CMSC423: Bioinformatic Algorithms, Databases and Tools

Exact string matching: introduction

Sequence alignment: exact matching

```
ACAGGTACAGTTCCCTCGACACCTACTACCTAAG     Text
CCTACT                                  Pattern
 CCTACT
  CCTACT
   CCTACT
```

```
for i = 0 .. len(Text) {
  for j = 0 .. len(Pattern) {
     if (Pattern[j] != Text[i]) go to next i
  }
  if we got there pattern matches at i in Text
}
```

Running time = O(len(Text) * len(Pattern)) = O(mn)

What string achieves worst case?

Worst case?

AAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAAA

AAAAAAAAAAAAT

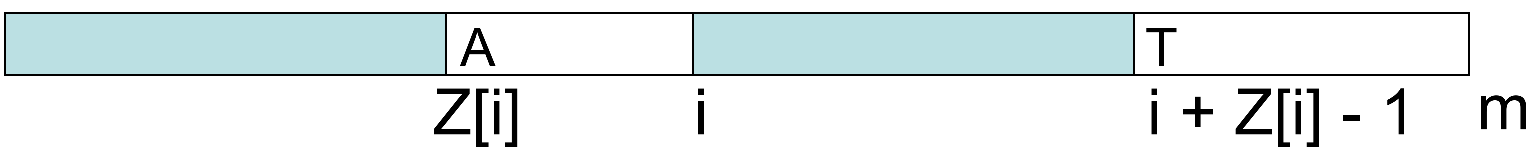
$(m - n + 1) * n$ comparisons

Can we do better?

the Z algorithm (Gusfield)

For a string T, Z[i] is the length of the longest prefix of T[i..m] that matches a prefix of T. Z[i] = 0 if the prefixes don't match.

T[0 .. Z[i]] = T[i .. i+Z[i] -1]

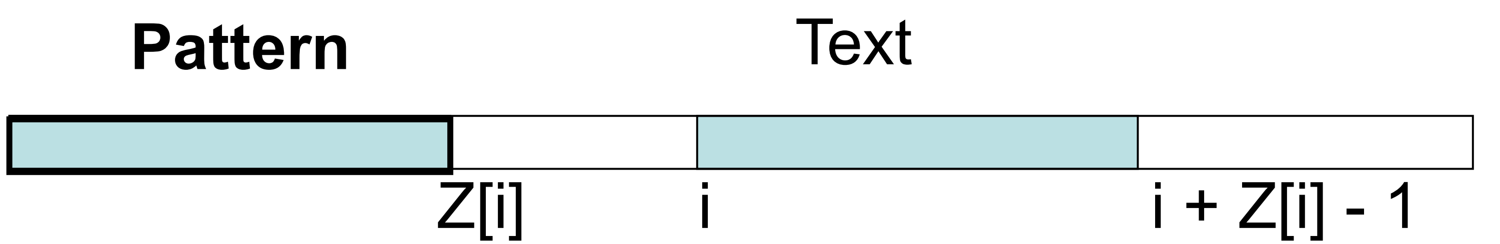


Example Z values

```
ACAGGTACAGTTCCCTCGACACCTACTACCTAAG
0010004010000000000302000200200001 10
```

Can the Z values help in matching?

Create string Pattern$Text where $ is not in the alphabet



If there exists i, s.t. Z[i] = length(Pattern)
Pattern occurs in the Text starting at i

example matching

```
CCTACT$ACAGGTACAGTTCCCTCGACACCTACTACCTAAG
01001000100000100002310100106100100410000
```

- What is the largest Z value possible?

Can Z values be computed in linear time?

AAAGGTACAGTTCCCTCGACACCTACTACCTAAG

Z[1]? compare T[1] with T[0], T[2] with T[1], etc. until mismatch

Z[1] = 2

This simple process is still expensive:

T[2] is compared when computing both Z[1] and Z[2].

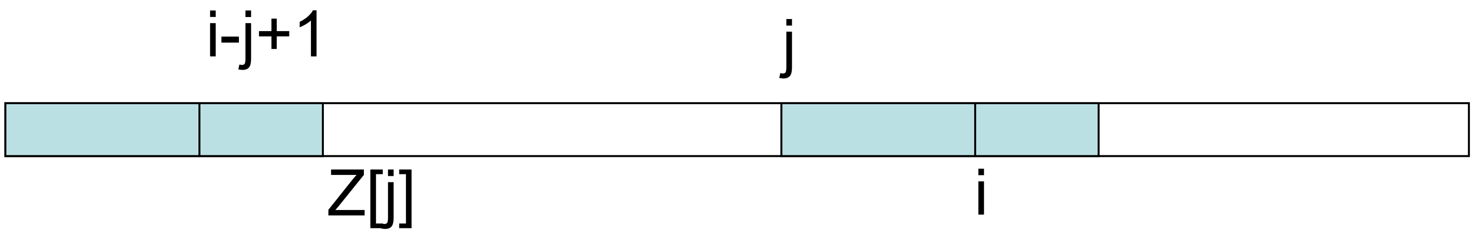
Trick to computing Z values in linear time:

each comparison must involve a character that was not compared before

Since there are only m characters in the string, the overall # of comparisons will be O(m).

Basic idea: 1-D dynamic programming

Can Z[i] be computed with the help of Z[j] for j < i?



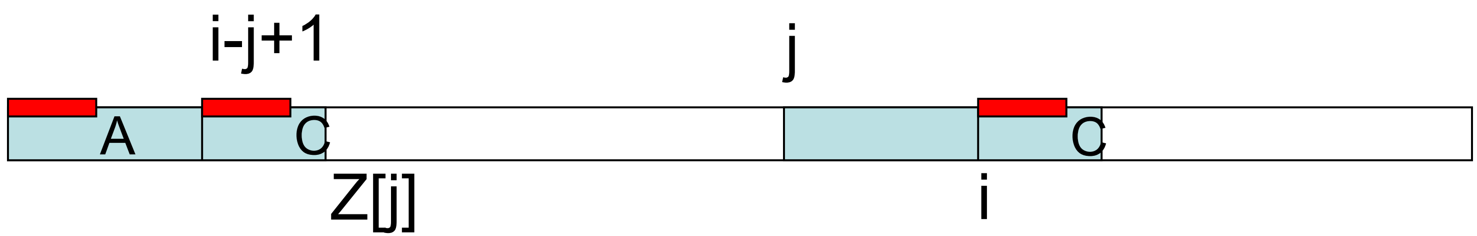
Assume there exists j < i, s.t. j + Z[j] – 1 > i
then Z[i – j + 1] provides information about Z[i]

If there is no such j, simply compare characters T[i..] to T[0..]
since they have not been seen before.

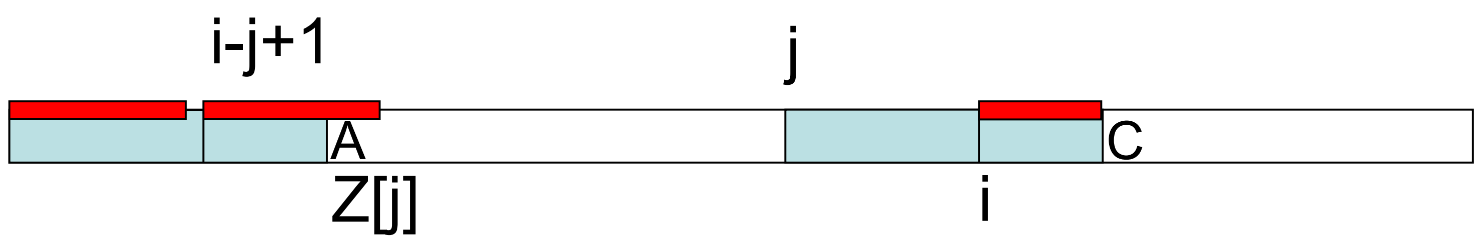
Three cases

Let $j < i$ be the coordinate that maximizes $j + Z[j] - 1$
(intuitively, the $Z[j]$ that extends the furthest)

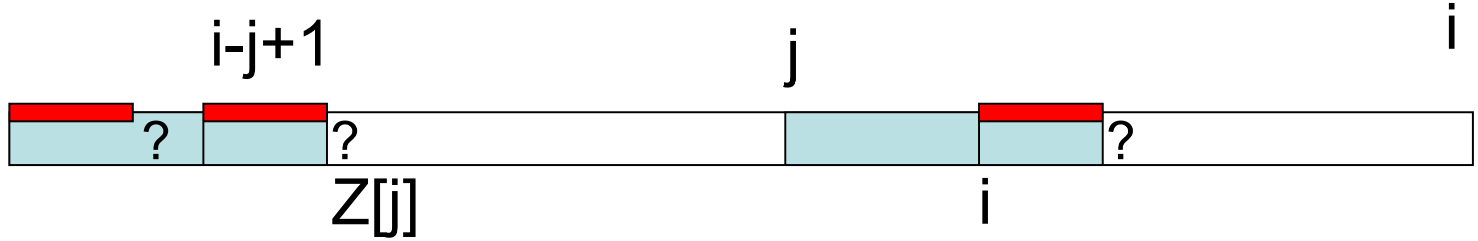
I. $Z[i - j + 1] < Z[j] - i + j - 1 \Rightarrow Z[i] = Z[i - j + 1]$



II. $Z[i - j + 1] > Z[j] - i + j - 1 \Rightarrow Z[i] = Z[j] - i + j - 1$



III. $Z[i - j + 1] = Z[j] - i + j - 1 \Rightarrow Z[i] = ??$, compare from $i + Z[i - j + 1]$



Time complexity analysis

- Why do these tricks save us time?

1. Cases I and II take constant time per Z-value computed – total time spent in these cases is O(n)
2. Case III might involve 1 or more comparisons per Z-value

   however:

   - every successful comparison (match) shifts the rightmost character that has been visited
   - every unsuccessful comparison terminates the “round” and algorithm moves on to the next Z-value

   total time spent in III cannot be more than # of characters in the text

Overall running time is O(n)

Space complexity?

- If using Z algorithm for matching, how many Z values do we need to store?

PPPPPPPPPP$TTTTTTTTTTTTTTTTTTTTTTTT

Some questions

- What are the Z-values for the following string:

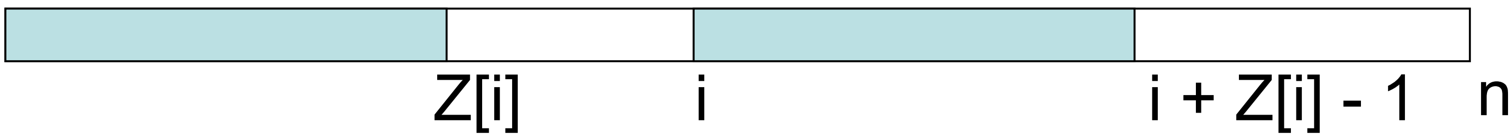
  TTAGGATAGCCATTAGCCTCATTAGGGATTAGGAT

- In the string above, what is the longest prefix that is repeated somewhere else in the string?

- Trace through the execution of the linear-time algorithm for computing the Z values for the string listed above. How many times do rules I, II, and III apply?

Z algorithm, not just for matching

- Lempel-Ziv compression (e.g. gzip)

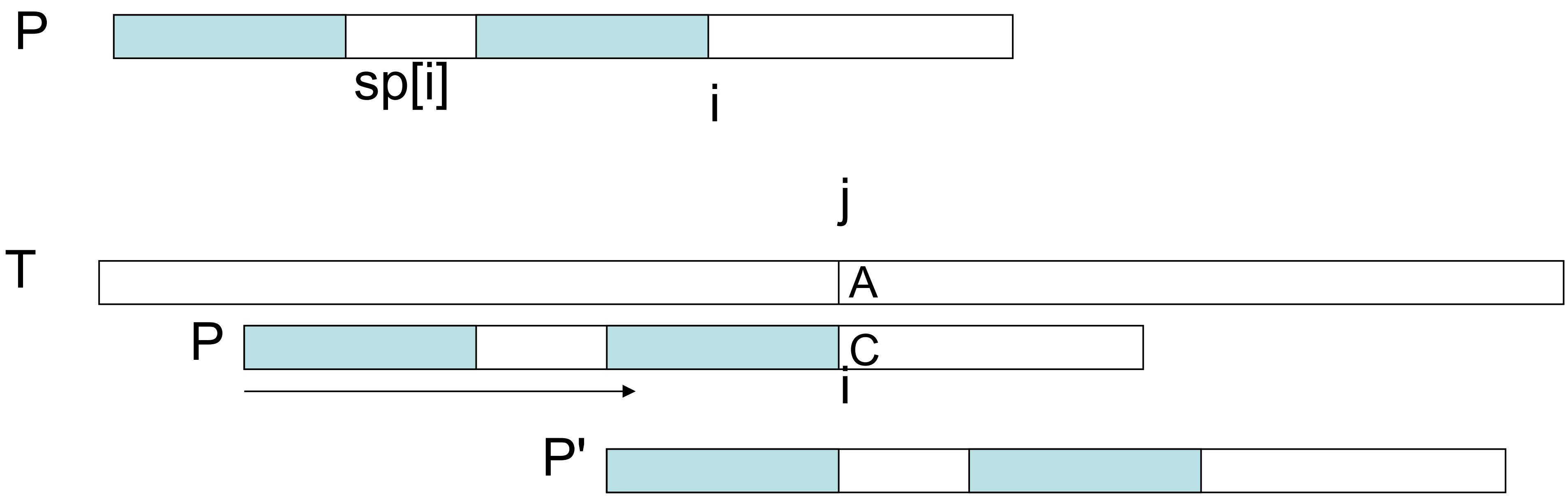


  if Z[i] = 0, just send/store the character T[i], otherwise, instead of sending T[i..i+Z[i] – 1] (Z[i] – 1 characters/bytes) simply send Z[i] (one number)

- Note: other exact matching algorithms used for data compression (e.g. Burrows-Wheeler transform relates to suffix arrays)

Knuth-Morris-Pratt algorithm

Given a Pattern and a Text, preprocess the Pattern to compute sp[i] = length of longest prefix of P that matches a suffix of P[0..i]



- Compare P with T until finding a mis-match
  (at coordinate i + 1 in P and j + 1 in T).
- Shift P such that first sp[i] characters match T[j – sp[i] + 1 .. j].
- Continue matching from T[i+1], P[sp[i]+1]

```
index:   0123456
pattern: AAAAAAA
sp:      0123456

index:   0123456
pattern: AAAAAAB
sp:      0123450
```

AAAAABAAAAAABAAAAAAA

```
index:    0123456
pattern: ABACABC
sp:      0010120
```

ABABBABAABABACABC

KMP

- Does it work?
- Can you miss a match by shifting too far?
- How do you prove that?

KMP – speed

- How many character comparisons are made during the execution?
- If a character in the text matches a character in the pattern, do we have to look at it again?
- How many times can a character in the text fail to match the pattern?

KMP – computing sp values

- Can sp values be computed efficiently?

- Can you use Z values?
- (aside – sp' values)

- Can you use induction as for the Z values?