PHIL309P
Methods in Philosophy, Politics and Economics

Eric Pacuit
University of Maryland

Ordinal Utility Theory

Utility Function

A **utility function** on a set X is a function $u : X \to \mathbb{R}$

Utility Function

A **utility function** on a set X is a function $u : X \to \mathbb{R}$

A preference ordering is **represented** by a utility function iff x is (weakly) preferred to y provided $u(x) \geq u(y)$

Utility Function

A **utility function** on a set X is a function $u : X \to \mathbb{R}$

A preference ordering is **represented** by a utility function iff x is (weakly) preferred to y provided $u(x) \geq u(y)$

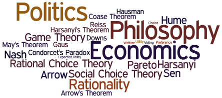
What properties does such a preference ordering have?

Ordinal Utility Theory

Fact. Suppose that X is finite and $\succeq$ is a complete and transitive ordering over X, then there is a utility function $u : X \rightarrow \Re$ that represents $\succeq$

(i.e., $x \succeq y$ iff $u(x) \geq u(y)$)

Ordinal Utility Theory

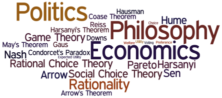


Fact. Suppose that X is finite and $\succeq$ is a complete and transitive ordering over X, then there is a utility function $u : X \to \Re$ that represents $\succeq$

(i.e., $x \succeq y$ iff $u(x) \geq u(y)$)

Utility is *defined* in terms of preference (so it is an error to say that the agent prefers x to y *because* she assigns a higher utility to x than to y).

Important



All three of the utility functions represent the preference $x \succ y \succ z$

Item	u_1	u_2	u_3
x	3	10	1000
y	2	5	99
z	1	0	1

$x \succ y \succ z$ is represented by both $(3, 2, 1)$ and $(1000, 999, 1)$, so one cannot say that y is “closer” to x than to z.

$$X = \{M, C, P, L\}$$

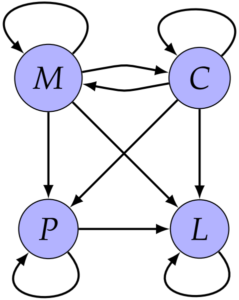
$$X = \{M, C, P, L\}$$

$M\ C\ P\ L$	$M\ C$	$P\ L$
$M\ P\ L$	$M\ P$	M
$C\ P\ L$	$M\ L$	C
$M\ C\ P$	$C\ P$	P
$M\ C\ L$	$C\ L$	L

$$X = \{M, C, P, L\}$$

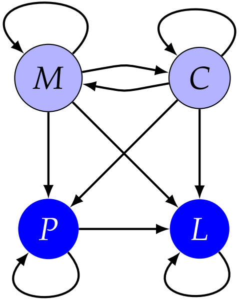
$\boxed{M\ C}\ P\ L$	$\boxed{M\ C}$	$\boxed{P}\ L$
$\boxed{M}\ P\ L$	$\boxed{M}\ P$	$\boxed{M}$
$\boxed{C}\ P\ L$	$\boxed{M}\ L$	$\boxed{C}$
$\boxed{M\ C}\ P$	$\boxed{C}\ P$	$\boxed{P}$
$\boxed{M\ C}\ L$	$\boxed{C}\ L$	$\boxed{L}$

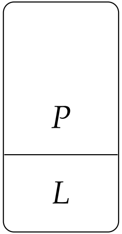
$$X = \{M, C, P, L\}$$



$$\succeq = \{(M, C), (C, M), (M, P), (M, L), (C, P), (C, L), (P, L), (M, M), (P, P), (C, C), (L, L)\}$$

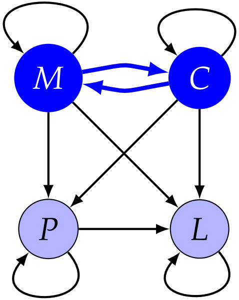
$$X = \{M, C, P, L\}$$

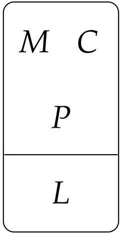




$$\succeq = \{(M,C),(C,M),(M,P),(M,L),(C,P),(C,L),(P,L),\\(M,M),(P,P),(C,C),(L,L)\}$$

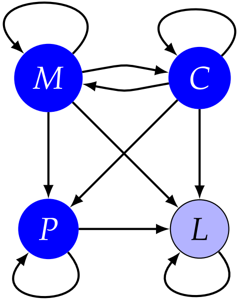
$$X = \{M, C, P, L\}$$





$$\succeq = \{(M,C),(C,M),(M,P),(M,L),(C,P),(C,L),(P,L),\\ (M,M),(P,P),(C,C),(L,L)\}$$

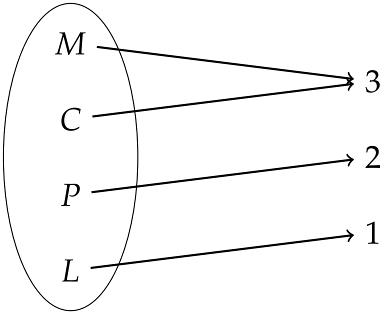
$$X = \{M, C, P, L\}$$



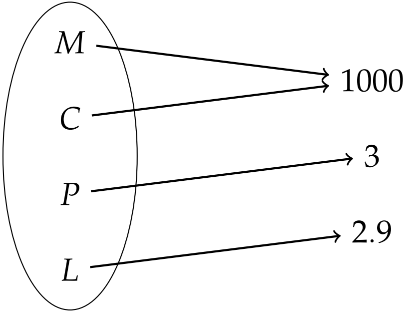
$M \quad C$
P
L

$$\succeq = \{(M, C), (C, M), (M, P), (M, L), (C, P), (C, L), (P, L), (M, M), (P, P), (C, C), (L, L)\}$$

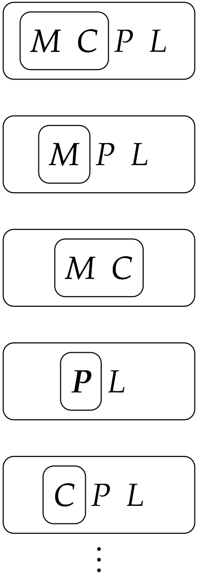
$$X = \{M, C, P, L\}$$

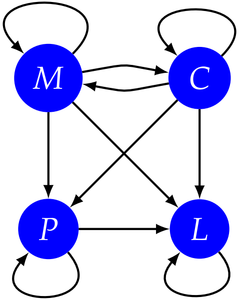


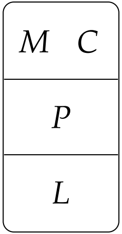
$$X = \{M, C, P, L\}$$

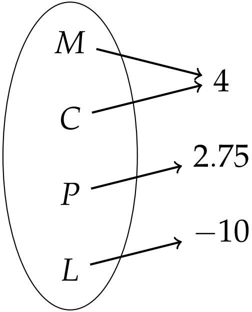


$$X = \{M, C, P, L\}$$









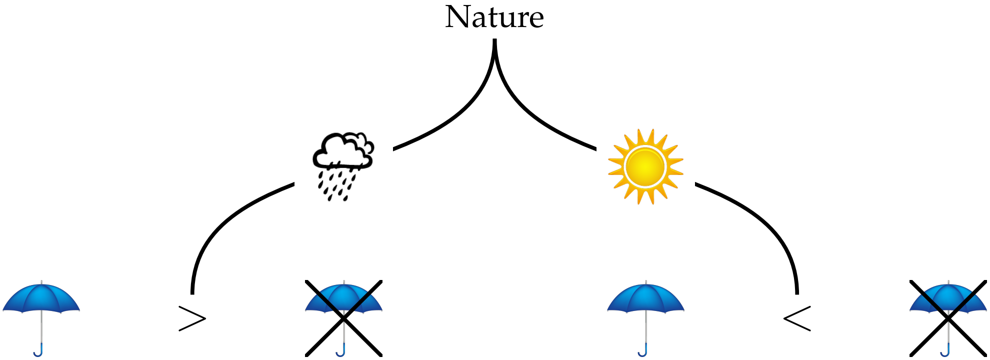
What is *utility*?

- usefulness
- from *Principle of Utility*: an object's "tendency to produce benefit, advantage, pleasure, good, or happiness" (Broome, p19) for all people
- a person's personal, subjective good
- "the value of a function that *represents* a person's preferences" (Reiss, p21)

Economists primarily use the last sense of utility (as will we), which is not problematic, however, “[i]f...you use ‘utility’ to stand for a representation of a person’s preferences, and at the same time for the person’s good, you cannot even express the question [of whether or not persons always act so as to maximize their utility]. You will say: by definition, what a person prefers has more utility for her, so how can it fail to have more utility for her? The ambiguity is intolerable.” (Reiss, p. 21)

Individual decision-making (against nature)

☂️ > ~~☂️~~ ☂️ < ~~☂️~~


Nature
>
<

		
	encumbered, dry	encumbered, dry
	wet	free, dry

States: it rains; it does not rain

Outcomes: encumbered, dry; wet; free, dry

Actions: take umbrella; leave umbrella

		
	encumbered, dry	encumbered, dry
	wet	free, dry

States: it rains; it does not rain

Outcomes: encumbered, dry; wet; free, dry

Actions: take umbrella; leave umbrella

		
	encumbered, dry	encumbered, dry
	wet	free, dry

States: it rains; it does not rain

Outcomes: encumbered, dry; wet; free, dry

Actions: take umbrella; leave umbrella

		
	encumbered, dry	encumbered, dry
	wet	free, dry

States: it rains; it does not rain

Outcomes: encumbered, dry; wet; free, dry

Actions: take umbrella; leave umbrella

		
	encumbered, dry	encumbered, dry
	wet	free, dry

States: it rains; it does not rain

Outcomes: encumbered, dry; wet; free, dry

Actions: take umbrella; leave umbrella

		
	encumbered, dry	encumbered, dry
	wet	free, dry

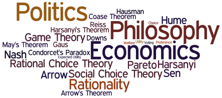
States: it rains; it does not rain

Outcomes: encumbered, dry; wet; free, dry

Actions: take umbrella; leave umbrella

Decision Problems

In many circumstances the decision maker doesn't get to choose outcomes directly, but rather chooses an instrument that affects what outcome actually occurs.



Decision Problems

In many circumstances the decision maker doesn't get to choose outcomes directly, but rather chooses an instrument that affects what outcome actually occurs.

Choice under

- *certainty*: highly confident about the relationship between actions and outcomes
- *risk*: clear sense of possibilities and their likelihoods
- *uncertainty*: the relationship between actions and outcomes is so imprecise that it is not possible to assign likelihoods

Lotteries

Suppose that X is a set of outcomes.

A **(simple) lottery** over X is denoted $[x_1 : p_1, x_2 : p_2, \ldots, x_n : p_n]$ where for $i = 1, \ldots, n$, $x_i \in X$ and $p_i \in [0, 1]$, and $\sum_i p_i = 1$.

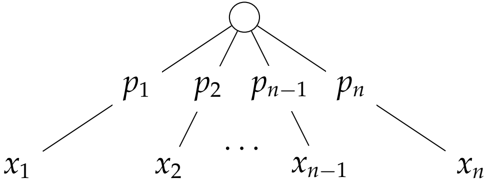
Let $\mathcal{L}$ be the set of (simple) lotteries over X. We identify elements $x \in X$ with the lottery $[x : 1]$.

Lotteries

Suppose that $X = \{x_1, \ldots, x_n\}$ is a set of outcomes. A **lottery** over X is a tuple $[p_1 : x_1, \ldots, p_n : x_n]$ where $\sum_i p_i = 1$.

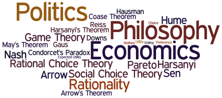
Lotteries

Suppose that $X = \{x_1, \ldots, x_n\}$ is a set of outcomes. A **lottery** over X is a tuple $[p_1 : x_1, \ldots, p_n : x_n]$ where $\sum_i p_i = 1$.



Let $\mathcal{L}$ be the set of lotteries.

Expected Value of a Lottery



Suppose that the outcomes of a lottery are monetary values. So, $L = [x_1 : p_1, x_2 : p_2, \ldots, x_n : p_n]$, where each x_i is an amount of money. Then,

$$EV(L) = \sum_i p_i \times x_i$$

Expected Value of a Lottery

Suppose that the outcomes of a lottery are monetary values. So, $L = [x_1 : p_1, x_2 : p_2, \ldots, x_n : p_n]$, where each x_i is an amount of money. Then,

$$EV(L) = \sum_i p_i \times x_i$$

E.g., if $L = [\$100 : 0.55, \$50 : 0.25, \$0 : 0.20]$, then

$$EV(L) = 0.55 * 100 + 0.25 * 50 + 0.2 * 0 = 80$$

You are given a choice between two lotteries L_1 and L_2. The outcome of the lotteries is determined by flipping a fair coin. The payoff for the two lotteries are given in the following table:

	Heads	Tails
L_1	\$1M	\$1M
L_2	\$3M	\$0

Which of the two lotteries would you choose?

1. L_1
2. L_2
3. I am indifferent between the two lotteries

Comments on Expected Utility

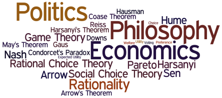
Options	$1/2$	$1/2$
L_1	$1M$	$1M$
L_2	$3M$	$0M$

Comments on Expected Utility

Options	1/2	1/2
L_1	$1M$	$1M$
L_2	$3M$	$0M$

$$EVM(L_1) = 1/2 \cdot 1 + 1/2 \cdot 1 = 1$$
$$EVM(L_1) = 1/2 \cdot 3 + 1/2 \cdot 0 = 1.5$$

Comments on Expected Utility

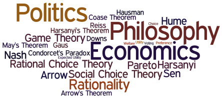


Options	1/2	1/2
L_1	$1M$	$1M$
L_2	$3M$	$0M$

$$EVM(L_1) = 1/2 \cdot 1 + 1/2 \cdot 1 = 1$$
$$EVM(L_1) = 1/2 \cdot 3 + 1/2 \cdot 0 = 1.5$$

What numbers should we use in place of monetary value?

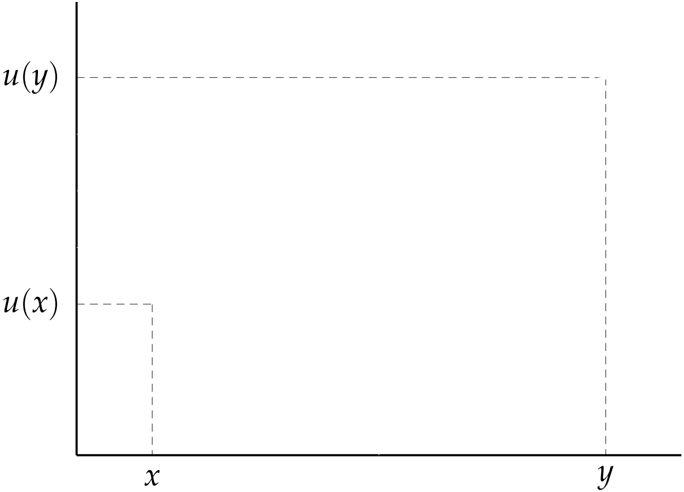
Comments on Expected Utility

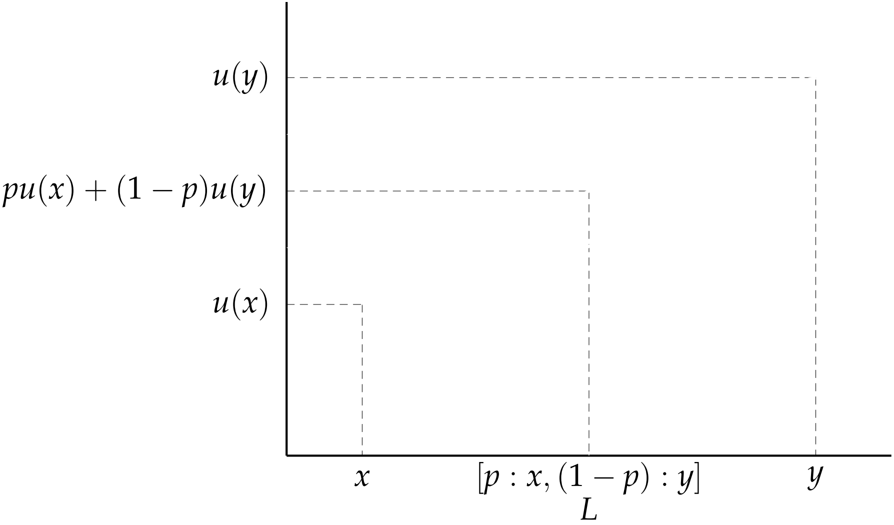


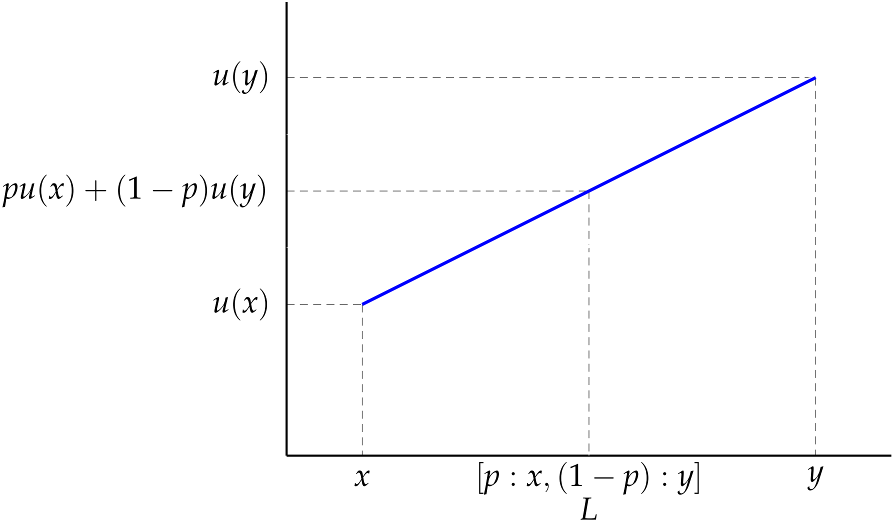
Options	1/2	1/2
L_1	$1M$	$1M$
L_2	$3M$	$0M$

$$EVM(L_1) = 1/2 \cdot 1 + 1/2 \cdot 1 = 1$$
$$EVM(L_1) = 1/2 \cdot 3 + 1/2 \cdot 0 = 1.5$$

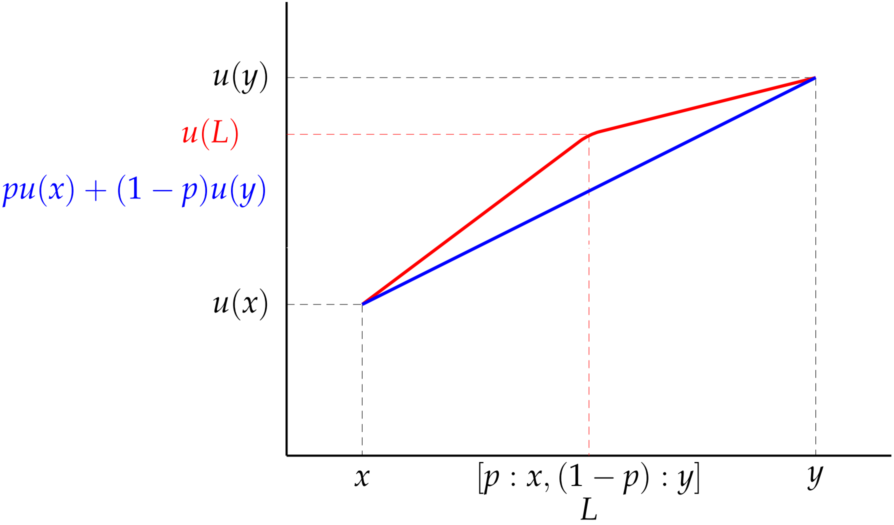
What numbers should we use in place of monetary value? (moral) value? personal utility?


$u(y)$
$u(x)$
x
y

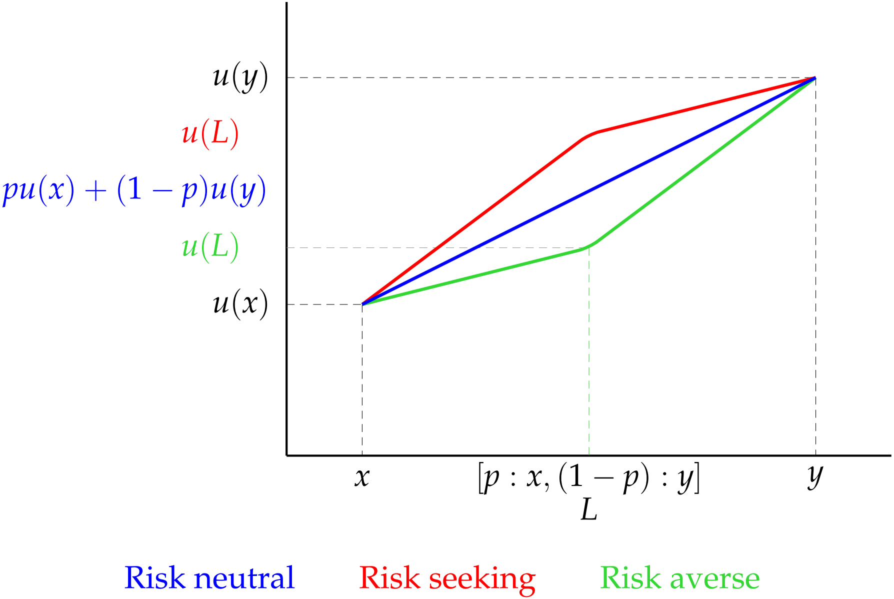

$u(y)$
$pu(x) + (1-p)u(y)$
$u(x)$
x
$[p : x, (1-p) : y]$
L
y



Risk neutral



Risk neutral Risk seeking



$u(y)$
$u(L)$
$pu(x) + (1-p)u(y)$
$u(L)$
$u(x)$
x
$[p : x, (1-p) : y]$
L
y
Risk neutral
Risk seeking
Risk averse