Introduction to Logics of Knowledge and Belief

Eric Pacuit

University of Maryland

pacuit.org

epacuit@umd.edu

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Knowledge, Questions and Issues

J. van Benthem and S. Minica. *Toward a Dynamic Logic of Questions*. Journal of Philosophical Logic, 41(4), pp. 633 - 669, 2012.

A. Baltag, R. Boddy and S. Smets. *Group Knowledge in Interrogative Epistemology*. in *Jaakko Hintikka on Knowledge and Game-Theoretical Semantics*, pp. 131-164.

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Every family of questions $Quest \subseteq Quest_W$ can be 'compressed' into one big 'conjunctive' question: this is the least refined partition that refines every question in $Quest$, $\approx_{Quest} = \bigcap\{\approx_Q \mid Q \in Quest\}$

For $i \in \mathcal{A}$, let $\approx_i$ represent i's, *total question*.

"van Benthem and Minica call $\approx_i$ the agent i's *issue relation*.... it essentially captures agent i's conceptual indistinguishability relation, since it specifies the finest relevant world-distinctions that agent i makes....Two worlds $s \approx_i t$ are conceptually indistinguishable for agent i (since the answers to all i's questions are the same in both worlds): one can say that s and t will correspond to the same world in agent i's own "subjective model"." (Baltag et al.)

Epistemic Issue Model

$\mathcal{M} = \langle W, \{\rightarrow_i\}_{i\in\mathcal{A}}, \{\approx_i\}_{i\in\mathcal{A}}, V\rangle$, where

- W is a non-empty set of states
- For $i \in \mathcal{A}$, $\approx_i \subseteq W \times W$ is an equivalence relation (the issue relation)
- For $i \in \mathcal{A}$, $\rightarrow_i \subseteq W \times W$ is reflexive (the epistemic alternative relation)
- $V : \mathrm{At} \rightarrow \wp(W)$ is a valuation funciton

For $s \in W$, $s(i) = \{s' \mid s \rightarrow_i s'\}$ is the set of epistemic possibilities for i at s.

Open questions: The restriction $\approx_{i|_{s(a)}} = \approx_i \cap (s(a) \times s(a))$ represents i's current open isues at world s.

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Suppose that $P \subseteq W$ is a proposition. Then,

$K_i P = \{s \mid s \in W, s(i) \subseteq P\}$

$CP = \{s \mid \text{ for all } t, \text{ if } s(\bigcup_i \to_i)^+ t, \text{ then } t \in P\}$

$DP = \{s \mid \text{ for all } t, \text{ if } s(\bigcap_i \to_i) t, \text{ then } t \in P\}$

$Q_i P = \{s \mid \text{ for all } t, \text{ if } s \approx_i t, \text{ then } t \in P\}$

Conceptual indistinguishability implies epistemic indistinguishability: For all $i \in \mathcal{A}$, $\approx_i \subseteq \rightarrow_i$.

For all φ, $K_i\varphi \Rightarrow Q_i\varphi$

To know is to know the answer to a question: For all $i \in \mathcal{A}$, $\rightarrow_i \approx_i \subseteq \rightarrow_i$

For all φ, $K_i\varphi \Rightarrow K_iQ_i\varphi$

Selective Public Announcement

Principle of Selective Learning. When confronted with information, agents come to know only the information that is relevant for their issues.

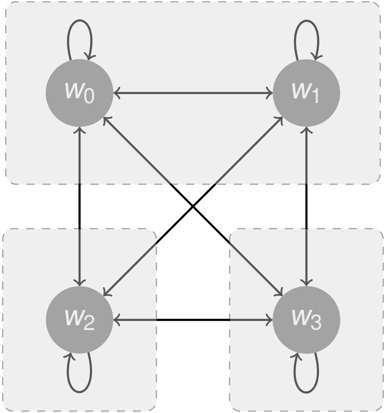
For any proposition $P \subseteq W$ and $i \in \mathcal{A}$, let P_i the *strongest i-relevant proposition entailed by* P:

$$P_i = \{s \in W \mid s \approx_i s' \text{ for some } s' \in P\}$$

Selective Public Announcement

Suppose that $\mathcal{M} = \langle W, \{\to_i\}_{i\in\mathcal{A}}, \{\approx_i\}_{i\in\mathcal{A}}, V\rangle$ is an epistemic issue model and $P \subseteq W$ is a proposition. A **selective public announcement** $!P$ is an action that changes $\mathcal{M}$ to $\mathcal{M}^P = \langle W^P, \{\to_i^P\}_{i\in\mathcal{A}}, \{\approx_i^P\}_{i\in\mathcal{A}}, V\rangle$, where

- $W^P = W$
- $\to_i^P = \to_i \cap \approx^{P_i}$
- $\approx_i^P = \approx_i$
- For all $p \in \mathrm{At}$, $V^P(p) = V(p)$.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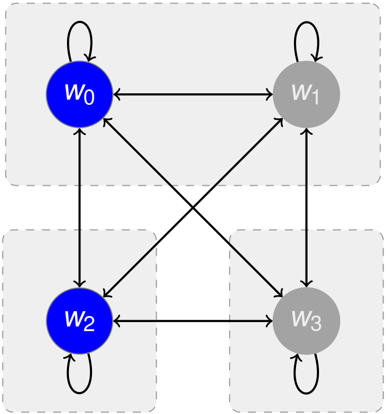
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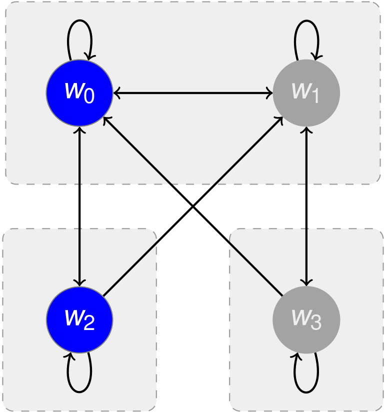

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I. Ciardelli and F. Roelofsen. *Inquisitive dynamic epistemic logic*. Synthese, 2015.

I. Ciardelli. *Modalities in the realm of questions: axiomatizing inquisitive epistemic logic*. Advances in Modal Logic, 2014.

Ivano Ciardelli, Jeroen Groenendijk, and Floris Roelofsen. *Inquisitive Semantics*. Oxford University Press, 2018.

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An **inquisitive model** is a tuple $\langle W, (\Sigma_i)_{i\in\mathcal{A}}, V\rangle$ where

- W is a non-empty set of possible worlds
- $V : W \to \wp(\mathrm{At})$ is a valuation function
- $\Sigma_i : W \to \Pi$ where $\Sigma_i(w)$ is an issue, satisfying:

  Factivity For all $w \in W$, $w \in \sigma_i(w)$
  Introspection For any $w, v \in W$ if $v \in \sigma_i(w)$, then $\Sigma_i(v) = \Sigma_i(w)$.

  where $\sigma_i(w) := \bigcup\Sigma_i(w)$ represents the information state of agent i in w.

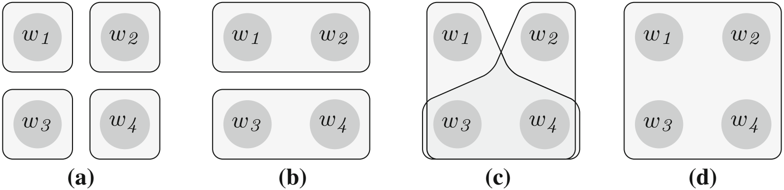

Fig. 1 Issues over the state $\{w1, w2, w3, w4\}$

1. For all $p \in \mathrm{At}$, $p \in \mathcal{L}_!$
2. For all $\bot \in \mathcal{L}_!$
3. If $\alpha_1, \ldots, \alpha_n \in \mathcal{L}_!$, then $?\{\alpha_1, \ldots, \alpha_n\} \in \mathcal{L}_?$
4. If $\varphi \in \mathcal{L}_\circ$ and $\psi \in \mathcal{L}_\circ$, then $\varphi \wedge \psi \in \mathcal{L}_\circ$
5. If $\alpha \in \mathcal{L}_!$ and $\psi \in \mathcal{L}_\circ$, then $\alpha \to \psi \in \mathcal{L}_\circ$
6. If $\varphi \in \mathcal{L}_\circ$, then $E_i\varphi \in \mathcal{L}_!$
7. If $\varphi \in \mathcal{L}_\circ$, then $K_i\varphi \in \mathcal{L}_!$

Interrogative: $?\{\alpha_1, \dots, \alpha_n\}$.

$?p$ means $?\{p, \neg p\}$

$K_i\varphi$: i knows that φ is true

$E_i\varphi$: i entertains φ being true

$K_i?p$ means "i knows whether p is true

$K_i?K_j?p$ "i knows whether j knows whether p is true

The following definition specifies recursively when a sentence is **supported** by a state s. Intuitively, for declaratives being supported amounts to being established, or true everywhere in s, while for interrogatives it amounts to being resolved in s.

1. $\mathcal{M}, s \models p$ iff $p \in V(w)$ for all $w \in s$.
2. $\mathcal{M}, s \models \bot$ iff $s = \emptyset$.
3. $\mathcal{M}, s \models ?\{\alpha_1, \ldots, \alpha_n\}$ iff $\mathcal{M}, s \models \alpha_i$ for some $1 \leq i \leq n$.
4. $\mathcal{M}, s \models \varphi \wedge \psi$ iff $\mathcal{M}, s \models \varphi$ and $\mathcal{M}, s \models \psi$.
5. $\mathcal{M}, s \models \alpha \rightarrow \varphi$ iff for any $t \subseteq s$, if $\mathcal{M}, t \models \alpha$, then $\mathcal{M}, t \models \varphi$.
6. $\mathcal{M}, s \models K_i\varphi$ iff for any $w \in s$, $\mathcal{M}, \sigma_i(w) \models \varphi$.
7. $\mathcal{M}, s \models E_i\varphi$ iff for any $w \in s$, for any $t \in \Sigma_i(w)$, $\mathcal{M}, t \models \varphi$.

Fact 1 (Persistency of support) If $\mathcal{M}, s \models \varphi$ and $t \subseteq s$, then $\mathcal{M}, t \models \varphi$.

Fact 2 (The empty state supports everything) For any $\mathcal{M}$ and any φ, $\mathcal{M}, \emptyset \models \varphi$

Fact 3 (Support for negation, disjunction, and polar interrogatives)

- $\mathcal{M}, s \models \neg\alpha$ iff for any non-empty $t \subseteq s$, $\mathcal{M}, t \not\models \alpha$
- $\mathcal{M}, s \models \alpha \vee \beta$ iff there are t_1, t_2 such that $s = t_1 \cup t_2$, and $\mathcal{M}, t_1 \models \alpha$ and $\mathcal{M}, t_2 \models \beta$
- $\mathcal{M}, s \models ?\alpha$ iff $\mathcal{M}, t \models \alpha$ or $\mathcal{M}, t \models \neg\alpha$

We say that a sentence φ **entails** ψ, notation $\varphi \models \psi$, just in case for all models $\mathcal{M}$ and states s, if $\mathcal{M}, s \models \varphi$ then $\mathcal{M}, s \models \psi$.

We say that a sentence φ is **valid** in case it is supported by all states in all models.

We say that two sentences φ and ψ are **equivalent**, notation $\varphi \equiv \psi$, just in case for all models $\mathcal{M}$ and states s, $\mathcal{M}, s \models \varphi$ iff $\mathcal{M}, s \models \psi$.

φ is **true** at w in $\mathcal{M}$ iff φ is supported by $\{w\}$ in $\mathcal{M}$

The **truth set** of a sentence φ in a model $\mathcal{M}$, denoted $|\varphi|_{\mathcal{M}}$, is defined as the set of worlds in $\mathcal{M}$ where φ is true:
$|\varphi|_{\mathcal{M}} := \{w \in W \mid \mathcal{M}, w \models \varphi\}$

The **proposition** $[\varphi]_{\mathcal{M}}$ expressed by a sentence φ in a model $\mathcal{M}$ is the set of all states in $\mathcal{M}$ that support φ:
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We have that $|?p|_{\mathcal{M}} = |?q|_{\mathcal{M}}$, but $[?p]_{\mathcal{M}} \neq [?q]_{\mathcal{M}}$

Fact: For any φ and any model $\mathcal{M}$, $|\varphi|_{\mathcal{M}} = \bigcup[\varphi]_{\mathcal{M}}$

Fact (Truth and support) For any model $\mathcal{M}$, any state s and any declarative α, the following holds:

$$\mathcal{M}, s \models \alpha \text{ iff } \mathcal{M}, w \models \alpha \text{ for all } w \in s$$

$\mathcal{M}, s \models \alpha \rightarrow \varphi$ iff $\mathcal{M}, s \cap |\alpha|_{\mathcal{M}} \models \varphi$

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If Ann invites Bill to the party, will he go? ($p \rightarrow ?q$)

Answers:

- Yes, if Ann invites Bill, he will go. ($p \rightarrow q$)
- No, if Ann invites Bill, he will not go. ($p \rightarrow \neg q$)

Knowledge

For declaratives α, $K_i\alpha$ boils down to the usual definition of truth of a modality familiar from modal logic.

For interrogatives μ, $K_i\mu$ holds when μ is resolved in $\sigma_i(w)$, which means that $K_i\mu$ expresses the fact that i has sufficient information to resolve μ at w.

For instance, $K_i?p$ is true at w just in case that $\sigma_i(w)$ supports either p or $\neg p$. That is, when i knows whether p is true.

Entertaining

$E_i\varphi$ is true at w just in case φ is supported by any state $t \in \Sigma_i(w)$

Fact. For any φ, $K_i\varphi \models E_i\varphi$

Fact. For any declarative α, $K_i\alpha \equiv E_i\alpha$

$W_i\varphi$ means "*i* wonders about φ: $W_i\varphi := \neg K_i\varphi \wedge E_i\varphi$

- $\mathcal{M}, w \models K_i\varphi$ iff $\bigcup \Sigma_i(w) \in [\varphi]_{\mathcal{M}}$
- $\mathcal{M}, w \models E_i\varphi$ iff $\Sigma_i(w) \subseteq [\varphi]_{\mathcal{M}}$

Public Announcement

Given $\mathcal{M} = \langle W, (\Sigma_i)_{i \in \mathcal{A}}, V \rangle$, the public announcement of φ transform $\mathcal{M}$ to $\mathcal{M}^\varphi = \langle W^\varphi, (\Sigma_i^\varphi)_{i \in \mathcal{A}}, V^\varphi \rangle$, where

- $W^\varphi = W \cap |\varphi|_{\mathcal{M}}$
- $V^\varphi = V_{|W^\varphi}$
- For all $w \in W^\varphi$, $\Sigma_i^\varphi(w) = \Sigma_i(w) \cap [\varphi]_{\mathcal{M}}$

For any φ, $\sigma_i^\varphi(w) = \sigma_i(w) \cap |\varphi|_{\mathcal{M}}$

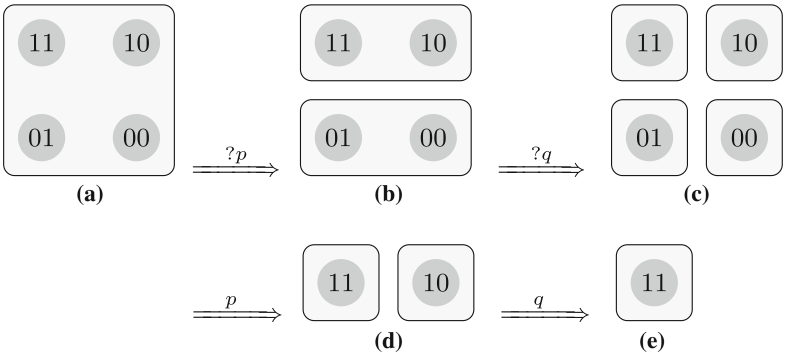

Fig. 2 The effects of a series of simple announcements on a state

Y. Wang. *Beyond knowing that: A new generation of epistemic logics*. 2016.

We have been studying “knowing that” expressions, but we often use the verb “know” with an embedded question such as:

- I know whether the claim is true.
- I know what your password is.
- I know how to swim.
- I know why he was late.
- I know who proved this theorem.
- I know where she has been.

Knowing Whether

$Kw_i\varphi$ means that i knows whether φ is true.

$Kw_i\varphi \leftrightarrow Kw_i\neg\varphi$ is valid
$Kw_iKw_j\varphi \rightarrow Kw_i\varphi$ is not valid

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$\triangle\varphi \ := \ \Box\varphi \vee \Box\neg\varphi$ means that φ is *not contingent*

S. Hart, A. Heifetz, and D. Samet. *Knowing whether, knowing that, and the cardinality of state spaces*. Journal of Economic Theory, 70(1):249 - 256, 1996.

L. Humberstone. *The logic of non-contingency*. Notre Dame Journal of Formal Logic, 36(2):214 - 229, 1995.

S. Kuhn. *Minimal non-contingency logic*. Notre Dame Journal of Formal Logic, 36(2):230 - 234, 1995..

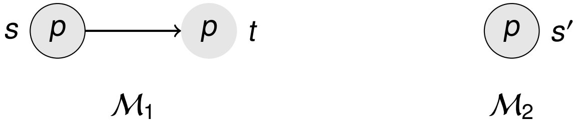
H. van Ditmarsch, J. Fan and Y. Wang. *Contingency and knowing whether*. Review of Symbolic Logic 8(1):75-107, 2015.

NCL Logic

$$\varphi \ ::= \ \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid \ \triangle_i \, \varphi$$

$\mathcal{M} = \langle W, (R_i)_{i \in \mathcal{A}}, V \rangle$ where

$\mathcal{M}, w \models \triangle_i \varphi$ iff for all v_1, v_2, if wR_iv_1 and wR_iv_2, then $\mathcal{M}, v_1 \models \varphi$ iff $\mathcal{M}, v_2 \models \varphi$



$\mathcal{M}_1, s$ and $\mathcal{M}_2, s'$ satisfy the *NCL* formulas, but can be distinguished by formulas of modal logic.

- $\neg \triangle_i \psi \rightarrow (\Box_i\varphi \leftrightarrow (\triangle_i\varphi \wedge \triangle_i(\psi \rightarrow \varphi)))$ is valid

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- It is impossible to use *NCL* formulas to capture frame properties.

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- *NCL* is not normal, e.g., $(\triangle_i(\varphi \rightarrow \psi) \wedge \triangle_i\varphi) \rightarrow \triangle_i\psi$ is not valid.

- $\neg \triangle_i \psi \rightarrow (\Box_i\varphi \leftrightarrow (\triangle_i\varphi \wedge \triangle_i(\psi \rightarrow \varphi)))$ is valid
- It is impossible to use *NCL* formulas to capture frame properties.
- *NCL* is not normal, e.g., $(\triangle_i(\varphi \rightarrow \psi) \wedge \triangle_i\varphi) \rightarrow \triangle_i\psi$ is not valid.
- *NCL* is not strictly weaker than modal logic, $\triangle_i\varphi \leftrightarrow \triangle_i\neg\varphi$ is valid.

- all instances of tautologies
- $(\triangle_i(q \to p) \wedge \triangle_i(\neg q \to p)) \to \triangle_i p$
- $(\triangle_i p \to (\triangle_i(p \to q) \vee \triangle_i(\neg p \to q))$
- $\triangle_i p \leftrightarrow \triangle_i \neg p$
- from $\varphi, \varphi \to \psi$, infer ψ
- from φ, infer $\triangle_i \varphi$
- from φ, infer $\varphi[p/\psi]$
- from $\varphi \leftrightarrow \psi$, infer $\triangle_i \varphi \leftrightarrow \triangle_i \psi$

Theorem. (Fan et al (2015)). The above axioms are sound and strongly complete over the class of arbitrary frames.

Public announcement logic is defined as usual.

$$[\varphi] \triangle_i \psi \leftrightarrow (\varphi \rightarrow (\triangle_i[\varphi]\psi \vee \triangle_i[\varphi]\neg\psi))$$

$$[?\varphi]\psi \leftrightarrow ([\varphi]\psi \wedge [\neg\varphi]\psi)$$

Knowing what

i knows what the value of c

$\exists x K_i(c = x)$

Knowing what

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_i\varphi \mid Kv_i c$$

where $p \in \mathrm{At}$ and $c \in \mathbf{C}$ (a set of constant symbols)

$\mathcal{M} = \langle W, D, (R_i)_{i\in\mathcal{A}}, V, V_C \rangle$

where $W \neq \emptyset$, each R_i is a relation on W, $V : \mathrm{At} \to \wp(W)$, D is the constant domain and $V_C : \mathbf{C} \times W \to D$ assigns to each $c \in \mathbf{C}$ and world w a value $d \in D$.

$$\mathcal{M}, w \models Kv_i c \text{ iff for any } v_1, v_2, \text{ if } wR_iv_1 \text{ and } wR_iv_2,$$

$$\text{then } V_C(v_1, c) = v_C(c, v_2)$$

$$K_i Kv_j c \wedge \neg Kv_j c \quad \text{vs.} \quad K_i K_j p \wedge \neg K_i p$$

$$K_i Kv_j c \wedge \neg Kv_j c \quad \text{vs.} \quad K_i K_j p \wedge \neg K_i p$$

$$\varphi ::= \top \mid p \mid \neg\varphi \mid (\varphi \wedge \varphi) \mid K_i\varphi \mid Kv_i c \mid [\varphi]\varphi$$

$(\langle p\rangle Kv_i c \wedge \langle q\rangle Kv_i c) \rightarrow \langle p \vee c\rangle Kv_i c$ is not derivable is S5 with recursion axioms.

Y. Wang and J. Fan. *Knowing that, knowing what, and public communication: Public announcement logic with Kv operators*. In: Proceedings of IJCAI?13, pp 1139 - 1146, 2013.

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Know how

J. Fantl. *Knowing-how and knowing-that*. Philosophy Compass, 3 (2008), 451 470.

M.P. Singh. *Know-how*. In Foundations of Rational Agency (1999), M. Woodridge and A. Rao, Eds., pp. 105 132.

Actions

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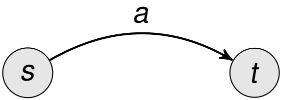

Actions

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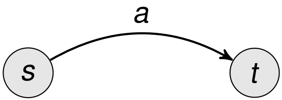
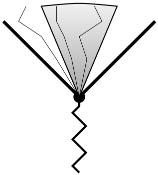
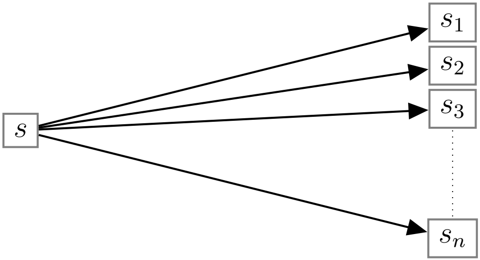


2. Actions *restrict* the set of possible future histories.



J. van Benthem, H. van Ditmarsch, J. van Eijck and J. Jaspers. *Chapter 6: Propositional Dynamic Logic*. Logic in Action Online Course Project, 2011.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Propositional Dynamic Logic

Language: The language of propositional dynamic logic is generated by the following grammar:

$$p \mid \neg\varphi \mid \varphi \wedge \psi \mid [\alpha]\varphi$$

where $p \in \mathrm{At}$ and α is generated by the following grammar:

$$a \mid \alpha \cup \beta \mid \alpha;\beta \mid \alpha^* \mid \varphi?$$

where $a \in \mathrm{Act}$ and φ is a formula.

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Semantics: $\mathcal{M} = \langle W, \{R_a \mid a \in \mathrm{P}\}, V\rangle$ where for each $a \in \mathrm{P}$, $R_a \subseteq W \times W$ and $V : \mathrm{At} \to \wp(W)$

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$[\alpha]\varphi$ means "after doing α, φ will be true"

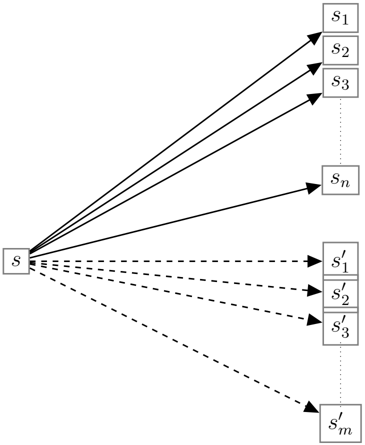
$\langle\alpha\rangle\varphi$ means "after doing α, φ may be true"

$\mathcal{M}, w \models [\alpha]\varphi$ iff for each v, if $wR_\alpha v$ then $\mathcal{M}, v \models \varphi$

$\mathcal{M}, w \models \langle\alpha\rangle\varphi$ iff there is a v such that $wR_\alpha v$ and $\mathcal{M}, v \models \varphi$

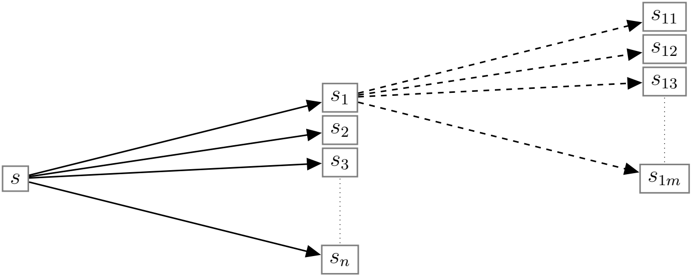
Union

$R_{\alpha\cup\beta} := R_\alpha \cup R_\beta$



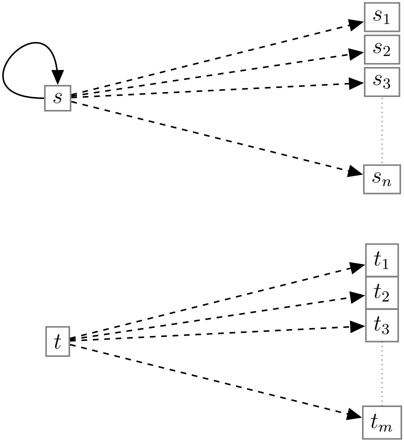
Sequence

$$R_{\alpha;\beta} := R_\alpha \circ R_\beta$$



Test

$$R_{\varphi?} = \{(w, w) \mid \mathcal{M}, w \models \varphi\}$$



Iteration

$$R_{\alpha^*} := \cup_{n\geq 0} R_\alpha^n$$

Propositional Dynamic Logic

1. Axioms of propositional logic
2. $[\alpha](\varphi \rightarrow \psi) \rightarrow ([\alpha]\varphi \rightarrow [\alpha]\psi)$
3. $[\alpha \cup \beta]\varphi \leftrightarrow [\alpha]\varphi \wedge [\beta]\varphi$
4. $[\alpha; \beta]\varphi \leftrightarrow [\alpha][\beta]\varphi$
5. $[\psi?]\varphi \leftrightarrow (\psi \rightarrow \varphi)$
6. $\varphi \wedge [\alpha][\alpha^*]\varphi \leftrightarrow [\alpha^*]\varphi$
7. $\varphi \wedge [\alpha^*](\varphi \rightarrow [\alpha]\varphi) \rightarrow [\alpha^*]\varphi$
8. Modus Ponens and Necessitation (for each program α)

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5. $[\psi?]\varphi \leftrightarrow (\psi \to \varphi)$
6. $\varphi \wedge [\alpha][\alpha^*]\varphi \leftrightarrow [\alpha^*]\varphi$ (Fixed-Point Axiom)
7. $\varphi \wedge [\alpha^*](\varphi \to [\alpha]\varphi) \to [\alpha^*]\varphi$ (Induction Axiom)
8. Modus Ponens and Necessitation (for each program α)

Actions and Ability

An early approach to interpret PDL as logic of actions was put forward by Krister Segerberg.

Segerberg adds an "agency" program to the PDL language δA where A is a formula.

K. Segerberg. *Bringing it about*. JPL, 1989.

Actions and Agency

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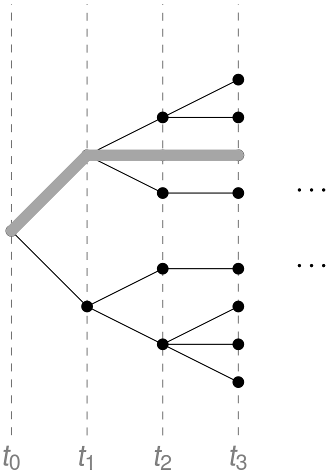
The axioms:

1. $[\delta A]A$
2. $[\delta A]B \to ([\delta B]C \to [\delta A]C)$

Actions and Agency in Branching Time

Alternative accounts of agency do not include explicit description of the actions:



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- Formulas are interpreted at history moment pairs.
- At each moment there is a choice available to the agent (partition of the histories through that moment)
- The key modality is $[i\ \textit{stit}]\varphi$ which is intended to mean that the agent *i* can "see to it that φ is true".
  - $[i\ \textit{stit}]\varphi$ is true at a history moment pair provided the agent can choose a (set of) branch(es) such that every future history-moment pair satisfies φ

STIT

We use the modality '$\Diamond$' to mean historic possibility.

$\Diamond[i\ \mathit{stit}]\varphi$: "the agent has the ability to bring about φ".

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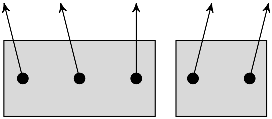
Many Agents

The previous model assumes there is *one* agent that “controls” the transition system.



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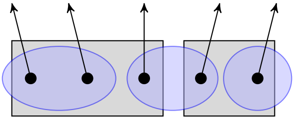

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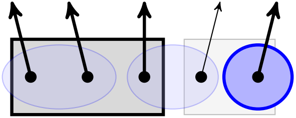

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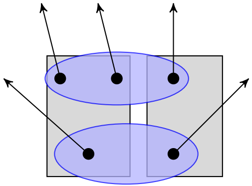

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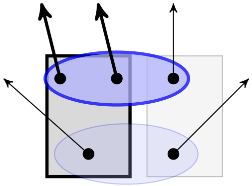

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$$\varphi \;=\; p \mid \neg\varphi \mid \varphi \wedge \psi \mid [i\ stit]\varphi \mid [i\ dstit:\ \varphi] \mid \Box\varphi$$

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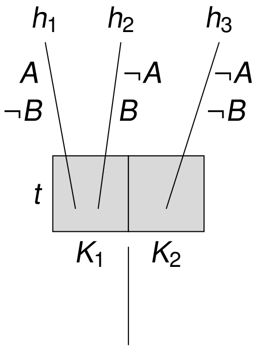
- $\mathcal{M}, t/h \models [i\ dstit]\varphi$ iff $\mathcal{M}, t/h' \models \varphi$ for all $h' \in Choice_i^t(h)$ and there is a $h'' \in H_t$ such that $\mathcal{M}, t/h \models \neg\varphi$

STIT: Example

The following are false: $A \to \Diamond[stit]A$ and $\Diamond[stit](A \vee B) \to \Diamond[stit]A \vee \Diamond[stit]B$.



J. Horty. *Agency and Deontic Logic*. 2001.

STIT: Axiomatics

- **S5** for $\Box$: $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$, $\Box\varphi \rightarrow \varphi$, $\Box\varphi \rightarrow \Box\Box\varphi$, $\neg\Box\varphi \rightarrow \Box\neg\Box\varphi$

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- $\Box\varphi \rightarrow [i\ \mathit{stit}]\varphi$
- $(\bigwedge_{i\in\mathcal{A}} \Diamond[i\ \mathit{stit}]\varphi_i) \rightarrow \Diamond(\bigwedge_{i\in\mathcal{A}}[i\ \mathit{stit}]\varphi_i)$
- Modus Ponens and Necessitation for $\Box$

M. Xu. *Axioms for deliberative STIT*. Journal of Philosophical Logic, Volume 27, pp. 505 - 552, 1998.

P. Balbiani, A. Herzig and N. Troquard. *Alternative axiomatics and complexity of deliberative STIT theories*. Journal of Philosophical Logic, 37:4, pp. 387 - 406, 2008.

Recap: Logics of Action and Ability

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- $\Diamond[i\ \mathit{stit}]\varphi$: the agent has the ability to bring about φ

Epistemizing logics of action and ability

Related Work: Knowing How to Execute a Plan

J. van Benthem. *Games in dynamic epistemic logic*. Bulletin of Economics Research 53, 4 (2001), 219 248..

J. Broersen. *A logical analysis of the interaction between Obligation-to- do and knowingly doing*. In Proceedings of DEON 2008.

Y. Lesperance, H. Levesque, F. Lin and R. Scherl. *Ability and Knowing How in the Situation Calculus*. Studia Logica 65, pgs. 165 - 186, 2000.

W. Jamroga and T. Agotnes. *Constructive Knowledge: What Agents can Achieve under Imperfect Information*. Journal of Applied Non-Classical Logics 17(4):423–425, 2007.

Knowledge, action, abilities

A. Herzig and N. Troquard. *Knowing how to play: uniform choices in logics of agency*. Proceedings of AAMAS 2006, pgs. 209 - 216.

A. Herzig. *Logics of knowledge and action: critical analysis and challenges*. Autonomous Agent and Multi-Agent Systems, 2014.

J. Broeresen, A. Herzig and N. Troquard. *What groups do, can do and know they can do: An analysis in normal modal logics*. Journal of Applied and Non-Classical Logics, 19:3, pgs. 261 - 289, 2009.

W. van der Hoek and M. Wooldridge. *Cooperation, knowledge and time: Alternating-time temporal epistemic logic and its applications*. Studia Logica, 75, pgs. 125 - 157, 2003.

Example

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Example

Ann, who is blind, is standing with her hand on a light switch. She has two options: toggle the switch (t) or do nothing (s):

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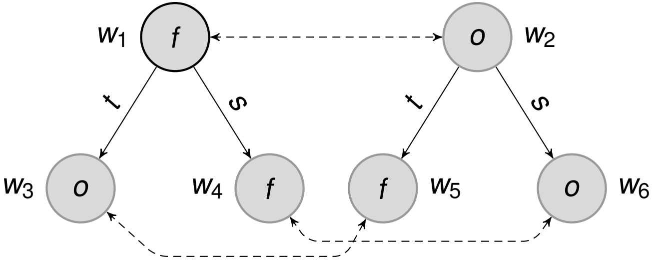

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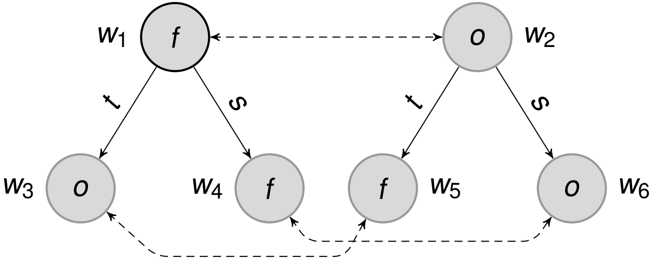
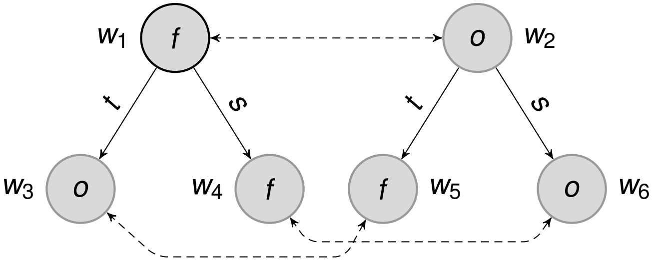


Does she have the *ability* to turn the light on? Is she *capable* of turning the light on? Does she *know how* to turn the light on?

Example



$w_1 \models \neg\Box f$: "Ann does not know the light is on"

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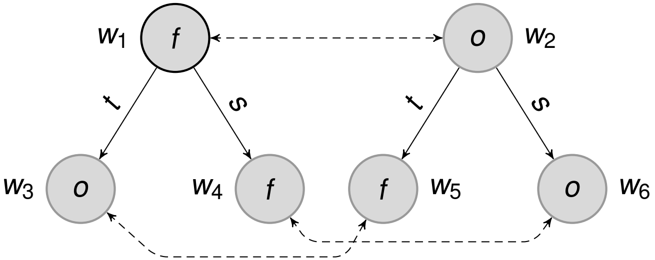

$w_1 \models \langle t \rangle o$ “after toggling the light switch, the light will be on”

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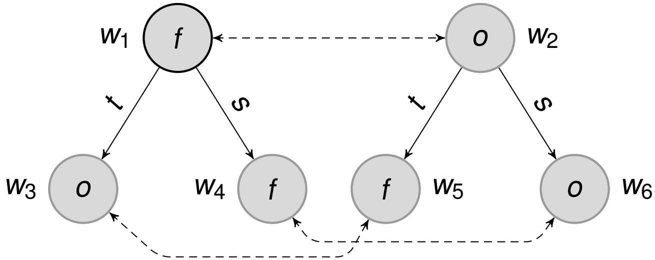
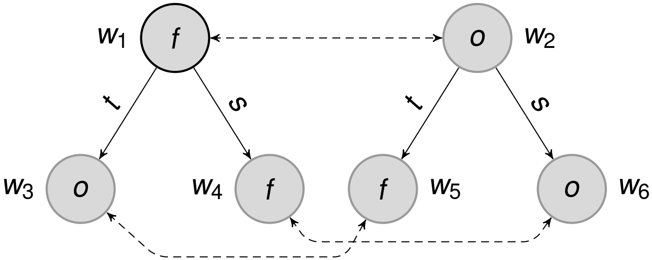


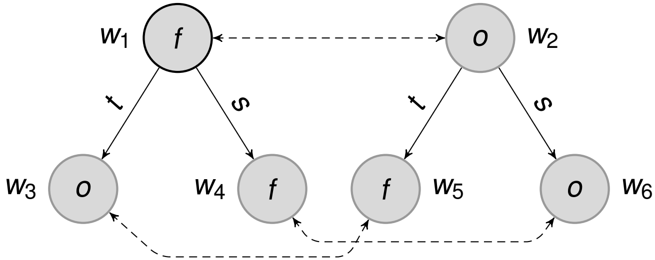
$w_1 \models \neg\Box\langle t\rangle o$: “Ann does not know that after toggling the light switch, the light will be on”

Example



$w_1 \models \Box(\langle t\rangle\top \wedge \langle s\rangle\top)$: "Ann knows that she can toggle the switch and she can do nothing"

Example



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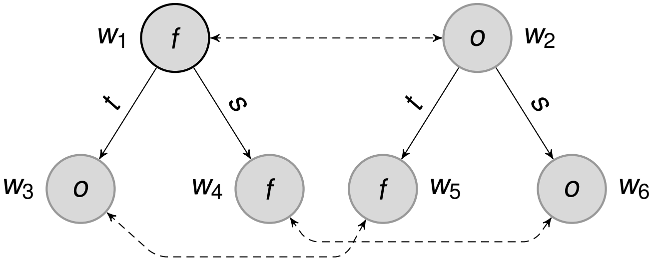
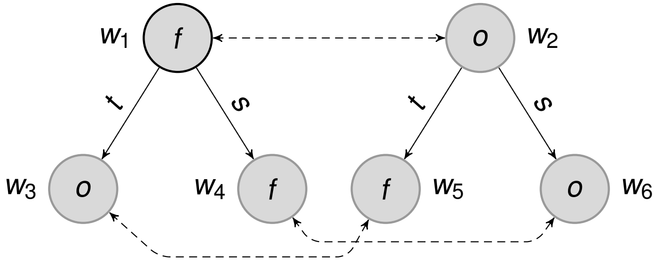


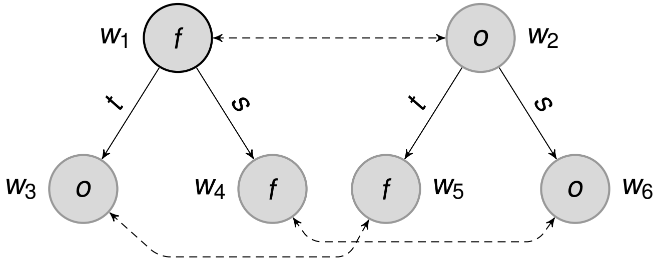
Let l be "turn the light on": a choice between t and s

Example



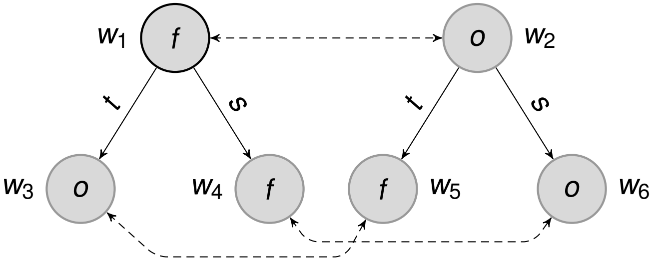
$w_1 \models \langle l \rangle^{\exists} o \wedge \neg \langle l \rangle^{\forall} o$: executing l can lead to a situation where the light is on, but this is not *guaranteed* (i.e., the plan may fail)

Example



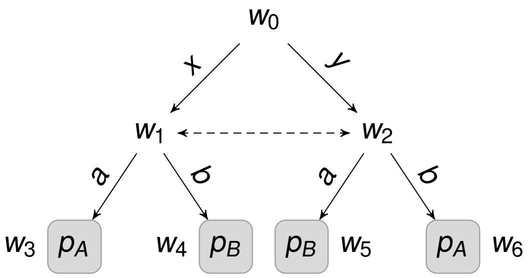
$w_1 \models \Box\langle l\rangle^{\exists} o$: Ann knows that she is capable of turning the light on. She has *de re* knowledge that she can turn the light on.

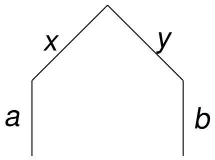
Example



$w_1 \models \neg\langle l\rangle^{\diamond} o$: Ann cannot knowingly turn on the light: there is no *subjective* path leading to states satisfying o (note that *all* elements of the last element of the subject path must satisfy o).

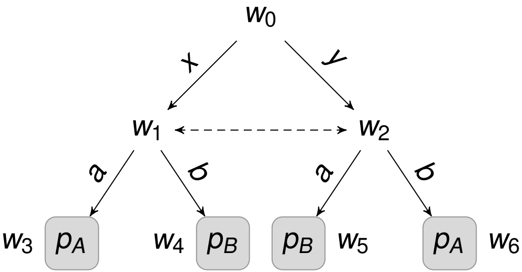
Knowing How to Win

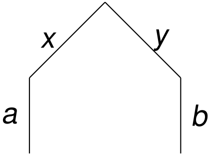




"the plan is a winning strategy for Ann."

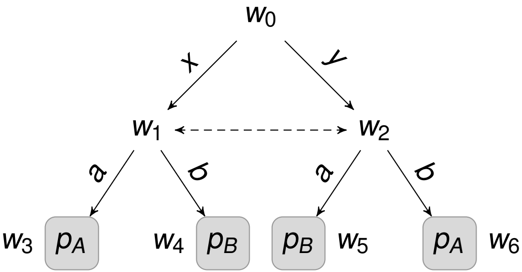
Knowing How to Win

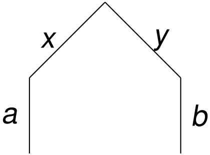




"Ann knows that the plan is a winning strategy."

Knowing How to Win





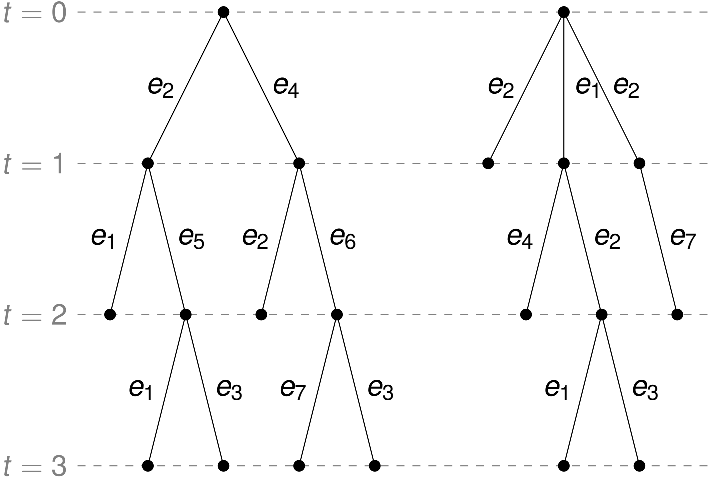
" the plan can be executed, but Ann does not know how to use it to win."

Epistemic Temporal Logic

R. Parikh and R. Ramanujam. *A Knowledge Based Semantics of Messages*. *Journal of Logic, Language and Information*, 12: 453 – 467, 1985, 2003.

FHMV. Reasoning about Knowledge. MIT Press, 1995.

The ‘Playground’



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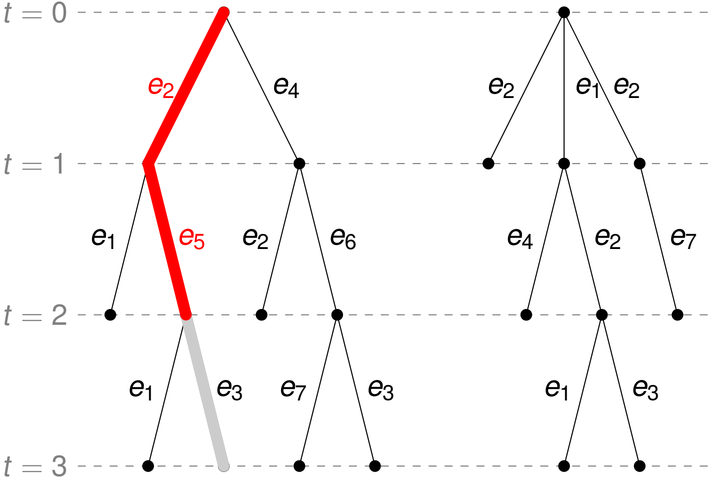

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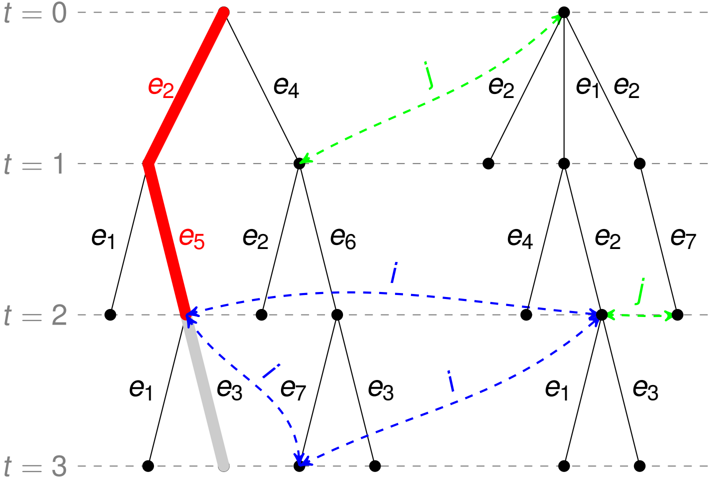

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- ϵ is the empty string and $\mathrm{FinPre}_{-\epsilon}(\mathcal{H}) = \mathrm{FinPre}(\mathcal{H}) - \{\epsilon\}$.

History-based Frames

Definition

Let Σ be any set of events. A set $\mathcal{H} \subseteq \Sigma^* \cup \Sigma^\omega$ is called a protocol provided $\text{FinPre}_{-\epsilon}(\mathcal{H}) \subseteq \mathcal{H}$. A rooted protocol is any set $\mathcal{H} \subseteq \Sigma^* \cup \Sigma^\omega$ where $\text{FinPre}(\mathcal{H}) \subseteq \mathcal{H}$.

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Definition

An ETL frame is a tuple $\langle \Sigma, \mathcal{H}, \{\sim_i\}_{i \in \mathcal{A}} \rangle$ where Σ is a (finite or infinite) set of events, $\mathcal{H}$ is a protocol, and for each $i \in \mathcal{A}$, $\sim_i$ is an equivalence relation on the set of finite strings in $\mathcal{H}$.

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Some assumptions:

1. If Σ is assumed to be finite, then we say that $\mathcal{F}$ is **finitely branching.**
2. If $\mathcal{H}$ is a rooted protocol, $\mathcal{F}$ is a **tree frame.**

Formal Languages

- $P\varphi$ (φ is true *sometime* in the past),
- $F\varphi$ (φ is true *sometime* in the future),
- $Y\varphi$ (φ is true at *the* previous moment),
- $N\varphi$ (φ is true at *the* next moment),
- $N_e\varphi$ (φ is true after event e)
- $K_i\varphi$ (agent i knows φ) and
- $C_B\varphi$ (the group $B \subseteq \mathcal{A}$ commonly knows φ).

History-based Models

An ETL **model** is a structure $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}, V\rangle$ where $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}\rangle$ is an ETL frame and

$V : \mathrm{At} \rightarrow 2^{\mathrm{finite}(\mathcal{H})}$ is a valuation function.

Formulas are interpreted at pairs H, t:

$$H, t \models \varphi$$

Truth in a Model

- $H, t \models P\varphi$ iff there exists $t' \leq t$ such that $H, t' \models \varphi$
- $H, t \models F\varphi$ iff there exists $t' \geq t$ such that $H, t' \models \varphi$
- $H, t \models N\varphi$ iff $H, t + 1 \models \varphi$
- $H, t \models Y\varphi$ iff $t > 1$ and $H, t - 1 \models \varphi$
- $H, t \models K_i\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_i H'_m$ then $H', m \models \varphi$
- $H, t \models C\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_* H'_m$ then $H', m \models \varphi$.

where $\sim_*$ is the reflexive transitive closure of the union of the $\sim_i$.

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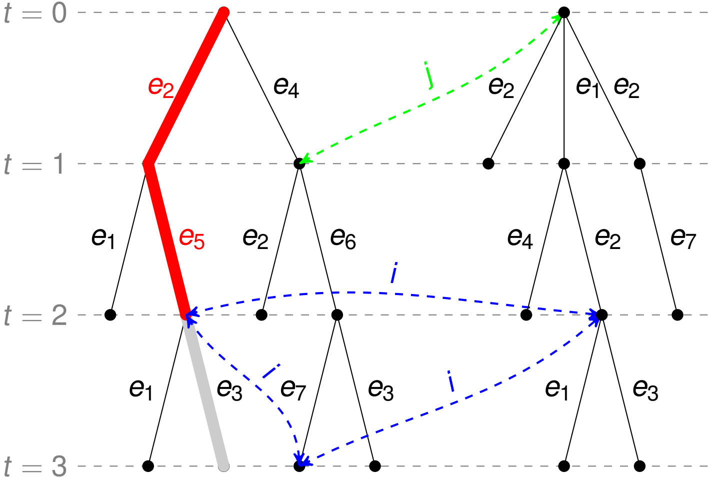

t = 0
t = 1
t = 2
t = 3
e2
e4
j
e2
e1
e2
e1
e5
e2
e6
e4
e2
e7
i
e1
e3
e7
e3
i
e1
e3

An Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

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There is a very simple procedure to solve Ann's problem: *have a (trusted) friend tell Bob the time and subject of her talk.*

Is this procedure correct?

An Example

Yes, if

1. Ann knows about the talk.

An Example

Yes, if

1. Ann knows about the talk.
2. Bob knows about the talk.

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1. Ann knows about the talk.
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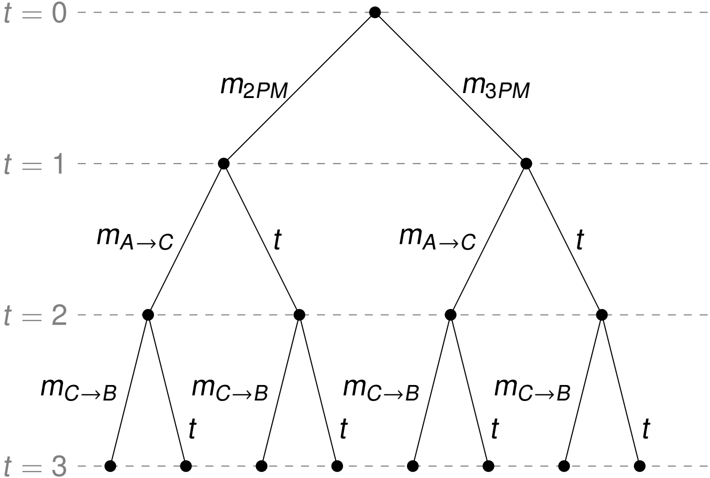
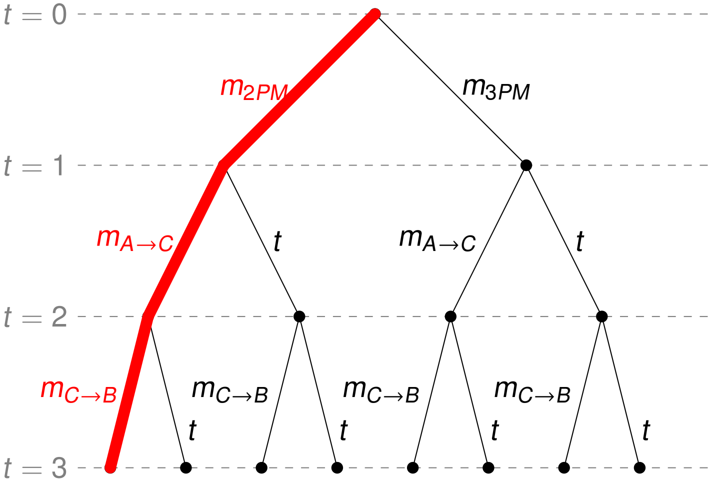
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.
4. Bob *does not* know that Ann knows that he knows about the talk.

An Example

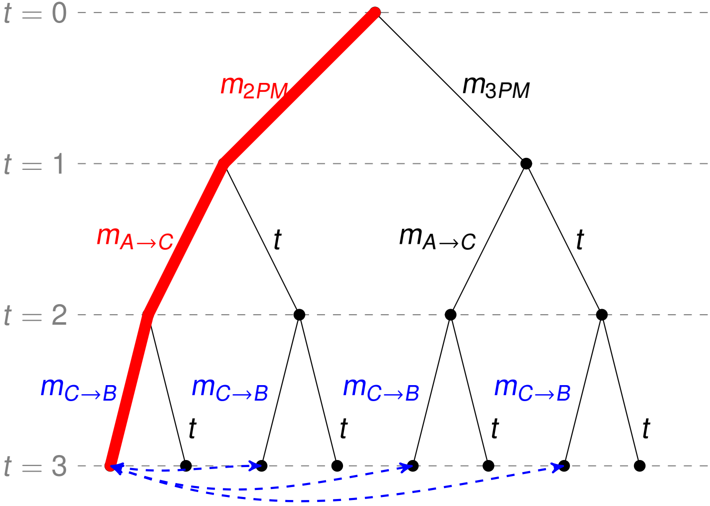
Yes, if

1. Ann knows about the talk.
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.
4. Bob *does not* know that Ann knows that he knows about the talk.
5. *And nothing else.*

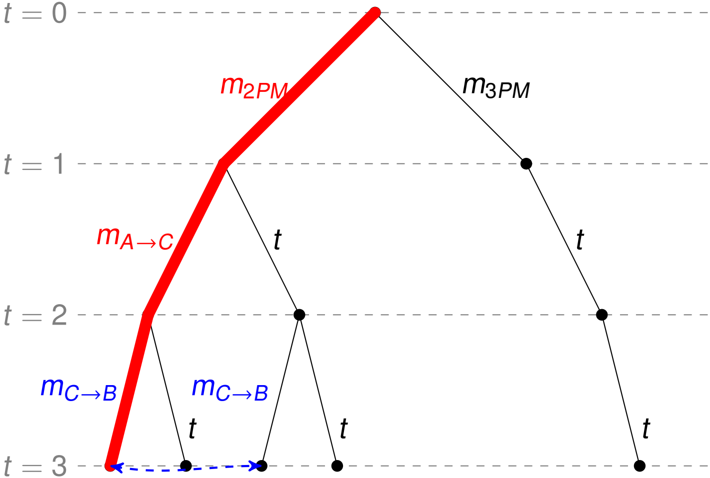

$t = 0$
$t = 1$
$t = 2$
$t = 3$
m_{2PM}
m_{3PM}
$m_{A\to C}$
t
$m_{A\to C}$
t
$m_{C\to B}$
t
$m_{C\to B}$
t
$m_{C\to B}$
t
$m_{C\to B}$
t



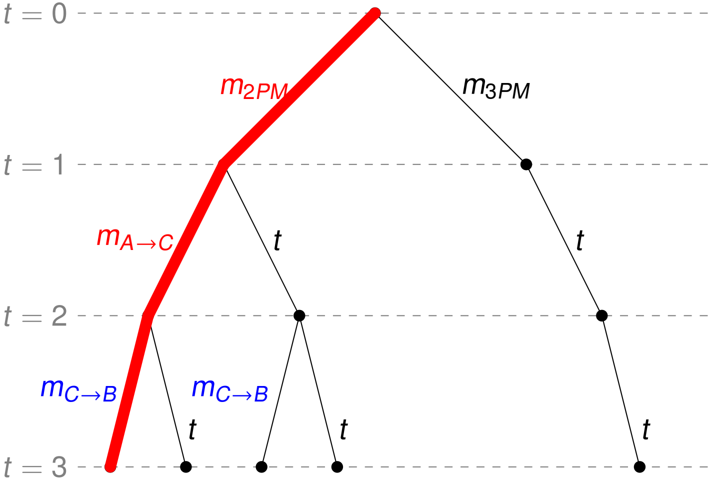
$$H, 3 \models \varphi$$



Bob's uncertainty: $H, 3 \models \neg K_B P_{2PM}$



Bob's uncertainty + 'Protocol information': $H, 3 \models K_B P_{2PM}$



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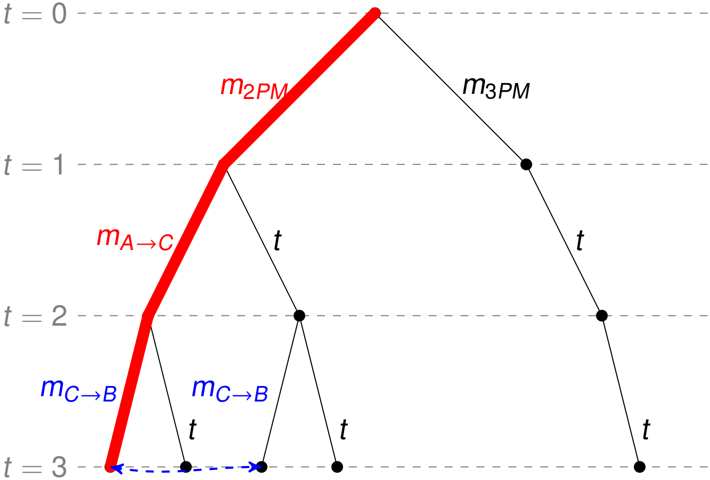

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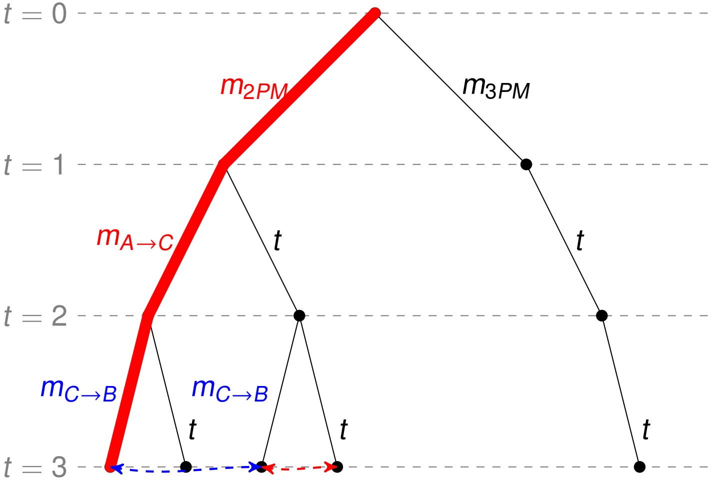

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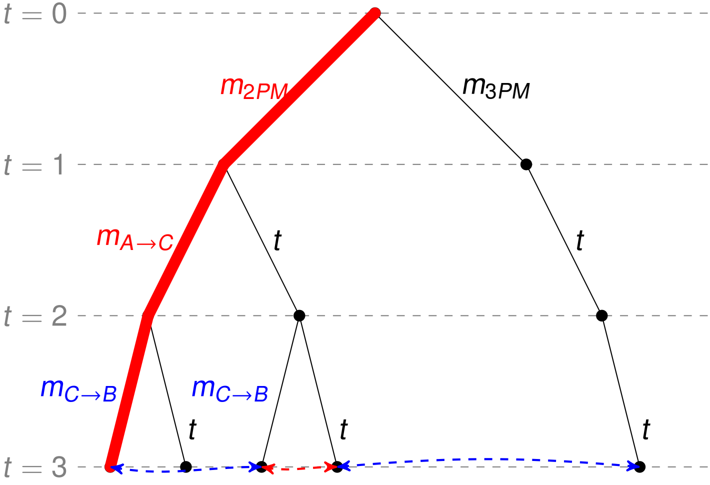

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2. Structural conditions on the underlying event structure. Do we restrict to protocol frames (finitely branching trees)? Finitely branching forests? Or, arbitrary ETL frames?

3. Conditions on the reasoning abilities of the agents. Do the agents satisfy perfect recall? No miracles? Do they agents' know what time it is?

Agent Oriented Properties:

- **No Miracles**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $H \sim_i H'$ then $He \sim_i H'e$.

- **Perfect Recall**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $He \sim_i H'e$ then $H \sim_i H'$.

- **Synchronous**: For all finite histories $H, H' \in \mathcal{H}$, if $H \sim_i H'$ then $\text{len}(H) = \text{len}(H')$.

Decidability in the Purely Temporal Setting

Theorem (Rabin)

The satisfiable problem for monadic second-order logic of the k-ary tree is decidable.

M. O. Rabin. *Decidability of Second-Order Theories and Automata on Infinite Trees*. *Transactions of the American Mathematical Society*, 141, 1969.

Theorem

The satisfiability problem for $\mathcal{L}_{TL}$ with respect to TL tree models (without epistemic structure) is decidable.

Arbitrary Agents

Theorem

The satisfiability problem (with respect to a language $\mathcal{L}_{ETL}$ with C,F, etc.) is decidable — EXPTIME-complete).

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- The theorem holds if we restrict to tree models.

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Assume there are two agents

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For example,

Theorem (Halpern & Vardi)

On interpreted systems that satisfy perfect recall or no learning, the satisfiability problem for $\mathcal{L}_{ETL}$ is Σ^1_1-complete.

(no learning: For $H, H' \in \mathcal{H}$, if $H_t \sim_i H'_{t'}$ then for all $k \geq t$ there exists $k' \geq t'$ such that $H_k \sim_i H'_{k'}$.)

J. Halpern and M. Vardi.. *The Complexity of Reasoning abut Knowledge and Time*. *J. Computer and Systems Sciences*, 38, 1989.

J. Horty and EP. *Action Types in Stit Semantics*. Review of Symbolic Logic, 2017.

Stit model

$$\langle \mathit{Tree}, <, \mathit{Agent}, \mathit{Choice}, V \rangle$$

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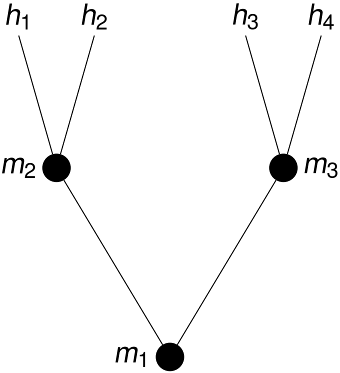

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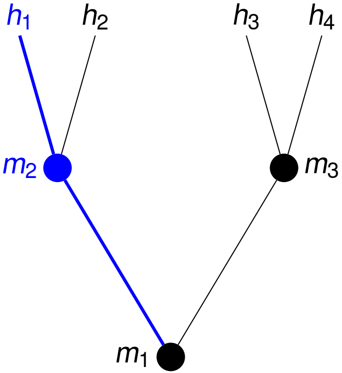

m/h denotes (m, h) with $m \in h$ is called an **index**

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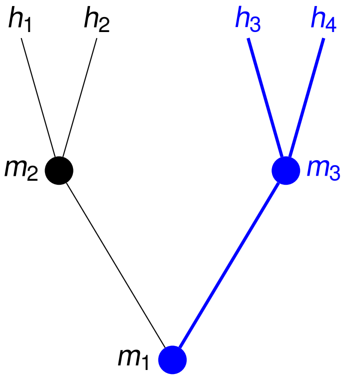

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$H^m = \{h \mid m \in h\}$

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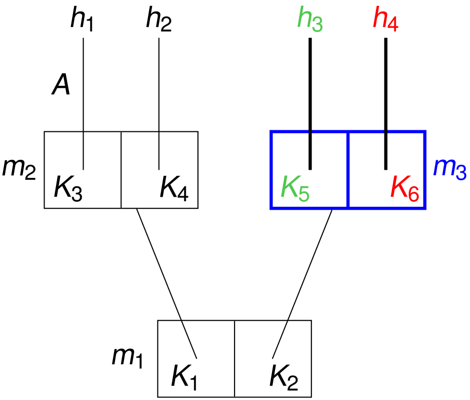

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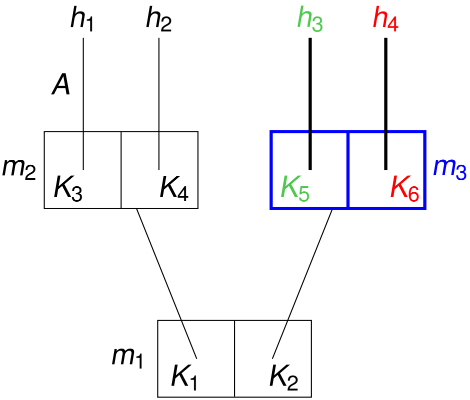

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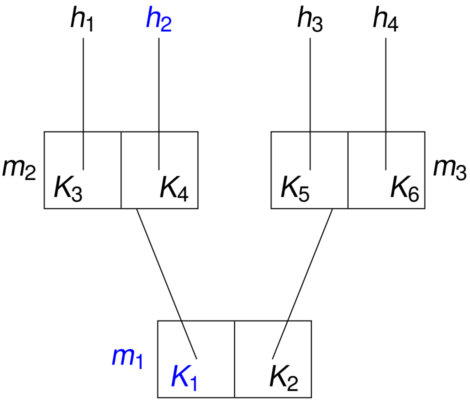

For $\alpha \in Agent$, $Choice^m_\alpha$ is a partition on H^m

$Choice^m_\alpha(h)$ is the particular action at m that contains h

Stit model

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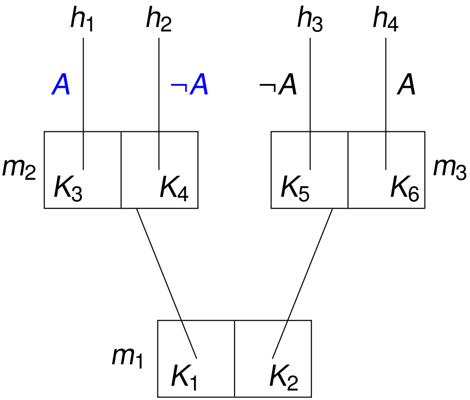
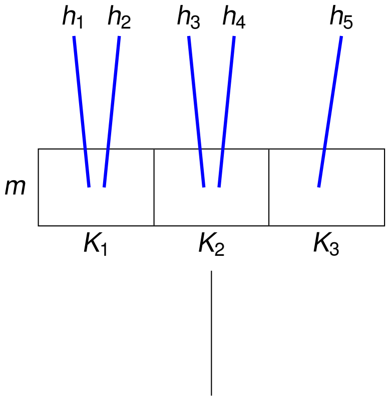
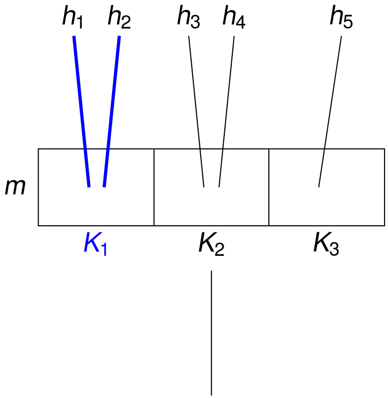


V assigns sets of indices to atomic propositions.

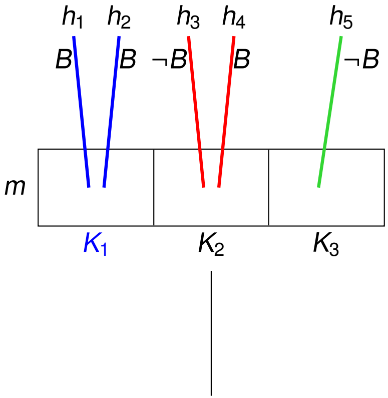
$m_2/h_1 \models A$ $\qquad$ $m_2/h_2 \not\models A$



- $\mathcal{M}, m/h \models \Box A$ if and only if $\mathcal{M}, m/h' \models A$ for all $h' \in H^m$,



- $\mathcal{M}, m/h \models \Box A$ if and only if $\mathcal{M}, m/h' \models A$,
- $\mathcal{M}, m/h \models [\alpha\ stit\colon A]$ if and only if $Choice^m_\alpha(h) \subseteq |A|^m_\mathcal{M}$



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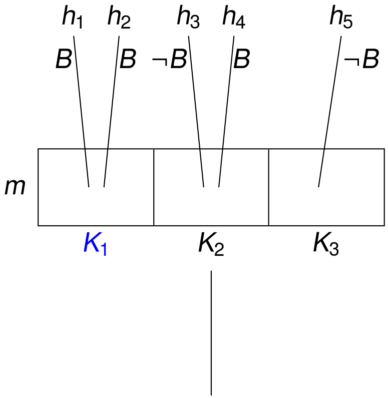
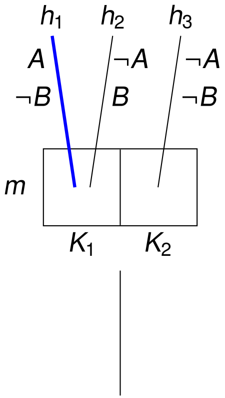
- $\mathcal{M}, m/h \models [\alpha\ stit\colon A]$ if and only if $Choice^m_\alpha(h) \subseteq |A|^m_\mathcal{M}$

$m/h_1 \models [\alpha\ stit\colon B]$, $m/h_3 \not\models [\alpha\ stit\colon B]$, $m/h_5 \models [\alpha\ stit\colon \neg B]$



- $\mathcal{M}, m/h \models \Box A$ if and only if $\mathcal{M}, m/h' \models A$,
- $\mathcal{M}, m/h \models [\alpha\ stit\colon A]$ if and only if $Choice^m_\alpha(h) \subseteq |A|^m_\mathcal{M}$
- Temporal modalities (P, F, ...)

Ability: $\Diamond[\alpha\ stit: A]$



- $m/h_1 \not\models A \supset \Diamond[\alpha\ stit: A]$
- $m/h_1 \not\models \Diamond[\alpha\ stit: A \vee B] \supset \Diamond[\alpha\ stit: A] \vee \Diamond[\alpha\ stit: B]$

$\Diamond[\alpha$ *stit*: $A]$ is a "causal sense" of ability. But, there is also an "epistemic sense" of ability...

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What needs to be added to stit models?

- Indistinguishability relation(s)

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What needs to be added to stit models?

- Indistinguishability relation(s)
- Action types

Epistemic stit models

A. Herzig. *Logics of knowledge and action: critical analysis and challenges*. Autonomous Agent and Multi-Agent Systems, 2014.

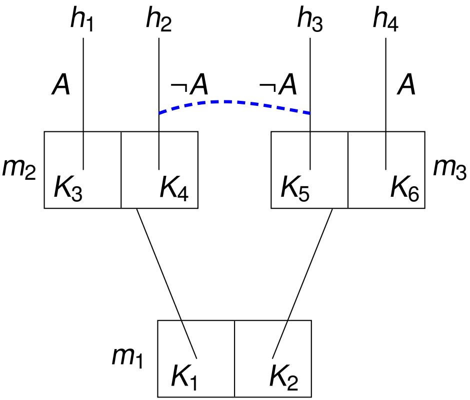
V. Goranko and EP. *Temporal aspects of the dynamics of knowledge*. in Johan van Benthem on Logic and Information Dynamics, Outstanding Contributions to Logic, (eds. Alexandru Baltag and Sonja Smets), pp. 235 - 266, 2014.

J. Broeresen, A. Herzig and N. Troquard. *What groups do, can do and know they can do: An analysis in normal modal logics*. Journal of Applied and Non-Classical Logics, 19:3, pgs. 261 - 289, 2009.

W. van der Hoek and M. Wooldridge. *Cooperation, knowledge and time: Alternating-time temporal epistemic logic and its applications*. Studia Logica, 75, pgs. 125 - 157, 2003.

Epistemic stit models

$$\langle Tree, <, Agent, Choice, \{\sim_\alpha\}_{\alpha \in Agent}, V \rangle$$

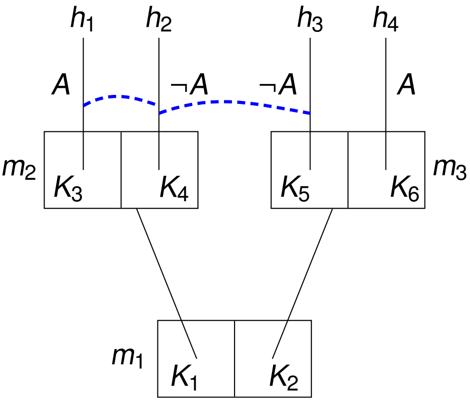


$\sim_\alpha$ is an equivalence relation on indices

$m/h \sim_\alpha m'/h'$: nothing α knows distinguishes m/h from m'/h', or m/h and m'/h' are indistinguishable

Epistemic stit models

$$\langle Tree, <, Agent, Choice, \{\sim_\alpha\}_{\alpha \in Agent}, V\rangle$$



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Epistemic stit models

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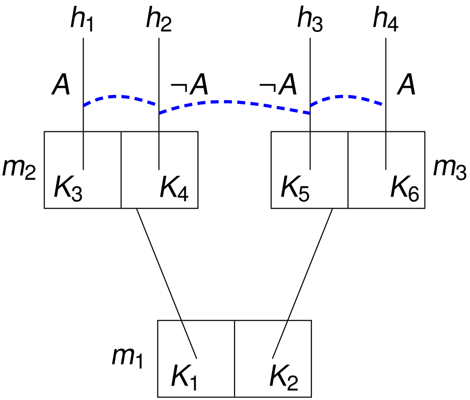

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Epistemic stit models

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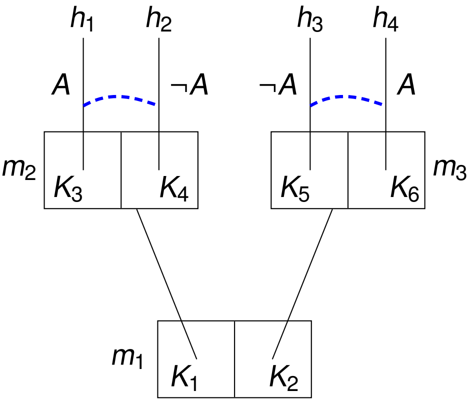

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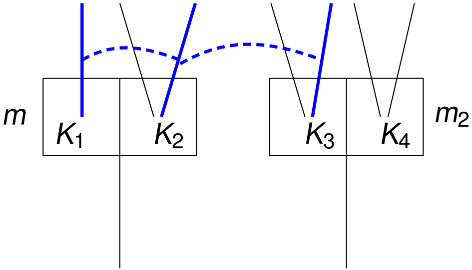
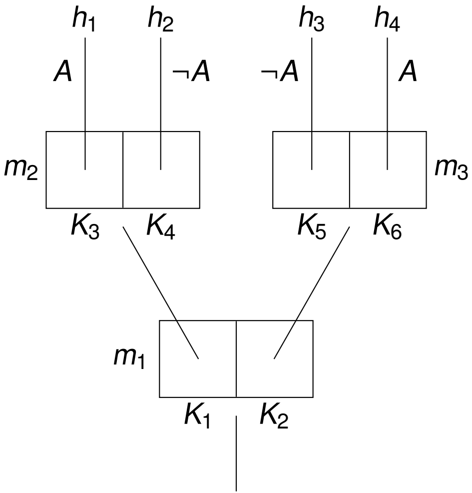
Epistemic stit models

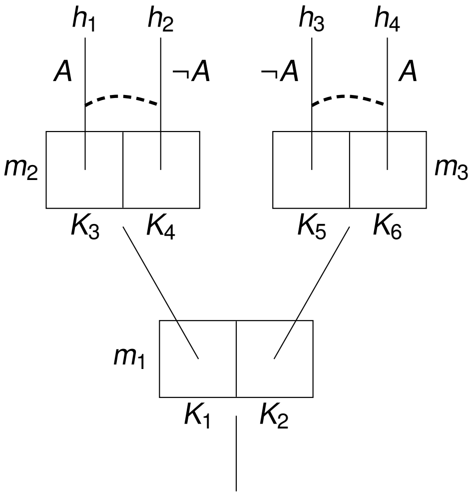


- $\mathcal{M}, m/h \models \mathsf{K}_\alpha A$ if and only if, for all m'/h', if $m/h \sim_\alpha m'/h'$, then $\mathcal{M}, m'/h' \models A$

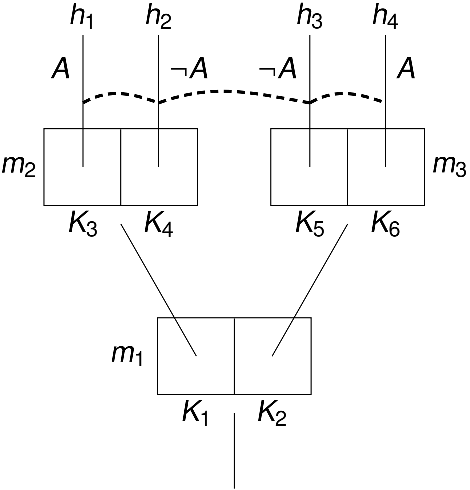
Coin game



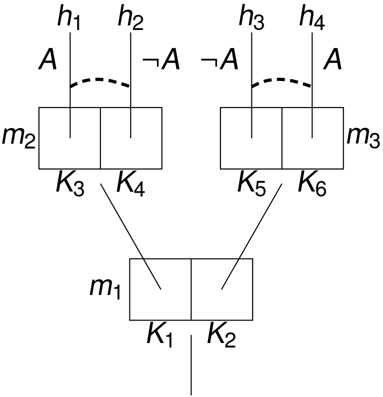
Coin game 1



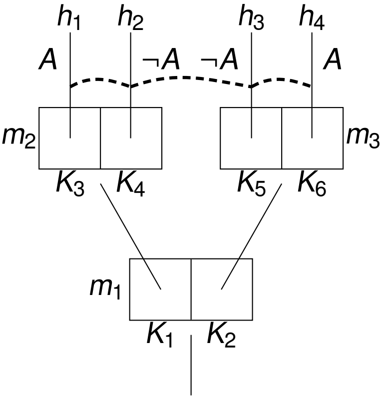
Coin game 2



Ability

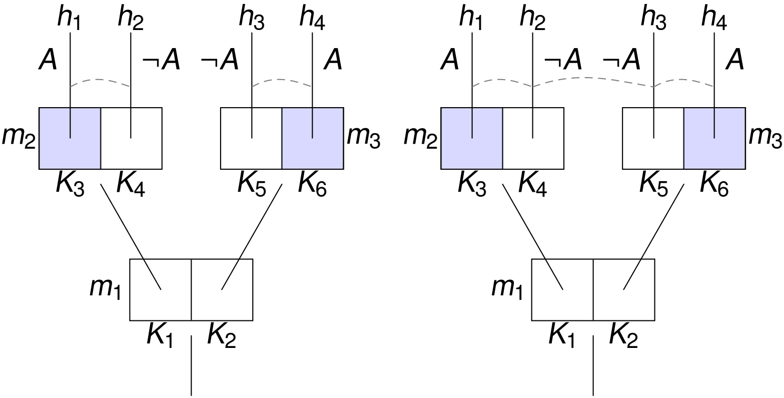


h_1
h_2
h_3
h_4
A
$\neg A$
$\neg A$
A
m_2
m_3
K_3
K_4
K_5
K_6
m_1
K_1
K_2



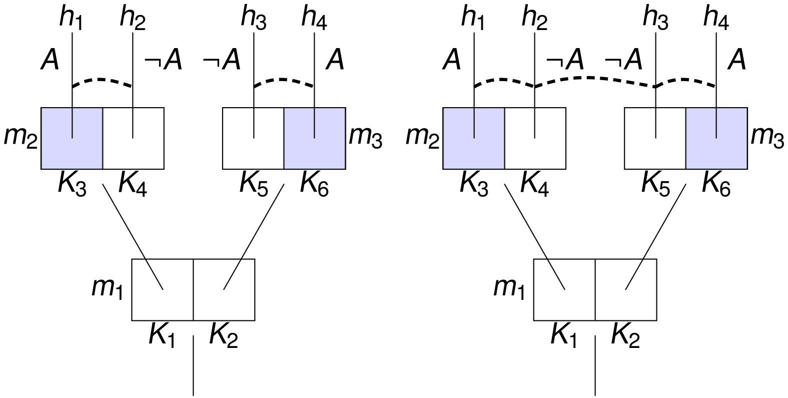
h_1
h_2
h_3
h_4
A
$\neg A$
$\neg A$
A
m_2
m_3
K_3
K_4
K_5
K_6
m_1
K_1
K_2

Ability



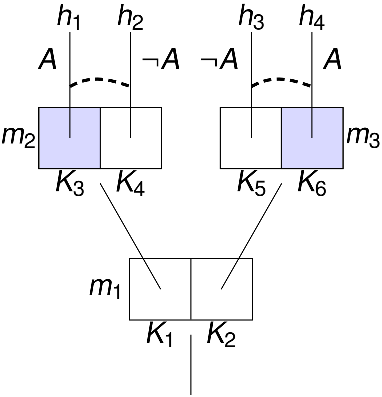
$\Diamond[\alpha$ *stit*: $A]$ is settled true in at m_2 and m_3 in both models.

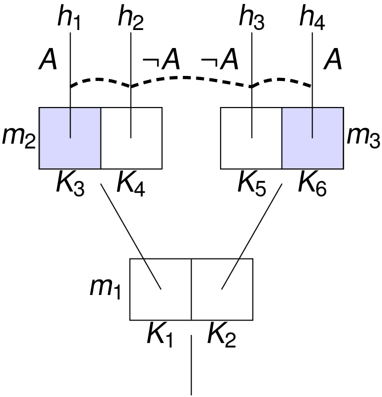
Ability



$\mathsf{K}_\alpha\Diamond[\alpha\ \textit{stit}\colon A]$ is settled true in at m_2 and m_3 in both models.

Ability



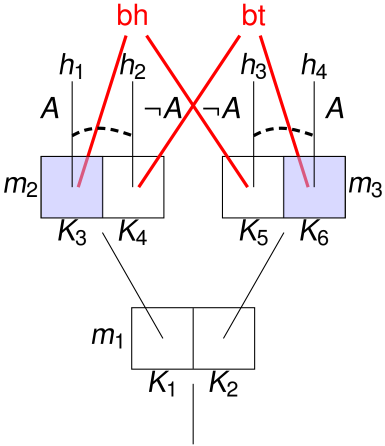


$\Diamond K_\alpha[\alpha$ *stit*: $A]$ is settled false in at m_2 and m_3 in both models.

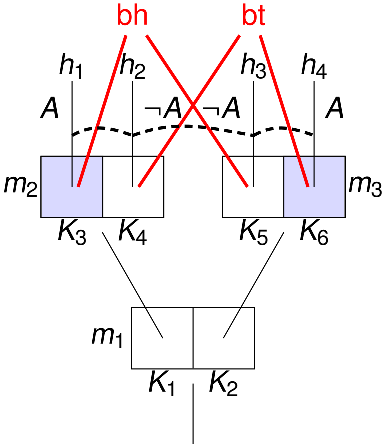
Ability

α has the ability to see to it that A in the epistemic sense just in case there is some action available to α that is known by α to guarantee the truth of A.

Ability

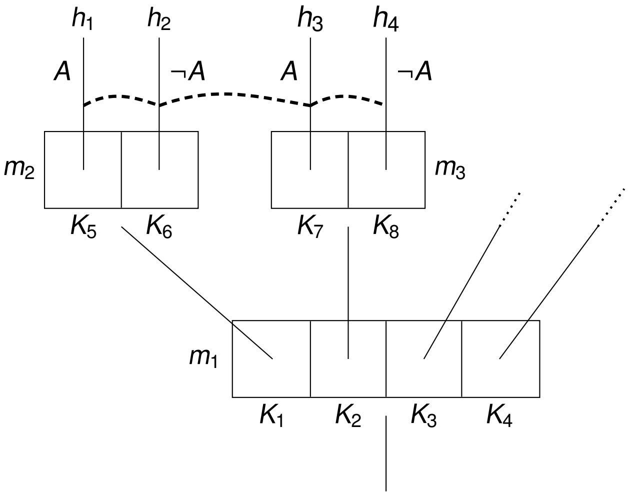


bh
bt
h_1
h_2
h_3
h_4
A
$\neg A$
$\neg A$
A
m_2
m_3
K_3
K_4
K_5
K_6
m_1
K_1
K_2



bh
bt
h_1
h_2
h_3
h_4
A
$\neg A$
$\neg A$
A
m_2
m_3
K_3
K_4
K_5
K_6
m_1
K_1
K_2

Coin game 3



Labeled stit model

$$\langle \mathit{Tree}, <, \mathit{Agent}, \mathit{Choice}, \{\sim_\alpha\}_{\alpha \in \mathit{Agent}}, \mathit{Type}, [\;], \mathit{Label}, V \rangle$$

Type $= \{\tau_1, \tau_2, \ldots\}$ is a finite set of action types—general kinds of action, as opposed to the concrete action tokens already present in stit logics.

Labeled stit model

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[ ] is a partial function mapping types to the particular action token $[\tau]^m_\alpha$ that results when τ is executed by α at *m*.

Labeled stit model

$$\langle \mathit{Tree}, <, \mathit{Agent}, \mathit{Choice}, \{\sim_\alpha\}_{\alpha \in \mathit{Agent}}, \mathit{Type}, [\,], \mathit{Label}, V \rangle$$

$\mathit{Type} = \{\tau_1, \tau_2, \ldots\}$ is a finite set of action types—general kinds of action, as opposed to the concrete action tokens already present in stit logics.

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- $[\tau]^m_\alpha \in \mathit{Choice}^m_\alpha$

Labeled stit model

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$\textit{Type} = \{\tau_1, \tau_2, \ldots\}$ is a finite set of action types—general kinds of action, as opposed to the concrete action tokens already present in stit logics.

$[\,]$ is a partial function mapping types to the particular action token $[\tau]_\alpha^m$ that results when τ is executed by α at m.

- $[\tau]_\alpha^m \in \textit{Choice}_\alpha^m$

Label is a 1-1 function mapping $\textit{Choice}_\alpha^m$ to action types.

Labeled stit model

$$\langle Tree, <, Agent, Choice, \{\sim_\alpha\}_{\alpha \in Agent}, Type, [\,], Label, V\rangle$$

Type $= \{\tau_1, \tau_2, \ldots\}$ is a finite set of action types—general kinds of action, as opposed to the concrete action tokens already present in stit logics.

[ ] is a partial function mapping types to the particular action token $[\tau]^m_\alpha$ that results when τ is executed by α at m.

- $[\tau]^m_\alpha \in Choice^m_\alpha$

Label is a 1-1 function mapping $Choice^m_\alpha$ to action types.

- If $K \in Choice^m_\alpha$, then $[Label(K)]^\alpha_m = K$
- If $\tau \in Type$ and $[\tau]^m_\alpha$ is defined, then $Label([\tau]^m_\alpha) = \tau$

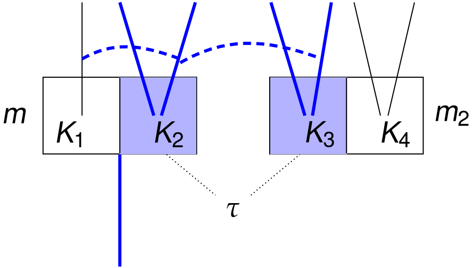
Labeled stit model, continued

$$\langle Tree, <, Agent, Choice, \{\sim_\alpha\}_{\alpha \in Agent}, Type, [\,], Label, V\rangle$$

$$Type^m_\alpha = \{Label(K) \mid K \in Choice^m_\alpha\}$$

$$Type^m_\alpha(h) = Label(Choice^m_\alpha(h))$$

kstit



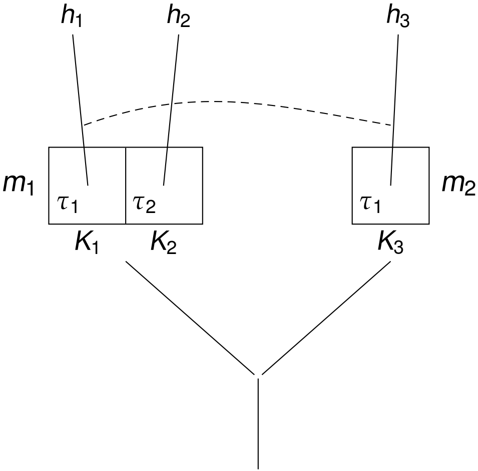
- $\mathcal{M}, m/h \models [\alpha\ kstit\colon A]$ if and only if $[Type^m_\alpha(h)]^{m'}_\alpha \subseteq |A|^{m'}_{\mathcal{M}}$ for all m'/h' such that $m'/h' \sim_\alpha m/h$.

The difference between $C1$ and $C2$

$(C1)$ If $m/h \sim_\alpha m'/h'$, then $\mathit{Type}_\alpha^m = \mathit{Type}_\alpha^{m'}$

$(C2)$ If $m/h \sim_\alpha m'/h'$, then $[\mathit{Type}_\alpha^m(h)]_\alpha^{m'}$ is defined.

Minimal Constraint



Knowledge of action types

Let A_α^τ be an atomic proposition carrying the intuitive meaning that the agent α executes the action type τ.

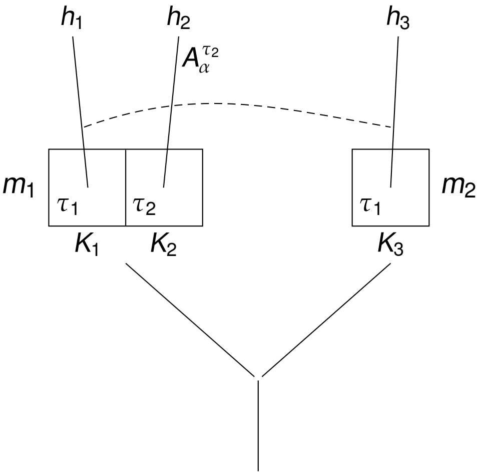
- $\mathcal{M}, m/h \models A_\alpha^\tau$ if and only if $Type_\alpha^m(h) = \tau$

Knowledge of action types

Let A_α^τ be an atomic proposition carrying the intuitive meaning that the agent α executes the action type τ.

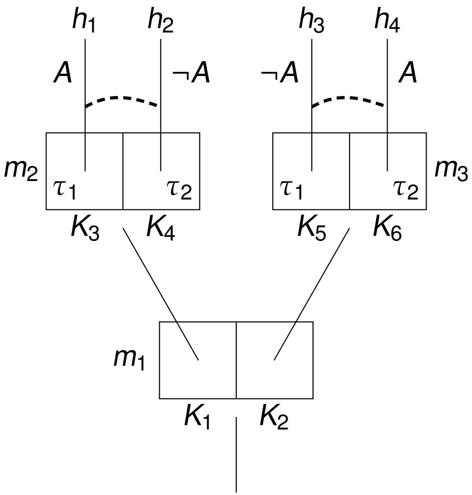
- $\mathcal{M}, m/h \models A_\alpha^\tau$ if and only if $Type_\alpha^m(h) = \tau$

C2 is satisfied iff $\Diamond A_\alpha^\tau \supset \mathsf{K}_\alpha \Diamond A_\alpha^\tau$ is valid.



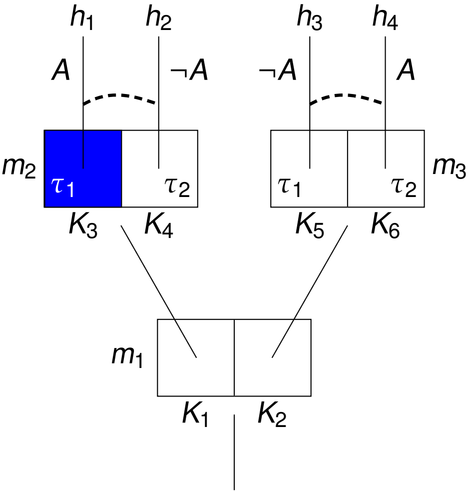
$m_1/h_1 \models \Diamond A_\alpha^{\tau_2}$ $\quad$ $m_1/h_1 \not\models \mathsf{K}_\alpha \Diamond A_\alpha^{\tau_2}$

Epistemic sense of ability



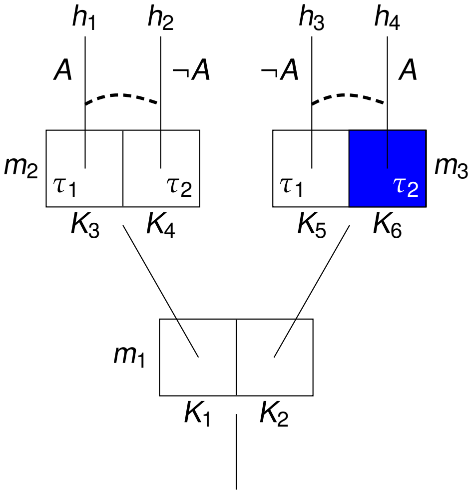
$\Diamond[\alpha$ *kstit*: $A]$ is settled true at m_2 and m_3.

Epistemic sense of ability



$\Diamond[\alpha\ kstit\colon A]$ is settled true at m_2 and m_3.

Epistemic sense of ability



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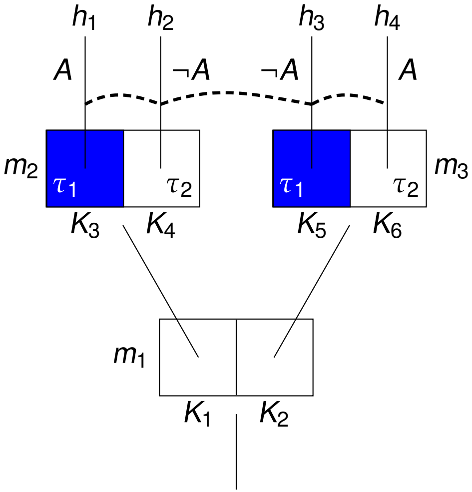
Epistemic sense of ability



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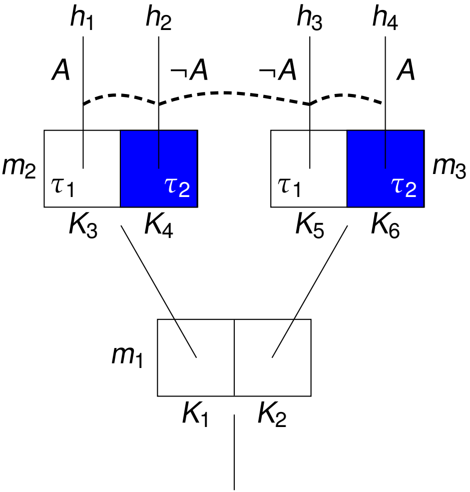
Epistemic sense of ability



$\Diamond[\alpha$ *kstit*: $A]$ is settled false at m_2 and m_3.

Discussion: Related Work

A. Herzig and N. Troquard. *Knowing how to play: uniform choices in logics of agency*. In Proceedings of the Fifth International Joint Conference on Autonomous Agents and Multi-agent Systems (AAMAS-06), pages 209 - 216. 2006..

J. Broersen. *Deontic epistemic stit logic distinguishing modes of* mens rea. Journal of Applied Logic, 9(2):127 - 152, 2011.

A. Herzig and E. Lorini. *A Dynamic Logic of Agency I: STIT, Capabilities and Powers*. Journal of Logic, Language and Information 19(1): 89-121, 2010.

EP, R. Parikh, and E. Cogan. *The logic of knowledge based obligation*. Synthese, 149:2, pp. 311 - 341, 2006.

M. Xu. *Combinations of stit and actions*. Journal of Logic, Language, and Information, 19:485 - 503, 2010.

Discussion

Validities:

- $\mathsf{K}_\alpha[\alpha \textit{ stit}: A] \supset [\alpha \textit{ kstit}: A]$
- $[\alpha \textit{ kstit}: A] \supset [\alpha \textit{ stit}: A]$

Discussion

Validities:

- $\mathsf{K}_\alpha[\alpha\ \textit{stit}: A] \supset [\alpha\ \textit{kstit}: A]$
- $[\alpha\ \textit{kstit}: A] \supset [\alpha\ \textit{stit}: A]$

Non-Validities:

- $\Diamond[\alpha\ \textit{kstit}: A] \supset \mathsf{K}_\alpha\Diamond[\alpha\ \textit{kstit}: A]$

Constraints

(*C*3) If $m/h \sim_\alpha m'/h'$, then $m = m'$

(*C*3) is satisfied iff [α *stit*: *A*] ≡ [α *kstit*: *A*] is valid.

$(C4) \qquad$ If $m/h \sim_\alpha m'/h'$, then $Type_\alpha^m(h) = Type_\alpha^{m'}(h')$

$(C4)$ is satisfied iff $A_\alpha^\tau \supset \mathsf{K}_\alpha A_\alpha^\tau$ is valid.

Deliberative perspective

(*C*5) If $m/h \sim_\alpha m'/h'$, then $m/h'' \sim_\alpha m'/h'''$ for all $h'' \in H^m$ and $h''' \in H^{m'}$

Indistinguishability between moments: $m \sim_\alpha m'$ iff $m/h \sim_\alpha m'/h'$ for all $h \in H^m$ and $h' \in H^{m'}$.

Discussion

- Language/validities

$$\Box A \supset [\alpha \; \textit{stit}\colon A]$$
$$\mathsf{K}_\alpha \Box A \supset [\alpha \; \textit{kstit}\colon A]$$
$$[\alpha \; \textit{kstit}\colon A] \equiv \mathsf{K}^{\mathsf{act}}_\alpha [\alpha \; \textit{stit}\colon A]$$
$$\ldots$$

- What do the agents know vs. What do the agents know *given what they are doing*.

- Equivalence between labeled stit models (cf. Thompson transformations specifying when two imperfect information games reduce to the same Normal form)