## PHIL309P

# Methods in Philosophy, Politics and Economics 

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## Ordinal Utility Theory

## Utility Function

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What properties does such a preference ordering have?

## Ordinal Utility Theory

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Fact. Suppose that $X$ is finite and $\succeq$ is a complete and transitive ordering over $X$, then there is a utility function $u: X \rightarrow \mathfrak{R}$ that represents $\succeq$ (i.e., $x \succeq y$ iff $u(x) \geq u(y)$ )

## Ordinal Utility Theory

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Utility is defined in terms of preference (so it is an error to say that the agent prefers $x$ to $y$ because she assigns a higher utility to $x$ than to $y$ ).

## Important

All three of the utility functions represent the preference $x \succ y \succ z$

| Item | $u_{1}$ | $u_{2}$ | $u_{3}$ |
| :---: | :---: | :---: | :---: |
| $x$ | 3 | 10 | 1000 |
| $y$ | 2 | 5 | 99 |
| $z$ | 1 | 0 | 1 |

$x \succ y \succ z$ is represented by both $(3,2,1)$ and $(1000,999,1)$, so one cannot say that $y$ is "closer" to $x$ than to $z$.

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X=\{M, C, P, L\}
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## $M \subset P L$

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$M P L$

$\boldsymbol{C}^{P L}$

M C P
$M \subset L$


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$$
\begin{gathered}
\succeq=\{(M, C),(C, M),(M, P),(M, L),(C, P),(C, L),(P, L), \\
(M, M),(P, P),(C, C),(L, L)\}
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## What is utility?

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- usefulness
- from Principle of Utility: an object's "tendency to produce benefit, advantage, pleasure, good, or happiness" (Broome, p19) for all people
- a person's personal, subjective good
- "the value of a function that represents a person's preferences" (Reiss, p21)

Economists primarily use the last sense of utility (as will we), which is not problematic, however, "[i]f...you use 'utility' to stand for a representation of a person's preferences, and at the same time for the person's good, you cannot even express the question [of whether or not persons always act so as to maximize their utility]. You will say: by definition, what a person prefers has more utility for her, so how can it fail to have more utility for her? The ambiguity is intolerable." (Reiss, p. 21)

## Individual decision-making (against nature)





States: it rains; it does not rain
Outcomes: encumbered, dry; wet; free, dry
Actions: take umbrella; leave umbrella

| encumbered, dry | encumbered, dry |
| :---: | :---: |
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## Decision Problems

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Arrows theorem

In many circumstances the decision maker doesn't get to choose outcomes directly, but rather chooses an instrument that affects what outcome actually occurs.

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In many circumstances the decision maker doesn't get to choose outcomes directly, but rather chooses an instrument that affects what outcome actually occurs.

Choice under

- certainty: highly confident about the relationship between actions and outcomes
- risk: clear sense of possibilities and their likelihoods
- uncertainty: the relationship between actions and outcomes is so imprecise that it is not possible to assign likelihoods


## Lotteries

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Suppose that $X$ is a set of outcomes.

A (simple) lottery over X is denoted $\left[x_{1}: p_{1}, x_{2}: p_{2}, \ldots, x_{n}: p_{n}\right]$ where for $i=1, \ldots, n, x_{i} \in X$ and $p_{i} \in[0,1]$, and $\sum_{i} p_{i}=1$.

Let $\mathcal{L}$ be the set of (simple) lotteries over $X$. We identify elements $x \in X$ with the lottery $[x: 1]$.

## Lotteries



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Suppose that $X=\left\{x_{1}, \ldots, x_{n}\right\}$ is a set of outcomes. A lottery over $X$ is a tuple $\left[p_{1}: x_{1}, \ldots, p_{n}: x_{n}\right]$ where $\sum_{i} p_{i}=1$.

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Let $\mathcal{L}$ be the set of lotteries.

## Expected Value of a Lottery

Suppose that the outcomes of a lottery are monetary values. So, $L=\left[x_{1}: p_{1}, x_{2}: p_{2}, \ldots, x_{n}: p_{n}\right]$, where each $x_{i}$ is an amount of money. Then,

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E V(L)=\sum_{i} p_{i} \times x_{i}
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E.g., if $L=[\$ 100: 0.55, \$ 50: 0.25, \$ 0: 0.20]$, then

$$
E V(L)=0.55 * 100+0.25 * 50+0.2 * 0=80
$$

You are given a choice between two lotteries $L_{1}$ and $L_{2}$. The outcome of the lotteries is determined by flipping a fair coin. The payoff for the two lotteries are given in the following table:

|  | Heads | Tails |
| :---: | :---: | :---: |
| $L_{1}$ | $\$ 1 \mathrm{M}$ | $\$ 1 \mathrm{M}$ |
| $L_{2}$ | $\$ 3 \mathrm{M}$ | $\$ 0$ |

Which of the two lotteries would you choose?

1. $L_{1}$
2. $L_{2}$
3. I am indifferent between the two lotteries

## Comments on Expected Utility

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| Options | $1 / 2$ | $1 / 2$ |
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\begin{aligned}
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What numbers should we use in place of monetary value? (moral) value? personal utility?




Risk neutral


Risk neutral Risk seeking


Risk neutral Risk seeking Risk averse

