Economics

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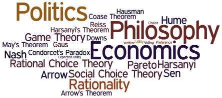
University of Maryland

- Beliefs: How should we represent the decision makers **beliefs** about the decision problems (e.g., the available outcomes, menu items, consequences of actions, etc.). What makes a belief rational or reasonable?

- Preferences: How should we represent the decision maker's **preferences** about the available choices? What makes a preference rational or reasonable?

Preferences



Preferring or choosing x is different that "liking" x or "having a taste for x": one can prefer x to y but *dislike* both options

Preferences are always understood as comparative: "preference" is more like "bigger" than "big"

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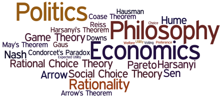

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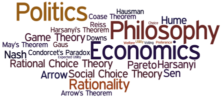

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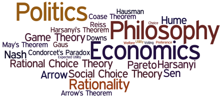

1. Lauren drank water rather than wine with dinner, despite preferring to drink wine, because she promised her husband she would stay sober.

2. Lauren drank water with dinner because she preferred to do so. But for the promise she made her husband to stay sober, she would have preferred to drink wine rather than water with dinner.

Preferences will be understood as *mental rankings* of alternatives “all things considered”.

Mathematically describing preferences

Mathematical background: Relations

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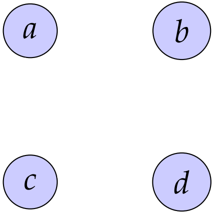

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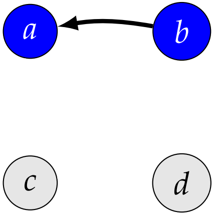

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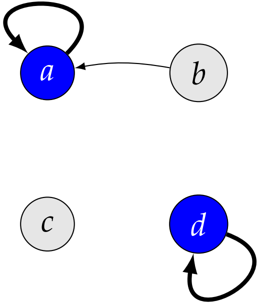

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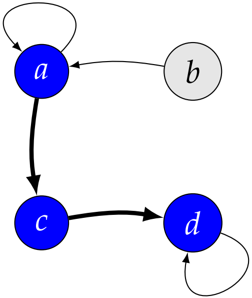

$a \; R \; a$
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$c \; R \; d$
$a \; R \; c$
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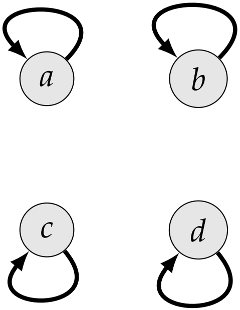

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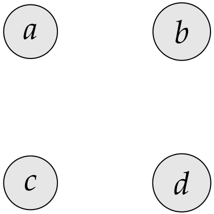

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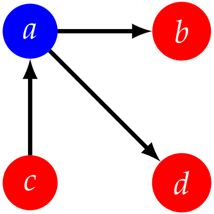

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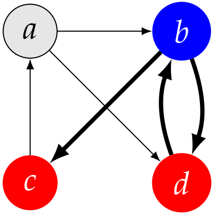

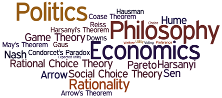

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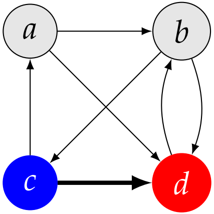

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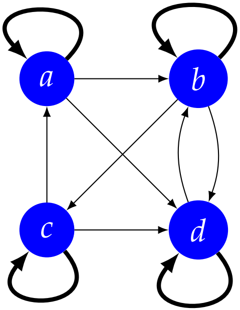

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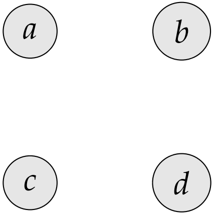

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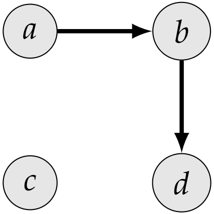

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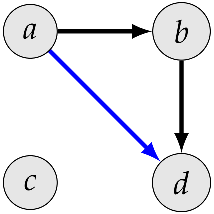

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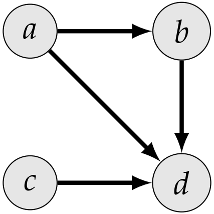

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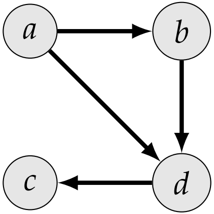

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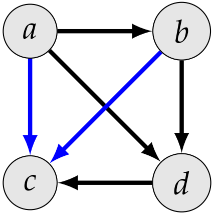

Representing Preferences

Let X be a set of options/outcomes. A decision maker's *preference* over X is represented by a *relation* $\succeq \subseteq X \times X$.

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Representing Preferences

Suppose that $\succeq$ is a relation on X (called the **weak preference**). Then, define the following:

- **Strict preference**: $x \succ y$ iff $x \succeq y$ and $y \not\succeq x$
- **Indifference**: $x \sim y$ iff $x \succeq y$ and $y \succeq x$
- **Non-comparability** $x \, N \, y$ iff $x \not\succeq y$ and $y \not\succeq x$

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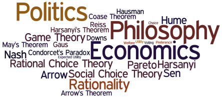

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What properties *should* weak/strict preference, indifference, non-comparability satisfy?

Rational preferences

A relation $\succeq \subseteq X \times X$ is a **rational preference relation** (for a decision maker) provided that

1. $\succeq$ is complete (and hence reflexive)
2. $\succeq$ is transitive

- What is the relationship between choice and preference?
- What makes a preference *rational*?
- *Should* a decision maker's preference be complete and transitive?
- *Are* people's preferences complete and transitive?

Choices

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Decisions are between beliefs and desires on the one hand and actions on the other.

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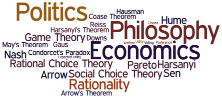
The verb "to prefer" can either mean "to choose" or "to like better," and these two senses are frequently confused in economic literature. That fact that an individual chooses A rather than B is far from conclusive evidence that he likes A better. But whether he likes A better or not should be completely irrelevant to the theory of price. (Little, 1949).

Preferences and Choices

Preferences are closely related to choices: preferences may *cause* and to help to *explain* choices; preferences may be invoked to *justify* choices, in fortuitous circumstances, we can use preference data to make *predictions* about choice. But to identify the two would be a mistake.

Preferences and Choices



- We have preferences over vastly more states of affairs than we can ever hope (or dread) to be in the position to choose.

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- What about *counter-preferential choice*?
- Preferences must be *stable* over a reasonable amount of time in a way that (observed) choices aren't (needed to predict and explain choices).
- Beliefs and expectations over future states of affairs are needed in addition to preferences in order to explain choices. To banish preferences understood as mental rankings because they are unobservable or subjective would mean that beliefs and expectations would have to be banished as well.

Revealed Preference Theory

Standard economics focuses on revealed preference because economic data comes in this form. Economic data can—at best—reveal what the agent wants (or has chosen) in a particular situation. Such data do not enable the economist to distinguish between what the agent intended to choose and what he ended up choosing; what he chose and what he ought to have chosen. (Gul and Pesendorfer, 2008)

Given some choices of a decision maker, in what circumtances can we understand those choices as being made by a *rational* decision maker?

Sen's α Condition

R: red wine

W: white wine

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If the world champion is American, then she must be a US champion too.

Observations of actual choices will only partially constrain preference attribution. That someone chooses red wine when white wine is available does not allow one to conclude that the choice of an white wine was ruled out by her preferences, only that her preferences ruled the red wine in.

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If some American is a world champion, then all champions of America must be world champions.

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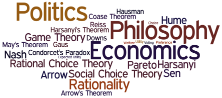

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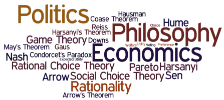

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Revelation Theorem. A decision maker's choices satisfy Sen's α and β if and only if the decision maker's choices are **rationalizable**.

Choice Functions

Suppose X is a set of options. And consider $B \subseteq X$ as a choice problem. A **choice function** is any function where $C(B) \subseteq B$. B is sometimes called a menu and $C(B)$ the set of "rational" or "desired" choices.

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Sen's β: If $x, y \in C(A)$, $A \subseteq B$ and $y \in C(B)$ then $x \in C(B)$.

Invoking someone's preferences will suffice to explain why some choices were not made (i.e. in terms of rational impermissibility) but not typically why some particular choice was made. To take up the slack, explanations must draw on factors other than preference: psychological one such as the framing of the choice problem or the saliency of particular options, or sociological ones such as the existence of norms or conventions governing choices of the relevant kind.

Ordinal Utility Theory

Utility Function

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What properties does such a preference ordering have?

Ordinal Utility Theory

Fact. Suppose that X is finite and $\succeq$ is a complete and transitive ordering over X, then there is a utility function $u : X \to \Re$ that represents $\succeq$

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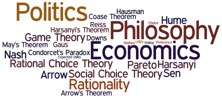
Fact. Suppose that X is finite and $\succeq$ is a complete and transitive ordering over X, then there is a utility function $u : X \to \mathfrak{R}$ that represents $\succeq$
(i.e., $x \succeq y$ iff $u(x) \geq u(y)$)

Utility is *defined* in terms of preference (so it is an error to say that the agent prefers x to y *because* she assigns a higher utility to x than to y).

Important



All three of the utility functions represent the preference $x \succ y \succ z$

Item	u_1	u_2	u_3
x	3	10	1000
y	2	5	99
z	1	0	1

$x \succ y \succ z$ is represented by both $(3, 2, 1)$ and $(1000, 999, 1)$, so one cannot say that y is "closer" to x than to z.

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$M\ C\ P\ L$	$M\ C$	$P\ L$
$M\ P\ L$	$M\ P$	M
$C\ P\ L$	$M\ L$	C
$M\ C\ P$	$C\ P$	P
$M\ C\ L$	$C\ L$	L

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$\boxed{M\ C}\ P\ L$	$\boxed{M\ C}$	$\boxed{P}\ L$
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$\boxed{C}\ P\ L$	$\boxed{M}\ L$	$\boxed{C}$
$\boxed{M\ C}\ P$	$\boxed{C}\ P$	$\boxed{P}$
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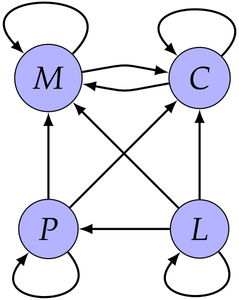

$$\succeq = \{(M,C),(C,M),(M,P),(M,L),(C,P),(C,L),(P,L),$$
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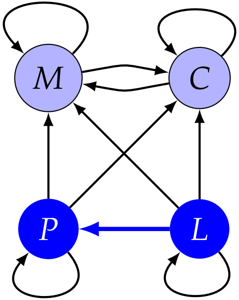

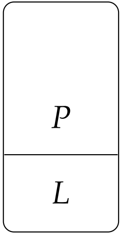

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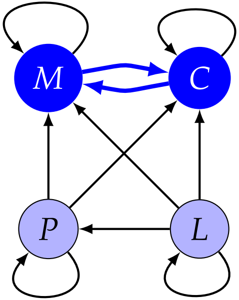

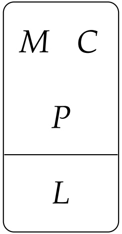

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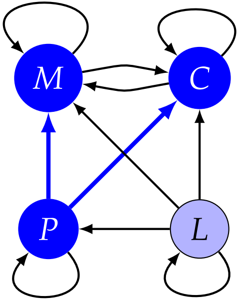

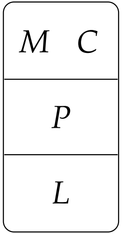

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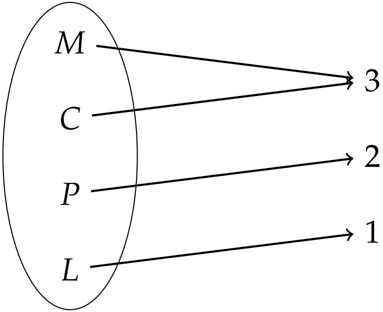

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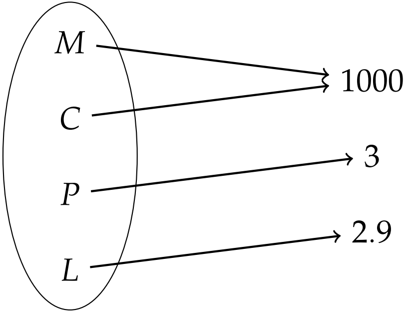

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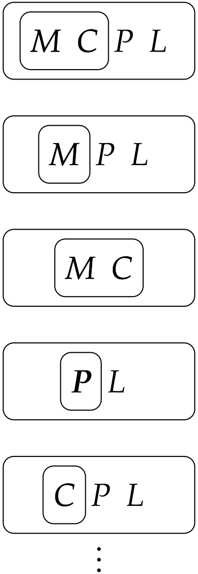

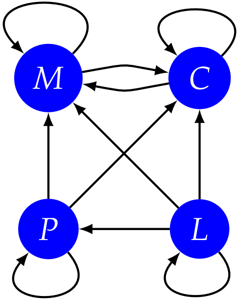

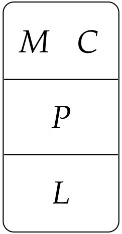

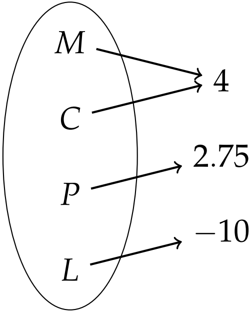

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A. Sen. *Maximization and the Act of Choice*. Econometrica, Vol. 65, No. 4, 1997, 745 - 779.

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"The formulation of maximizing behavior in economics has often parallels the modeling of maximization in physics an related disciplines. But maximizing *behavior* differs from nonvolitional maximization because of the fundamental relevance of the choice act, which has to be placed in a central position in analyzing maximizing behavior. A person's preferences over *comprehensive* outcomes (including the choice process) have to be distinguished form the conditional preferences over *culmination* outcomes *given* the act of choice." (pg. 745)

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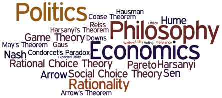

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