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PARALLEL INTERPOLATION, SPLITTING, AND RELEVANCE IN BELIEF CHANGE

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Abstract. The splitting theorem says that any set of formulae has a finest representation as a family of letter-disjoint sets. Parikh formulated this for classical propositional logic, proved it in the finite case, used it to formulate a criterion for relevance in belief change, and showed that AGM partial meet revision can fail the criterion. In this paper we make three further contributions. We begin by establishing a new version of the well-known interpolation theorem, which we call parallel interpolation, use it to prove the splitting theorem in the infinite case, and show how AGM belief change operations may be modified, if desired, so as to ensure satisfaction of Parikh's relevance criterion.

Notational preliminaries. We use lower case $a, b, \ldots, x, y, \ldots,$ to range over *formulae* of classical propositional logic. To facilitate smooth formulation of properties such as interpolation, we include the zero-ary falsum $\bot$ among the primitive connectives. *Sets of formulae* are denoted by upper case letters $A, B, \ldots, X, Y, \ldots,$ reserving L for the set of all *formulae*, E for the set of all *elementary letters* (alias propositional variables) and $F, G, \ldots$ for subsets of the elementary letters. For any formula a, we write $E(a)$ to mean the set of all elementary letters occurring in a; similarly for sets A of formulae. For any set A of formulae, $L(A)$ stands for the sublanguage generated by $E(A)$, i.e., the set of all formulae x with $E(x) \subseteq E(A)$. Thus immediately $L(A) = L(E(A))$, and also $E(A) \subseteq E(B)$ iff $L(A) \subseteq L(B)$. Note that since the zero-ary $\bot$ is a primitive connective, $L(\emptyset)$ is not empty, containing both $\bot$ and $\neg\bot$.

Classical consequence is written as $\vdash$ when treated as a relation over $2^L \times L$, *Cn* when viewed as an operation on 2^L into itself. The relation of *classical equivalence* is written $\dashv\vdash$.

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To lighten notation, we follow the common convention of writing A, x for $A \cup \{x\}$. $A \vdash B$ is short for: $A \vdash b$ for all $b \in B$. Also, $v(A) = 1$ is short for: $v(a) = 1$ for all $a \in A$, while $v(A) = 0$ abbreviates: $v(a) = 0$ for some $a \in A$.

§1. Standard versus parallel interpolation. The interpolation theorem (also known as Craig's Lemma—see Craig [4, 5], or textbook presentations such as Hodges [7]) tells us, in the case of classical propositional logic, that whenever $A \vdash x$ there is a formula b such that $E(b) \subseteq E(A) \cap E(x)$ and $A \vdash b \vdash x$. Equivalently given compactness: whenever $A \vdash x$ there is a set B of formulae such that $E(B) \subseteq E(A) \cap E(x)$ and $A \vdash B \vdash x$. Equivalently again given both compactness and monotony: whenever $A \vdash x$ then $Cn(A) \cap L(A) \cap L(x) \vdash x$.

Now consider the case that $A = \bigcup\{A_i\}_{i \in I}$ where the sets $E(A_i)$ of elementary letters are pairwise disjoint. Any set A has this form for singleton I, but the interesting case is when I has more than one element. Suppose $\bigcup\{A_i\}_{i \in I} \vdash x$. We know from interpolation that there is a formula b such that $E(b) \subseteq E(A) \cap E(x)$ and $\bigcup\{A_i\}_{i \in I} = A \vdash b \vdash x$. But since the sets A_i do not separately imply x, interpolation does not tell us immediately whether we may treat the A_i in parallel, i.e., whether we can find formulae b_i such that each $E(b_i) \subseteq E(A_i) \cap E(x)$, $A_i \vdash b_i$, and $\{b_i\}_{i \in I} \vdash x$, as illustrated for the case $|I| = 2$ in Figure 1 below.

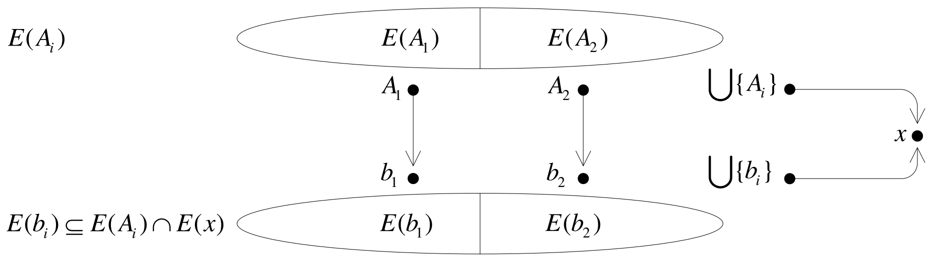

FIGURE 1. Parallel interpolation: case $|I| = 2$

THEOREM 1.1. *Let $A = \bigcup\{A_i\}_{i \in I}$ where the letter sets $E(A_i)$ are pairwise disjoint, and suppose $\bigcup\{A_i\}_{i \in I} \vdash x$. Then there are formulae b_i such that each $E(b_i) \subseteq E(A_i) \cap E(x)$, $A_i \vdash b_i$, and $\{b_i\}_{i \in I} \vdash x$.*

Remarks on the Proof. (a) The requirement that the letter sets are disjoint is essential. (b) It is possible to give a direct proof of the theorem, but simpler to derive it by iterating standard interpolation as in the following argument due to Georg Gottlob (personal communication).

PROOF OF THEOREM 1.1. By the compactness of classical consequence, when the union of a family of sets A_i implies a formula x, then there is a finite subfamily of finite subsets of the A_i, the conjunction of whose elements implies x. It thus suffices to prove the theorem for finite I and each A_i a singleton $\{a_i\}$.

Suppose $a_1 \wedge a_2 \wedge \ldots \wedge a_n \vdash x$. Then $a_1 \vdash (a_2 \wedge \ldots \wedge a_n \rightarrow x)$ and by standard interpolation there is a formula b_1 with $a_1 \vdash b_1 \vdash (a_2 \wedge \ldots \wedge a_n \rightarrow x)$ and $E(b_1) \subseteq E(a_1) \cap E(a_2 \wedge \ldots \wedge a_n \rightarrow x)$. But $E(a_1) \cap E(a_2 \wedge \ldots \wedge a_n \rightarrow x) = E(a_1) \cap (E(a_2) \cup \ldots \cup E(a_n) \cup E(x)) = E(a_1) \cap E(x)$ since the sets $E(a_i)$ are pairwise disjoint, so that $E(b_1) \subseteq E(a_1) \cap E(x)$. Since $b_1 \vdash (a_2 \wedge \ldots \wedge a_n \rightarrow x)$ we have $b_1 \wedge a_2 \wedge \ldots \wedge a_n \vdash x$ and the sets $E(b_1), E(a_2), \ldots, E(a_n)$ are pairwise disjoint. We may therefore repeat the procedure for a_2, obtaining a suitable interpolant b_2, and so on with n applications of standard interpolation to obtain finally $b_1 \wedge b_2 \wedge \ldots \wedge b_n \vdash x$ where each $a_i \vdash b_i$ and each $E(b_i) \subseteq E(a_i) \cap E(x)$. $\dashv$

§**2. Application to obtain least splitting theorem in infinite case.** We now extend Parikh's finest splitting theorem to the infinite case, using parallel interpolation in the proof. We begin by recalling the definitions. Let K be any set of formulae.

DEFINITION 2.1 (Splitting). Let $\mathbf{E} = \{E_i\}_{i \in I}$ be any partition of the set E of all elementary letters of the language. Extending the definition of Parikh [9] to the case that E is infinite, we say that $\mathbf{E}$ *is a splitting of* K iff there is a family $\{B_i\}_{i \in I}$ with each $E(B_i) \subseteq E_i$ such that $\bigcup\{B_i\}_{i \in I} \dashv\vdash K$.

The idea of the definition may be illustrated by the following diagram.

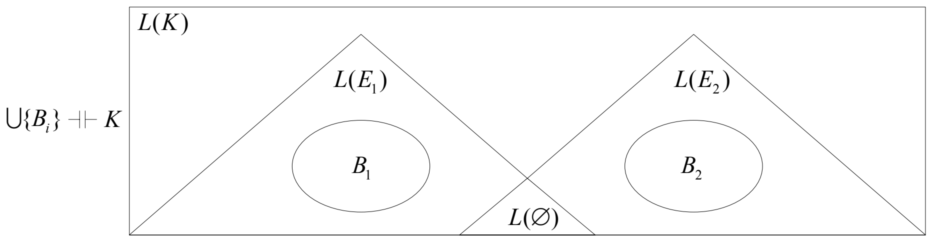

FIGURE 2. Splitting: outline diagram for case $|I| = 2$

DEFINITION 2.2 (Fineness of a partition). Following customary terminology, we say that a partition $\mathbf{E} = \{E_i\}_{i \in I}$ of E is *at least as fine* as another partition $\mathbf{F}$ of E, and we write $\mathbf{E} \leqslant \mathbf{F}$, iff every cell of $\mathbf{F}$ is the union of cells of $\mathbf{E}$. Equivalently, iff $R_{\mathbf{E}} \subseteq R_{\mathbf{F}}$, where $R_{\mathbf{E}}$ resp. $R_{\mathbf{F}}$ is the equivalence relation over E associated with $\mathbf{E}$ resp. $\mathbf{F}$.

Parikh [9] showed that when E is finite, then every set K of formulae has a unique finest splitting. In this section we extend his result to the infinite case, using a streamlined proof with parallel interpolation.

We begin with a convenient characterization of splitting. In effect, it permits us to focus more on individual formulae in K and less on K itself, making it easier to show that a given partition is a splitting.

LEMMA 2.3. *Let* $\mathbf{E} = \{E_i\}_{i \in I}$ *be a partition of the set of elementary letters. Then* $\mathbf{E}$ *splits* K *iff for every* $x \in K$ *there are formulae* $b_1, \ldots, b_n$ *and sets* $E_1, \ldots, E_n \in \mathbf{E}$ (*the integer* n *depending on the choice of* x) *such that*:

(1) *Each* $E(b_i) \subseteq E_i$ $(i \leqslant n)$
(2) $K \vdash b_i$ *for each* $i \leqslant n$
(3) $\{b_i\}_{i \leqslant n} \vdash x$.

PROOF OF LEMMA 2.3. Suppose $\mathbf{E} = \{E_i\}_{i \in I}$ is a partition of the set of elementary letters and $\mathbf{E}$ splits K. Then there is a family $\{B_i\}_{i \in I}$ of sets of formulae with each

$E(B_i) \subseteq E_i$ and $\bigcup\{B_i\}_{i\in I} \dashv\vdash K$. Take any $x \in K$. Then $\bigcup\{B_i\}_{i\in I} \vdash x$ so by compactness there are formulae $b_1, \dots, b_n$ and $B_1, \dots, B_n \in \{B_i\}_{i\in I}$ such that $\{b_i\}_{i\leqslant n} \vdash x$ and each $b_i (i \leqslant n)$ is the conjunction of finitely many elements of B_i. From the former, (3) is satisfied. From the latter we have immediately that both $E(b_i) \subseteq E(B_i) \subseteq E_i$ so that (1) holds, and $K \vdash \bigcup\{B_i\}_{i\in I} \vdash b_i$ so that (2) also holds.

Conversely, suppose $\mathbf{E} = \{E_i\}_{i\in I}$ is a partition of the set E of elementary letters that satisfies the condition of the lemma; we need to show that $\mathbf{E}$ splits K. For each $i \in I$ put B_i to be the set of all formulae b with $K \vdash b$ and $E(b) \subseteq E_i$. Then $K \vdash B_i$ and $E(B_i) \subseteq E_i$ for each $i \in I$. To complete the proof we need only show that $\{B_i\}_{i\in I} \dashv\vdash K$. Since $K \vdash B_i$ for each B_i, we have $K \vdash \bigcup\{B_i\}_{i\in I}$. Conversely, let $x \in K$; we need to show $\bigcup\{B_i\}_{i\in I} \vdash x$. Choose any $b_1, \dots, b_n$ and $E_1, \dots, E_n$ satisfying (1), (2), (3); their existence is guaranteed by our supposition. By (1) and (2) we have each $b_i \in B_i (i \leqslant n)$, so each $b_i \in B_1 \cup \cdots \cup B_n \subseteq \bigcup\{B_i\}_{i\in I}$ giving us $\bigcup\{B_i\}_{i\in I} \vdash x$ by (3) as desired. $\dashv$

THEOREM 2.4. *Every set K of formulae has a unique finest splitting.*

PROOF OF THEOREM 2.4. Let $\mathbf{E}$ be the partition of the elementary letters that is the infimum of the family $\mathscr{E}$ of all partitions that split K. It suffices to show that $\mathbf{E}$ also splits K. Let $x \in K$. By Lemma 2.3, it suffices to find formulae $b_1, \dots, b_n$ and sets $E_1, \dots, E_n \in \mathbf{E}$ (the integer n depending on the choice of x) satisfying conditions (1), (2), (3) of the Lemma.

Since x is a single formula, it contains only finitely many elementary letters. Hence there is a partition $\mathbf{F} \in \mathscr{E}$ such that no partition in $\mathscr{E}$ that is finer than $\mathbf{F}$ separates any two elementary letters in $E(x)$ that are not already separated by $\mathbf{F}$. Since $\mathbf{F} \in \mathscr{E}$, we know by Lemma 2.3 that there are formulae $a_1, \dots, a_n$ and sets $F_1, \dots, F_n \in \mathbf{F}$ such that:

(a1) Each $E(a_i) \subseteq F_i$ $(i \leqslant n)$
(a2) $K \vdash a_i$ for each $i \leqslant n$
(a3) $\{a_i\}_{i\leqslant n} \vdash x$.

Recalling that the cells $F \in \mathbf{F}$ are disjoint, and conjoining the a_i when their letters come from the same cell in $\mathbf{F}$, we may assume without loss of generality that in addition:

(a0) The letter sets $E(a_i)$ are disjoint $(i \leqslant n)$.

Applying parallel interpolation to (a3), (a0) we get formulae $b_1, \dots, b_n$ with:

(b1) Each $E(b_i) \subseteq E(a_i) \cap E(x) \subseteq F_i \cap E(x)$ $(i \leqslant n)$, using also (a1)
(b2) $a_i \vdash b_i$ for each $i \leqslant n$
(b3) $\{b_i\}_{i\leqslant n} \vdash x$.

It remains to check that the properties (1)–(3) of Lemma 2.3 are satisfied. Property (3) is just (b3). Property (2) is immediate from (b2) with (a2). For property (1), consider any $i \leqslant n$. By (b1) we have $E(b_i) \subseteq F_i \cap E(x)$. Since $E(b_i) \subseteq E(x)$, the infimum partition $\mathbf{E}$ does not separate any two elementary letters in b_i that are not already separated by $\mathbf{F}$. But since also $E(b_i) \subseteq F_i$, no distinct letters in b_i are separated by $\mathbf{F}$. Hence there is an $E_i \in \mathbf{E}$ with $E(b_i) \subseteq E_i$ and we are done. $\dashv$

§3. Parikh's relevance criterion. Parikh has applied the finest splitting theorem to the logic of belief revision, specifically to formulate a criterion of relevance in belief change.

Suppose we begin with a belief set K (it does not matter for what follows whether or not it is closed under classical consequence), and wish to revise it by introducing a formula x, which in the principal case is inconsistent with K. As is well known, Alchourrón, Gärdenfors and Makinson [1] introduced regularity conditions that such a process may plausibly be taken to satisfy (for arbitrary fixed K), and showed that those conditions are characterized by a specific kind of construction known as partial meet revision.

However, as Parikh [9] observed, revisions effected in this manner may discard more than perhaps they should, by eliminating from K items that are in a certain sense irrelevant to the inconsistency of K with x. The definition of relevance is formulated in terms of finest splitting, as follows.

DEFINITION 3.1 (Irrelevant formulae). Let K be any consistent set of formulae, with x a formula that is a candidate for contracting from K or integrating into K by a process of revision. Let $\boldsymbol{E} = \{E_i\}_{i \in I}$ be the unique finest splitting of K. We say that a formula a is *irrelevant* to the contraction or revision of K by x (briefly: a is irrelevant to x modulo K) iff there is no cell $E_i \in \mathbf{E}$ such that each of $E_i \cap E(a)$ and $E_i \cap E(x)$ is non-empty. Equivalently, iff $\bigcup \{E_j\}_{j \in J} \cap E(a) = \emptyset$, where $\{E_j\}_{j \in J}$ is the subfamily of cells in $\mathbf{E}$ that share some elementary letter with $E(x)$.

The following diagram illustrates what is meant in the case that $|I| = 4$ and $|J| = 2$.

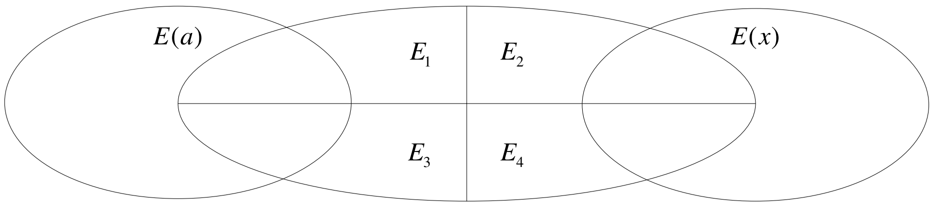

FIGURE 3. Example of irrelevance

We note that this concept is preserved under classical equivalence in the argument K. That is, if K, K' are classically equivalent, then they have the same finest splitting and so a is irrelevant to x modulo K iff it is so modulo K'. Note too that the relation of irrelevance (and thus also its complement, relevance) is symmetric in the variables a and x.

The *relevance criterion* may be put as follows: whenever an element a of K is irrelevant to x modulo K, then it remains an element of the result of contracting or revising K by x. As Parikh observed, AGM partial meet revision can fail this criterion, and the same can happen for contraction. We give a simple example for contraction.

Let p, q be two distinct elementary letters, and put $K = Cn(p, q)$. Then there is an AGM partial meet contraction (in fact, a maxichoice contraction) that puts $K - p$ to be $Cn(p \leftrightarrow q)$, thus eliminating not only p but also q from K. However, the letter q is irrelevant to p modulo K. This is because the representation of K by $\{p, q\}$ puts $E_1 = \{p\}$, $E_2 = \{q\}$, and neither of these two sets contains *both of* the letters p and q.

The example is robust in the sense that it goes through even if we work with belief bases rather than belief sets already closed under consequence; such robustness is indeed guaranteed by the fact, already noted, that relevance is preserved between classically equivalent belief sets. Put $K_0 = \{p \leftrightarrow q, q\}$, so that $Cn(K_0) = K$ above. Then one of the AGM maxichoice base contractions puts $K_0 - p$ to be $\{p \leftrightarrow q\}$, which eliminates q. However, the letter q is irrelevant to p modulo K_0 because there is *another* representation of K_0 as $\{p, q\}$, which puts $E_1 = \{p\}$, $E_2 = \{q\}$, and neither of these two sets contains both of the letters p and q.

Parikh and collaborators have studied the problem of modifying the notion of partial meet revision so as to ensure that it satisfies the relevance criterion. They have done so *syntactically*, i.e., by examining what further postulates may be added to the standard ones of AGM to ensure respect of relevance. This work was carried out in Parikh [9], Chopra, Georgatos and Parikh [2], Chopra and Parikh [3], Peppas, Chopra and Foo [10]. However one may also approach the situation from a more *semantic* angle, asking how we might modify the definitions of partial meet contraction and revision so as to ensure that they respect relevance. That is our purpose in the next section; but before doing so we comment on the criterion itself.

Is AGM's failure to satisfy Parikh's relevance criterion really a shortcoming? In our view, this question does not have a simple answer: violation of this kind of relevance may be undesirable in some contexts but perfectly acceptable in others, depending on the epistemic policy guiding the contraction.

Consider the same example as above, where we are contracting the letter p from the closed belief set $K = Cn(p, q)$, or from its base $K_0 = \{p \leftrightarrow q, q\}$. If we are working with the base, we may perhaps regard it as supplying us with epistemic information, namely that its elements are particularly important items that deserve to be protected more than other items not appearing in it. But even so, the base gives us no information to discriminate *between* its elements. In the example $K_0 = \{p \leftrightarrow q, q\}$, we are not told explicitly that the elementary letter q deserves protection more than the biconditional $p \leftrightarrow q$. Indeed, we may wish to allow for the possibility that the latter is more deeply entrenched, less vulnerable, than the former. In that case, when we discard p we will jettison the letter q and keep the biconditional, regardless of the fact that q is irrelevant to p modulo K in the sense that Parikh has defined.

On the other hand, there may be occasions in which we wish to treat elementary letters as the only carriers of epistemic significance. In our view, this policy is difficult to justify in theoretical terms, but it sometimes appears to be adopted in contexts of artificial intelligence for reasons of computational convenience. Under such a policy, compound formulae such as the biconditional $p \leftrightarrow q$ are of no epistemic significance, and when changing a belief set we would want to minimize change in the status of elementary letters and preserve relevance in the sense of Parikh.

In any case, it remains an interesting question whether the AGM partial meet operations may elegantly be modified so as to ensure that relevance is satisfied. Our answer in the next section is positive.

§4. Normalized partial meet operations. If we regard the finest splitting of a belief set as providing us with a canonical form for it, then instead of applying an AGM contraction or revision operation directly to K, we may first of all massage K into its canonical form K' and apply an AGM operation to K'. It turns out that with this simple manipulation, giving us what may be called *normalized* belief change operations, relevance is respected. We do not need to add any further conditions to the AGM postulates, *provided* contraction or revision is applied to the canonical form K' of the belief set K rather than directly to K itself.

Note that irrespective of whether or not K is closed under classical consequence, i.e., whether $K = Cn(K)$, its canonical form K' is not in general so closed. In other words, contraction and revision on K' are what are commonly called *belief base* operations.

We begin by considering contraction and obtain the corresponding result for revision as a corollary. We write $K - x$ for any partial meet contraction,[1] and $K * x$ for the corresponding partial meet revision.

THEOREM 4.1. *Let K be any consistent set of formulae. Let $\mathbf{E} = \{E_i\}_{i\in I}$ be the unique finest splitting of K, with $\{B_i\}_{i\in I}$ a family such that $K' = \bigcup\{B_i\}_{i\in I} \dashv\vdash K$ and $E(B_i) \subseteq E_i$ for each $i \in I$. Let x be any formula, and $K' - x$ a partial meet contraction of x from K'. Then, whenever $a \in K'$ is irrelevant to x (modulo K'), we have $a \in K' - x$.*

In other words, partial meet contraction applied to the finest splitting of a consistent belief set respects relevance, in the sense that it never eliminates any formula to which the discarded formula is irrelevant. In the statement of the theorem, the prime should not be overlooked: the essential feature of the contraction is that it is carried out on $K' = \bigcup\{B_i\}_{i\in I}$ rather than on K.

At the suggestion of a referee, before giving a formal proof of Theorem 4.1 we explain in rough terms how the preliminary passage from K to K' prevents the elimination of irrelevant formulae, despite the classical equivalence of K and K'. When $K' - x$ is the result of a partial meet contraction of x from K' then it is the intersection of some family of maximal x-nonimplying subsets of K'. But if there is no cell E_i of the splitting that contains letters from both x and a formula $a \in K'$, then the addition of a to any x-nonimplying subset of K' will leave it x-nonimplying, so that a must be in all the maximal x-nonimplying subsets of K', and so in $K' - x$. Thus in our trivial example where $K = Cn(p,q)$ while $K' = \{p, q\}$, q is not in all the maximal p-nonimplying subsets of the former, but is in the (unique) maximal p-nonimplying subset of the latter.

PROOF OF THEOREM 4.1. Suppose $a \in K'$ but $a \notin K' - x$ while a is irrelevant to x (modulo K'); we derive a contradiction. Let $\{E_j\}_{j\in J}$ be the subfamily of cells in $\mathbf{E}$ that share some elementary letter with $E(x)$. By the irrelevance, $\bigcup\{E_j\}_{j\in J} \cap$

[1] Partial meet contraction in brief: $K - x = \bigcap\gamma(K\perp x)$ where γ is a selection function that singles out the class of the 'most important' maximal subsets of K that do not imply x. Partial meet revision may be defined using the Levi identity, i.e., putting $K * x = Cn((K - \neg x) \cup \{x\})$, or, if we are working with sets K not closed under classical consequence, simply $K * x = (K - \neg x) \cup \{x\}$. Partial meet contraction has a central role in theory change and is the core of the AGM paradigm of belief revision. Much of its power is in its generality and properties; but the selection functions make it computationally demanding. For more details see the classic paper Alchourrón, Gärdenfors and Makinson [1] or more recent textbook presentations such as Hansson [6] and Rott [11].

$E(a) = \emptyset$. Since $a \notin K' - x$, we have by the definition of partial meet contraction that $a \notin A$ for some set A that maximally satisfies the conditions that $K' - x \subseteq A \subseteq K'$ and $A \nvdash x$ (i.e., A satisfies those conditions and is not a proper subset of any set satisfying them). Since $a \in K'$ but $a \notin A$ the maximality of A gives us $A \cup \{a\} \vdash x$. Put $K_1 = \bigcup\{B_j\}_{j \in J}$ and $K_2 = \bigcup\{B_i\}_{i \in I \setminus J}$. Then since $A \subseteq K' = \bigcup\{B_i\}_{i \in I} = K_1 \cup K_2$ we have $(A \cap K_1) \cup (A \cap K_2) \cup \{a\} = A \cap (K_1 \cup K_2) \cup \{a\} = A \cup \{a\} \vdash x$. Hence by compactness, $\{b_1, \ldots, b_m\} \cup \{c_1, \ldots, c_n\} \cup \{a\} \vdash x$ where $b_1, \ldots, b_m$ are elements of $A \cap K_1$ and $c_1, \ldots, c_n$ are elements of $A \cap K_2$. Thus: $\{b_1, \ldots, b_m\} \cup \{\neg x\} \vdash \neg(c_1 \wedge \cdots \wedge c_n \wedge a)$.

We claim that the left and right sides have no letters in common. For by construction all letters occurring in formulae on the left are in $\bigcup\{E_j\}_{j \in J}$ while no letters occurring in formulae on the right are in $\bigcup\{E_j\}_{j \in J}$.

Since left and right have no letters in common, either the left side is inconsistent or the right side is a tautology. If the left side is inconsistent, $\{b_1, \ldots, b_m\} \vdash x$ so since $b_1, \ldots, b_m \in A$ we have $A \vdash x$ contradicting $A \nvdash x$. If the right side is a tautology, $\{c_1, \ldots, c_n, a\}$ is inconsistent so since all its elements are in K', K' is inconsistent and thus K is inconsistent, contrary to hypothesis. $\dashv$

We obtain as corollary the corresponding property of partial meet revision: inputting a formula for revision does not dislodge any elements of the belief set to which it is irrelevant.

Corollary 4.2. *Suppose the same conditions as for Theorem* 4.1, *with $K' * x$ a partial meet revision of K' by input x. Then, whenever $a \in K'$ is irrelevant to x (modulo K'), we have $a \in K' * x$. In brief: partial meet revision applied to the finest splitting of a consistent belief set respects relevance.*

Proof. By the definition of revision from contraction using the Levi identity, $K' - \neg x \subseteq K' * x$. Since $E(x) = E(\neg x)$ the preceding theorem tells us that $a \in K' - \neg x$, and we are done. $\dashv$

§5. Open questions. Theorem 4.1 guarantees that any AGM normalized contraction operation, i.e., defined on a set K' in canonical form, respects relevance in Parikh's sense. But it does not give us any guidance on which contraction operation on K' we might wish to choose if we are *already* given a specific contraction operation on K. So we may ask whether there is any natural way of choosing the former so that it departs as little as possible in its values from the latter. In particular:

- Is there any distinguished way of choosing a selection function γ' associated with K', given γ associated with K? Can this be done so that it preserves properties of the selection function such as relationality and transitive relationality? On the syntactic level, can we ensure that the supplementary contraction postulates (K-7) and (K-8) are preserved?

The concepts and results of this paper have been formulated in the context of classical propositional logic. Inspection of the proofs makes it clear that they all carry over to the classical first-order context, with 'elementary letters' understood as elementary predicate and function symbols. This leaves open the following question:

- How far can the results be established for subclassical (e.g., intuitionistic) consequence relations or supraclassical ones (e.g., preferential consequence relations or the relation of 'logical friendliness' of Makinson [8])?

REFERENCES

[1] CARLOS A. ALCHOURRÓN, PETER GÄRDENFORS, and DAVID MAKINSON, *On the logic of theory change: partial meet contraction and revision functions*, this JOURNAL, vol. 50 (1985), pp. 510–530.

[2] SAMIR CHOPRA, KONSTANTINOS GEORGATOS, and ROHIT PARIKH, *Relevance sensitive non-monotonic inference on belief sequences*, ***Journal of Applied Non-Classical Logics***, vol. 11 (2001), no. 1–2, pp. 131–150.

[3] SAMIR CHOPRA and ROHIT PARIKH, *Relevance sensitive belief structures*, ***Annals of Mathematics and Artificial Intelligence***, vol. 28 (2000), no. 1–4, pp. 259–285.

[4] WILLIAM CRAIG, *Linear reasoning. A new form of the Herbrand-Gentzen theorem*, this JOURNAL, vol. 22 (1957), no. 3, pp. 250–268.

[5] ———, *Three uses of the Herbrand-Gentzen theorem in relating model theory and proof theory*, this JOURNAL, vol. 22 (1957), no. 3, pp. 269–285.

[6] SVEN O. HANSSON, ***A textbook of belief dynamics: Theory change and database updating***, Applied Logic Series, Kluwer Academic Publishers, 1999.

[7] WILFRID HODGES, ***A shorter model theory***, Cambridge University Press, 1997.

[8] DAVID MAKINSON, *Friendliness for logicians*, ***We will show them! Essays in honour of Dov Gabbay*** (Sergei N. Artemov, Howard Barringer, Artur S. d'Avila Garcez, Luis C. Lamb, and John Woods, editors), vol. two, College Publications, 2005, pp. 259–292. Also appearing in ***Logica Universalis***, second edition, (J.–Y. Béziau, editor), Birkhäuser Verlag, Basel, 2007.

[9] ROHIT PARIKH, *Beliefs, belief revision, and splitting languages*, ***Logic, language, and computation*** (Stanford, California) (Lawrence Moss, Jonathan Ginzburg, and Maarten de Rijke, editors), vol. 2, CSLI Publications, 1999, pp. 266–278.

[10] PAVLOS PEPPAS, SAMIR CHOPRA, and NORMAN FOO, *Distance semantics for relevance-sensitive belief revision*, ***KR2004: Principles of knowledge representation and reasoning*** (Didier Dubois, Christopher A. Welty, and Mary-Anne Williams, editors), AAAI Press, Menlo Park, California, 2004, pp. 319–328.

[11] HANS ROTT, ***Change, choice and inference: A study of belief revision and nonmonotonic reasoning***, Oxford University Press, Oxford, 2001.

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