PHIL309P
Methods in Philosophy, Politics and Economics

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Representing Preferences

Suppose that $\succeq$ is a relation on X (called the **weak preference**). Then, define the following:

- **Strict preference**: $x \succ y$ iff $x \succeq y$ and $y \not\succeq x$
- **Indifference**: $x \sim y$ iff $x \succeq y$ and $y \succeq x$
- **Non-comparability** $x\ N\ y$ iff $x \not\succeq y$ and $y \not\succeq x$

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What properties *should* weak/strict preference, indifference, non-comparability satisfy?

Assumptions/Axioms of Preference Relations

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- Transitivity
- Completeness

The other two are more substantive and often implicit within economic models:

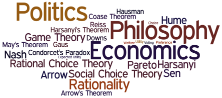
- Agents choose in accordance with their preferences (choice determination)
- Agents' preferences do not change over different choice contexts (context independence)

- What is the relationship between choice and preference?
- *Should* a decision maker's preference be complete and transitive?
- *Are* people's preferences complete and transitive?

Preferences and Choices

Preferences are closely related to choices: preferences may *cause* and to help to *explain* choices; preferences may be invoked to *justify* choices, in fortuitous circumstances, we can use preference data to make *predictions* about choice. But to identify the two would be a mistake.

Preferences and Choices



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- What about *counter-preferential choice*?

- Preferences must be *stable* over a reasonable amount of time in a way that (observed) choices aren't (needed to predict and explain choices).

Revealed Preference Theory

Standard economics focuses on revealed preference because economic data comes in this form. Economic data can—at best—reveal what the agent wants (or has chosen) in a particular situation. Such data do not enable the economist to distinguish between what the agent intended to choose and what he ended up choosing; what he chose and what he ought to have chosen. (Gul and Pesendorfer, 2008)

Given some choices of a decision maker, in what circumtances can we understand those choices as being made by a *rational* decision maker?

Sen's α Condition

R: red wine

W: white wine

L: lemonade

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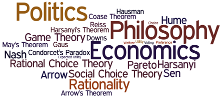
Sen's α Condition



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If the world champion is American, then she must be a US champion too.

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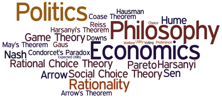
If some American is a world champion, then all champions of America must be world champions.

Revealed Preference Theory



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Revelation Theorem. A decision maker's choices satisfy Sen's α and β if and only if the decision maker's choices are **rationalizable**.

Choice Functions

Suppose X is a set of options. And consider $B \subseteq X$ as a choice problem. A **choice function** is any function where $C(B) \subseteq B$. B is sometimes called a menu and $C(B)$ the set of "rational" or "desired" choices.

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Sen's β: If $x, y \in C(A)$, $A \subseteq B$ and $y \in C(B)$ then $x \in C(B)$.

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- *Are* people's preferences complete and transitive?

- Acyclic Preferences: Money-pump argument
- Completeness: Incommensurable options

Transitivity

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Indifference: For all $x, y, z \in X$, if $x \sim y$ and $y \sim z$, then $x \sim z$.

- For example, you may be indifferent between a curry with x amount of cayenne pepper, and a curry with x plus one particle of cayenne pepper for any amount x. But you are not indifferent between a curry with no cayenne pepper and one with 1 lbs. of it!

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Strict preference: For all $x, y, z \in X$, if $x \succ y$ and $y \succ z$, then $x \succ z$.

Indifference is not transitive: $x_1 \sim x_2 \sim \cdots \sim x_n$, yet $x_1 \succ x_n$

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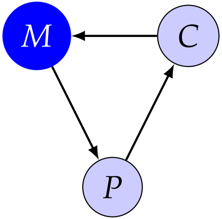
Cycle: $x_1 \succ x_2 \cdots \succ x_n$, yet $x_n \succ x_1$

Cyclic Preferences

I do not think we can clearly say what should convince us that a man at a given time (without change of mind) preferred *a* to *b*, *b* to *c* and *c* to *a*. The reason for our difficulty is that we cannot make good sense of an attribution of preference except against a background of coherent attitudes...My point is that if we are intelligibly to attribute attitudes and beliefs, or usefully to describe motions as behaviour, then we are committed to finding, in the pattern of behaviour, belief, and desire, a large degree of rationality and consistency. (Davidson 1974: p. 237)

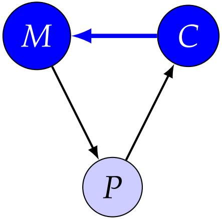
D. Davidson. *'Philosophy as psychology'*. In S. C. Brown (ed.), Philosophy of Psychology, 1974. Reprinted in his Essays on Actions and Events. Oxford: OUP 2001: pp. 229244.

Money-Pump Argument



(M)

Money-Pump Argument



$$(M) \implies (C, -1)$$

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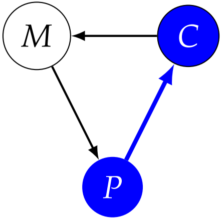
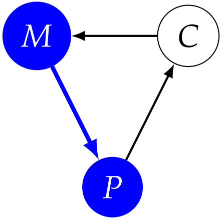


$$(M) \implies (C, -1) \implies (P, -2)$$

Money-Pump Argument



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Money-Pump Argument

$$
\begin{tikzcd}
M \arrow[dr] & & C \arrow[ll] \\
& P \arrow[ur] &
\end{tikzcd}
$$

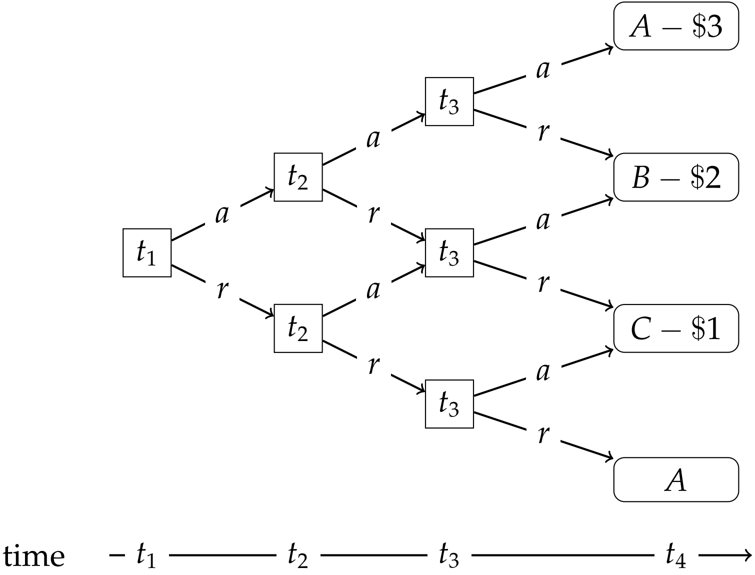
$$(M) \implies (C, -1) \implies (P, -2) \implies (M, -3) \implies (C, -4) \implies \cdots$$

Assumptions

- Ann prefers x to y, written $x \succ y$, iff Ann always takes x when y is the only alternative.
- If $x \succ y$, then $x + \$w \succ y + \$w$
- If $x \succ y$, then there is some $v > 0$ such that for all u,
$$x - \$u \succ y \text{ iff } u \leq v.$$
- $x + \$w \succ x + \$z$ iff $w > z$.

Note: $x - \$w$ means that you keep item x and pay $\$w$

- $A \succ B \succ C \succ A$
- Decision maker is faced with a choice over three days.
- " I will give you C for A, B for C, or A for B at a charge of $1"
- Each day, the decision maker can either accept (a) or reject (r) the offer.


$A - \$3$
a
t_3
a
r
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$B - \$2$
a
r
a
t_1
t_3
r
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$C - \$1$
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a
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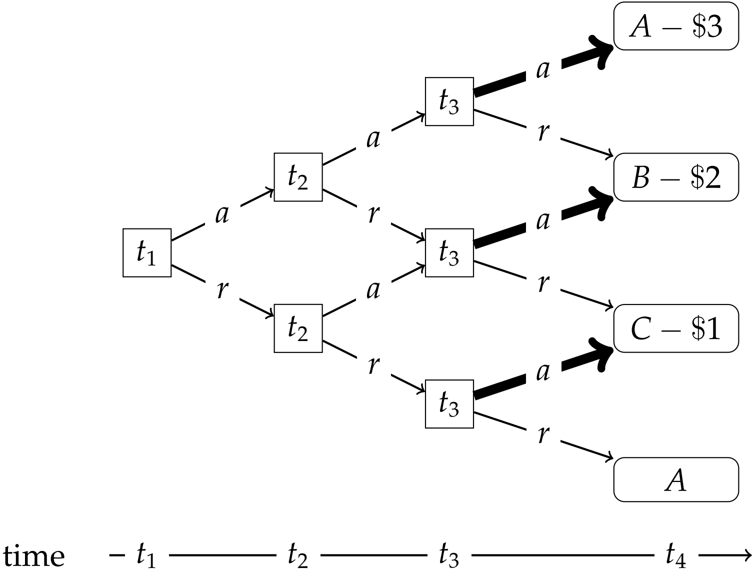
r
A
time t_1 t_2 t_3 t_4


t_1
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$C - \$1$
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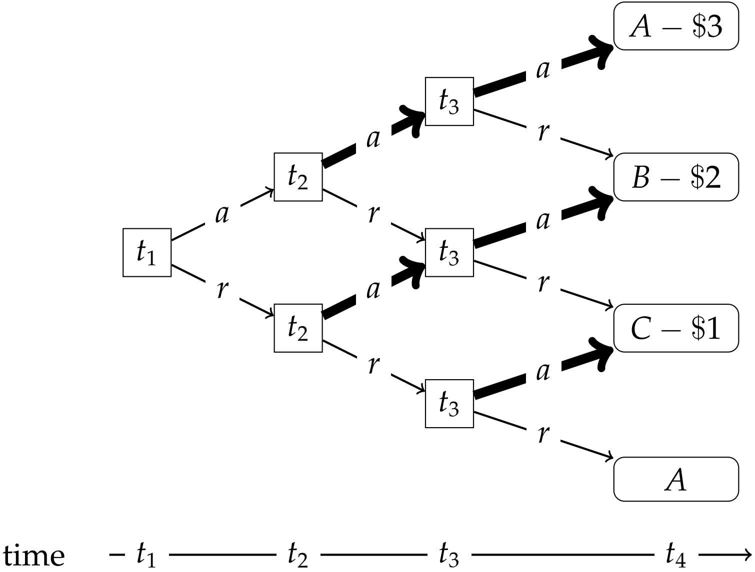
A – $3
B – $2
C – $1
A
t1
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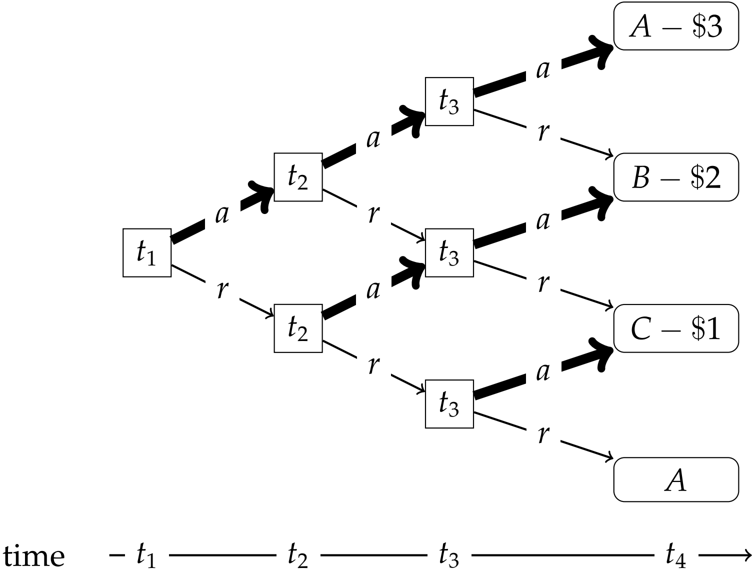
a
r
time – t1 — t2 — t3 — t4



A – $3
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A
t1
t2
t2
t3
t3
t3
a
r
time
t1
t2
t3
t4

Completeness

For all $x, y \in X$, one of the following obtains:

1. the decision maker strictly prefers x over y ($x \succ y$);
2. the decision maker strictly prefers y over x ($y \succ x$); or
3. the decision maker is indifferent between x over y ($y \sim x$)

Completeness

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[O]f all the axioms of utility theory, the completeness axiom is perhaps the most questionable. Like others, it is inaccurate as a description of real life; but unlike them we find it hard to accept even from the normative viewpoint.

(Aumann, 1962)

Context Independence

Context independence is a troublesome axiom, because some kinds of context dependence are common, and some kinds appear to be reasonable.

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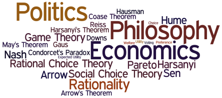

Context independence is a troublesome axiom, because some kinds of context dependence are common, and some kinds appear to be reasonable. One way to reconcile the existence of apparently context-dependent preferences...is to take the description of alternatives to include "everything that matters to the agent"

(Hausman, p16)

Context Independence

A. Sen. *Maximization and the Act of Choice*. Econometrica, Vol. 65, No. 4, 1997, 745 - 779.

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"The formulation of maximizing behavior in economics has often parallels the modeling of maximization in physics an related disciplines. But maximizing *behavior* differs from nonvolitional maximization because of the fundamental relevance of the choice act, which has to be placed in a central position in analyzing maximizing behavior. A person's preferences over *comprehensive* outcomes (including the choice process) have to be distinguished form the conditional preferences over *culmination* outcomes *given* the act of choice." (pg. 745)

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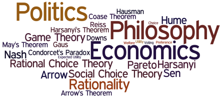
Should we see this as a violation of choice determination, or as a violation of context independence, or as a misdescription of the choice situation?



Rather than trying to provide instrumental or pragmatic justifications for the axioms of ordinal utility, it is better...to see them as constitutive of our conception of a fully rational agent....those disposed to blatantly ignore transitivity are unintelligible to us: we can't understand their pattern of actions as sensible. [Gaus], pg. 39

Ordinal Utility Theory

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A preference ordering is **represented** by a utility function iff x is (weakly) preferred to y provided $u(x) \geq u(y)$

What properties does such a preference ordering have?

Ordinal Utility Theory

Fact. Suppose that X is finite and $\succeq$ is a complete and transitive ordering over X, then there is a utility function $u : X \to \Re$ that represents $\succeq$

(i.e., $x \succeq y$ iff $u(x) \geq u(y)$)

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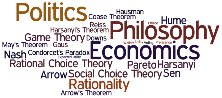

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(i.e., $x \succeq y$ iff $u(x) \geq u(y)$)

Utility is *defined* in terms of preference (so it is an error to say that the agent prefers x to y *because* she assigns a higher utility to x than to y).

Important

All three of the utility functions represent the preference $x \succ y \succ z$

Item	u_1	u_2	u_3
x	3	10	1000
y	2	5	99
z	1	0	1

$x \succ y \succ z$ is represented by both $(3, 2, 1)$ and $(1000, 999, 1)$, so one cannot say that y is "closer" to x than to z.