## PHIL309P

# Philosophy, Politics and Economics 

Eric Pacuit<br>University of Maryland, College Park<br>pacuit.org<br>Politics cases maxan Phion Nition ine Philosophy Game The May's Theorem Gaus Nash Condorcet's Paradox kneeted<br>Rational Choice Theory. ParetoHarsany<br>ArrowSocial Choice TheorySen<br>Rationality<br>Arrow's Theorem

## Announcements

- Course website

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https://myelms.umd.edu/courses/1133211
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- Reading
- Gaus, Ch. 5
- EP, Voting Methods (Stanford Encyclopedia of Philosophy)
- C. List, Social Choice Theory (Stanford Encyclopedia of Philosophy)
- M. Morreau, Arrow's Theorem (Stanford Encyclopedia of Philosophy)
- Online videos
- Quiz


## Arrow's Theorem

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Theorem (Arrow, 1951). Suppose that there are at least three candidates and finitely many voters. Any social welfare function that satisfies universal domain, independence of irrelevant alternatives and unanimity is a dictatorship.

- Infinitely many voters.
- Domain restrictions.
- Richer ballots.

Approval Voting: Each voter selects a subset of candidates. The candidate with the most "approvals" wins the election.
S. Brams and P. Fishburn. Approval Voting. Birkhauser, 1983.
J.-F. Laslier and M. R. Sanver (eds.). Handbook of Approval Voting. Studies in Social Choice and Welfare, 2010. wans same wesme Economics Nastional choice Theor Mepereot icsan Arrow Sociationality

Under Approval Voting (AV), voters are asked to select the candidates that the voter approves.

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Under ranking voting procedures (such as Borda Count), voters are asked to (linearly) rank the candidates.

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The two pieces of information are related, but not derivable from each other

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Under ranking voting procedures (such as Borda Count), voters are asked to (linearly) rank the candidates.

The two pieces of information are related, but not derivable from each other
Approving of a candidate is not (necessarily) the same as simply ranking the candidate first.

## Why Approval Voting?

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Rationality
www.electology.org/approval-voting
S. Brams and P. Fishburn. Going from Theory to Practice: The Mixed Success of Approval Voting. Handbook of Approval Voting, pgs. 19-37, 2010.

## Example

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| Voters | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 0 |
| 3 | 0 | 1 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 1 | 1 | 1 | 1 |

## Example

 We can irow Fonomics Nash Condorcets Paradox ECO ParetoHarsanyiRational Choice Theory
ArrowSocial Choice Theory Sen

| Voters | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 0 |
| 3 | 0 | 1 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 1 | 1 | 1 | 1 |


| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| A | B | D | D | A |
| B | C | B | C | B |
| C | A | C | B | D |
| D | D | A | A | C |

## Example

 We can irow Fonomics Nash Condorcets Parradox ECO P ParetoHarsanyiRational Choice Theory
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Rationality

| Voters | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 0 |
| 3 | 0 | 1 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 1 | 1 | 1 | 1 |


| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| A | B | D | D | A |
| B | C | B | C | B |
| C | A | C | B | D |
| D | D | A | A | C |

## Example

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| Voters | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 0 |
| 3 | 0 | 1 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 1 | 1 | 1 | 1 |


| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| A | B | D | D | A |
| B | C | B | C | B |
| C | A | C | B | D |
| D | D | A | A | C |

An AV ballot is sincere if, given the lowest-ranked candidate that a voter approves of, he or she also approves of all candidates ranked higher.

## Example

| Voters | A | B | C | D |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 1 | 1 | 1 | 2 | 3 | 4 | 5 |
| 2 | 0 | 1 | 1 | 0 | A | B | D | D | A |
| 3 | 0 | 1 | 0 | 0 | B | C | B | C | B |
| 4 | 0 | 0 | 0 | 0 | C | A | C | B | D |
| 5 |  |  |  |  | D | D | A | A | C |

An AV ballot is sincere if, given the lowest-ranked candidate that a voter approves of, he or she also approves of all candidates ranked higher.

## Example

| Voters | A | B | C | D |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 1 | 0 | 1 | 1 |
| 2 | 0 | 1 | 1 | 0 |
| 3 | 0 | 1 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 |
| 5 | 1 | 1 | 1 | 1 |


| 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: |
| A | B | D | D | A |
| B | C | B | C | B |
| C | A | C | B | D |
| D | D | A | A | C |

An AV ballot is sincere if, given the lowest-ranked candidate that a voter approves of, he or she also approves of all candidates ranked higher.

## Approval Voting is more flexible

 Game Theory Downsmars Theorem Gus.
Nash Consorcelts Paratoox ECOMOMICS
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Rationality

| \# voters | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | D | D | A |
|  | B | A | B |
|  | C | C | D |

The Condorcet winner is $A$.

## Approval Voting is more flexible

 Nash Condorcets Parasox ECO ParetoHarsanyi Arrowsocial Cholice

There is no fixed rule that always elects a unique Condorcet winner.

| \# voters | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | D | D | A |
|  | B | A | B |
|  | C | C | D |

The Condorcet winner is $A$.
Vote-for-1 elects $\{A, B\}$

## Approval Voting is more flexible


 Arrowsocial Cholice

There is no fixed rule that always elects a unique Condorcet winner.

| \# voters | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | D | D | A |
|  | B | A | B |
|  | C | C | D |

The Condorcet winner is $A$.
Vote-for-1 elects $\{A, B\}$, vote-for-2 elects $\{D\}$

## Approval Voting is more flexible


 Arrowsocial Cholice

There is no fixed rule that always elects a unique Condorcet winner.

| \# voters | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | D | D | A |
|  | B | A | B |
|  | C | C | D |

The Condorcet winner is $A$.
Vote-for-1 elects $\{A, B\}$, vote-for- 2 elects $\{D\}$, vote-for-3 elects $\{A, B\}$.

## Approval Voting is more flexible

 Game Theory Downsmars Theorem Gus.
Nash Consorcelts Paratoox ECOMOMICS Nash Condorcets Paradox ECO
Rational Choice Theory ParetoHarsanyi Arrow Sociaionality

AV may elect the Condorcet winner

| \# voters | 2 | 2 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | D | D | A |
|  | B | A | B |
|  | C | C | D |

The Condorcet winner is $A$.
( $\{A\},\{B\},\{C, A\}$ ) elects $A$ under AV.

## Possible Failure of Unanimity


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## Possible Failure of Unanimity

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Rationality

| \# voters | 1 | 1 | 1 |
| :---: | :---: | :---: | :---: |
|  | A | C | D |
|  | B | A | A |
|  | C | B | B |
|  | D | D | C |

Approval Winners: $A, B$

## Indeterminate or Responsive?

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| \# voters | 6 | 5 | 4 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | C | C | B |
|  | B | A | A |

Plurality winner: $A$, Borda and Condorcet winner: $C$.

## Indeterminate or Responsive?

 Mas seme temo Nashemanc cheone Tho $\underset{\text { Arows theovem }}{\text { Rationality }}$| \# voters | 6 | 5 | 4 |
| :---: | :---: | :---: | :---: |
|  | A | B | C |
|  | C | C | B |
|  | B | A | A |

Plurality winner: $A$, Borda and Condorcet winner: $C$.
Any combination of $A, B$ and $C$ can be an AV winner (or AV winners).

## Generalizing Approval Voting

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Rationality

Ask the voters to provide both a linear ranking of the candidates and the candidates that they approve.

## Generalizing Approval Voting

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Ask the voters to provide both a linear ranking of the candidates and the candidates that they approve.

Make the ballots more expressive: Dis\&Approval voting, RangeVoting, Majority Judgement

## Grading

In many group decision situations, people use measures or grades from a common language of evaluation to evaluate candidates or alternatives:

- in figure skating, diving and gymnastics competitions;
- in piano, flute and orchestra competitions;
- in classifying wines at wine competitions;
- in ranking university students;
- in classifying hotels and restaurants, e.g., the Michelin *


## Voting by Grading: Questions

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- What grading language should be used? (e.g., $A-F, 0-10, *-* * * *)$


## Voting by Grading: Questions

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- How should we aggregate the grades? (e.g., Average or Median)


## Voting by Grading: Questions

 waveneme weormeconomics Nash benacece sime ArrowSocial Choice- What grading language should be used? (e.g., $A-F, 0-10, *-* * * *)$
- How should we aggregate the grades? (e.g., Average or Median)
- Should there be a "no opinion" option?


## Voting by Grading: Questions

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- How should we aggregate the grades? (e.g., Average or Median)
- Should there be a "no opinion" option?


## Voting by Grading: Examples

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Approval Voting: voters can assign a single grade "approve" to the candidates

Dis\&Approval Voting: voters can approve or disapprove of the candidates
Majority Judgement, Score Voting: voters can assign any grade from a fixed set of grades to the candidates

## Score Voting/Range Voting

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Fixe a common grading language consisting of, for example, the integers $\{0,1,2, \ldots, 10\}$

The candidate with the largest average grade is declared the winner.

## Score Voting/Range Voting

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Rational Choice Theory ParetoHarsany Arrow Sacial Chality

Fixe a common grading language consisting of, for example, the integers $\{0,1,2, \ldots, 10\}$

The candidate with the largest average grade is declared the winner.

Suppose $A$ 's grades are $\{7,7,8,8,9,9,9,10\}$. The average grade is 8.375
Suppose $B^{\prime}$ 's grades are $\{9,9,9,9,9,10,10,10\}$. The average grade is 9.375
So, Score Vote (Range Vote) ranks $B$ above candidate $A$.

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So, Score Vote (Range Vote) ranks $B$ above candidate $A$.
www.electology.org/score-voting and rangevoting.org

## Majority Judgement

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Fix a common grading language. For example, $\{0,1,2, \ldots, 10\}$
The candidate with the largest median grade is declared the winner.
The median grade is the grade that is in the middle of the list when the grades are ordered (If there is an even number of judges, then the median grade is the lowest grade in the middle interval.)

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Suppose that $A$ 's grades are $\{6,6,7,7,7,8,9,10,10\}$ : The median grade is 7 .

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Suppose that $A$ 's grades are $\{6,6,7,7,7,8,9,10,10\}$ : The median grade is 7 .
Suppose $B^{\prime}$ 's grades are $\{6,6,6,6,9,9,9,10\}$ : The median grade is 6 .

## Majority Judgement

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The candidate with the largest median grade is declared the winner.
The median grade is the grade that is in the middle of the list when the grades are ordered (If there is an even number of judges, then the median grade is the lowest grade in the middle interval.)

Suppose that $A$ 's grades are $\{6,6,7,7,7,8,9,10,10\}$ : The median grade is 7 .
Suppose B's grades are $\{6,6,6,6,9,9,9,10\}$ : The median grade is 6 .
Majority Judgement ranks $B$ above $A$.

## Majority Judgement: Tie-breaking rules

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## Majority Judgement: Tie-breaking rules


 ArrowSocial Choice
Rationality

What happens when the median grades are the same?
A's grades: $\{7,9,9,11,11\}$
$B$ 's grades: $\{8,9,9,10,11\}$

## Majority Judgement: Tie-breaking rules

 Nash Condorcets Paradox
Rational Choice
Theory Arrow Rationality

What happens when the median grades are the same?
A's grades: $\{7,9,9,11,11\}$
B's grades: $\{8,9,9,10,11\}$
The second median grade is found:
A's grades: $\{7,9,9,11,11\}$
$B^{\prime}$ 's grades: $\{8,9,9,10,11\}$

## Majority Judgement: Tie-breaking rules

 Game Theory Downsmars Theorem Guss
Nash Consorests Paratox ECOOMOMCS Nash Condorcets Parasox
Rational Choice
Theory Arrow Rationality

What happens when the median grades are the same?
A's grades: $\{7,9,9,11,11\}$
$B$ 's grades: $\{8,9,9,10,11\}$
The second median grade is found:
A's grades: $\{7,9,9,11,11\}$
$B$ 's grades: $\{8,9,9,10,11\}$
The third median grade is found:
A's grades: $\{7,9,9,11,11\}$
$B^{\prime}$ 's grades: $\{8,9,9,10,11\}$

## Majority Judgement: Tie-breaking rules

 ArrowSocial Choice TheorySen $\underset{\text { Arows theorem }}{\text { Rationality }}$

What happens when the median grades are the same?
A's grades: $\{7,9,9,11,11\}$
$B$ 's grades: $\{8,9,9,10,11\}$
The second median grade is found:
A's grades: $\{7,9,9,11,11\}$
$B$ 's grades: $\{8,9,9,10,11\}$
The third median grade is found:
A's grades: $\{7,9,9,11,11\}$
$B^{\prime}$ 's grades: $\{8,9,9,10,11\}$
So, $A$ is ranked above $B$.

## Example

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## Example

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Rational Choice Theory ArrowSocial Choice TheorySen

Suppose that there are five voters, $1, \ldots, 5$ and three candidates I, II, and III. The grades are $A, B, C, D$, or $F$ (from best to worst).

|  | 1 | 2 | 3 | 4 | 5 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| I | A | A | C | D | D |
| II | B | B | F | B | F |
| III | D | C | B | A | D |

## Example

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Suppose that there are five voters, $1, \ldots, 5$ and three candidates I, II, and III. The grades are $A, B, C, D$, or $F$ (from best to worst).

|  | 1 | 2 | 3 | 4 | 5 | Median |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | A | A | C | D | D | C |
| II | B | B | F | B | F | B |
| III | D | C | B | A | D | C |

Candidate II is the majority judgement winner.

## Example

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Suppose that there are five voters, $1, \ldots, 5$ and three candidates $I, I I$, and III. The grades are $A, B, C, D$, or $F$ (from best to worst).

|  | 1 | 2 | 3 | 4 | 5 | Median |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | A | A | C | D | D | C |
| II | B | B | F | B | F | B |
| III | D | C | B | A | D | C |

Candidate II is the majority judgement winner. If asked about their preference, 4 voters would rank candidate I above candidate II

## Example

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Suppose that there are five voters, $1, \ldots, 5$ and three candidates I, II, and III. The grades are $A=4, B=3, C=2, D=1$, or $F=0$ (from best to worst).

|  | 1 | 2 | 3 | 4 | 5 | Average |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| I | 4 | 4 | 2 | 1 | 1 | 2.4 |
| II | 3 | 3 | 0 | 3 | 0 | 1.8 |
| III | 1 | 2 | 3 | 4 | 1 | 2.2 |

Candidate II is the Majority Judgement winner. Candidate I is the Score Voting winner

## More Information

 Mssemen werny conomics Arrow Sociaionality
M. Balinski and R. Laraki. Majority Judgement: Measuring, Ranking and Electing. The MIT Press, 2010.
W. D. Smith. www.rangevoting.org. .

 Arrow Sationality
S. Brams and R. Potthoff. The Paradox of Grading Systems. Manuscript, 2015.

A grading system is a voting system in which a voter can give any of $g$ grades, $\left\{w_{1}, \ldots, w_{g}\right\}$, to each candidates.

AG winner: Candidate(s) that receives the largest average grade

SG winner: compare each candidate's grades with the grades of all other candidates. Candidate $X$ beats candidate $Y$ if the number of voters who grade $X$ higher than $Y$ exceed the number of voters that grade $Y$ higher than $X$. The candidate(s) that beat every other candidate is(are) the SG winner(s).

## Weak Paradox of Grading Systems

Grades: $\{0,1,2\}$ nes nemen wem Economics

Candidates: $\{A, B, C\}$ 9 Voters

| \# voters | 2 | 3 | 4 | Avg |
| :---: | :--- | :--- | :--- | :--- |
| $A$ | 2 | 0 | $1 \mid$ |  |
| $B$ | 1 | 2 | 0 |  |
| $C$ | 0 | 1 | 2 |  |

Grades: $\{0,1,2\}$
Candidates: $\{A, B, C\}$ 9 Voters

| \# voters | 2 | 3 | 4 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 2 | 0 | $1 \mid$ | $8 / 9$ |
| $B$ | 1 | 2 | 0 | $8 / 9$ |
| $C$ | 0 | 1 | 2 | $11 / 9$ |

Grades: $\{0,1,2\}$
Candidates: $\{A, B, C\}$ 9 Voters

| \# voters | 2 | 3 | 4 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 2 | 0 | 1 | $8 / 9$ |
| $B$ | 1 | 2 | 0 | $8 / 9$ |
| $C$ | 0 | 1 | 2 | $11 / 9$ |

Average Grade Winner: C

Grades: $\{0,1,2\}$
Candidates: $\{A, B, C\}$

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Rationality 9 Voters

| \# voters | 2 | 3 | 4 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 2 | 0 | 1 |  |
| $B$ | 1 | 2 | 0 |  |
| $C$ | 0 | 1 | 2 |  |

Average Grade Winner: $C$

$$
A>B
$$

Grades: $\{0,1,2\}$
Candidates: $\{A, B, C\}$ Navshementereme Economics
 ArrowSocial Choice 9 Voters

| \# voters | 2 | 3 | 4 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 2 | 0 | 1 |  |
| $B$ | 1 | 2 | 0 |  |
| $C$ | 0 | 1 | 2 |  |

Average Grade Winner: C

$$
A>B>C
$$

Grades: $\{0,1,2\}$
Candidates: $\{A, B, C\}$

 ArrowSocial Choice 9 Voters

| \# voters | 2 | 3 | 4 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 2 | 0 | 1 |  |
| $B$ | 1 | 2 | 0 |  |
| $C$ | 0 | 1 | 2 |  |

Average Grade Winner: $C$

$$
A>B>C>A
$$

Grades: $\{0,1,2\}$
Candidates: $\{A, B, C\}$ waveneme weormeconomics
 ArrowSocial Choice
Rationality 9 Voters

| \# voters | 2 | 3 | 4 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 2 | 0 | 1 |  |
| $B$ | 1 | 2 | 0 |  |
| $C$ | 0 | 1 | 2 |  |

Average Grade Winner: C
Superior Grade Winners: $A, B, C$

## Strong Paradox of Grading Systems

Grades: $\{0,1,2,3\}$
Candidates: $\{A, B, C\}$ 3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :--- | :--- | :--- | :--- |
| $A$ | 3 | 2 | 0 |  |
| $B$ | 0 | 3 | 1 |  |
| $C$ | 0 | 3 | 1 |  |

Grades: $\{0,1,2,3\}$ wans ame ther
 Arrow Rationality
Candidates: $\{A, B, C\}$ 3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 3 | 2 | 0 | $5 / 3$ |
| $B$ | 0 | 3 | $1 \mid$ | $4 / 3$ |
| $C$ | 0 | 3 | 1 | $4 / 3$ |

Average Grade Winner: $A$

Grades: $\{0,1,2,3\}$

 ArrowSocial Choice
Rationality
Arrows theorem
Candidates: $\{A, B, C\}$
3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 3 | 2 | 0 |  |
| $B$ | 0 | 3 | 1 |  |
| $C$ | 0 | 3 | 1 |  |

Average Grade Winner: $A$

$$
B>A
$$

Grades: $\{0,1,2,3\}$
Candidates: $\{A, B, C\}$

 ArrowSocial Choice
Rationality 3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 3 | 2 | 0 |  |
| $B$ | 0 | 3 | 1 |  |
| $C$ | 0 | 3 | 1 |  |

Average Grade Winner: $A$

$$
C \sim B>A
$$

Grades: $\{0,1,2,3\}$ Mens shemen wem Economics
 ArrowSocial Choice
Candidates: $\{A, B, C\}$ $\underset{\text { Arows theorem }}{\text { Rationality }}$

3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 3 | 2 | 0 |  |
| $B$ | 0 | 3 | 1 |  |
| $C$ | 0 | 3 | 1 |  |

Average Grade Winner: $A$

$$
C \sim B>A
$$

Grades: $\{0,1,2,3\}$
Candidates: $\{A, B, C\}$

## 3 Voters

| \# voters | 1 | 1 | 1 | Avg |
| :---: | :---: | :---: | :---: | :---: |
| $A$ | 3 | 2 | 0 |  |
| $B$ | 0 | 3 | 1 |  |
| $C$ | 0 | 3 | 1 |  |

Average Grade Winner: $A$
Superior Grade Winners: $C, B$

Grades: $\{0,1,2,3,4,5\}$
Candidates: $\{A, B, C\}$ 5 Voters

| \# voters | 1 | 4 | Avg |
| :---: | :---: | :---: | :---: |
| $A$ | 5 | 0 | $5 / 5$ |
| $B$ | 0 | 1 | $4 / 5$ |
| $C$ | 0 | 1 | $4 / 5$ |

Average Grade Winner: $A$
Superior Grade Winner: B, C

To conclude, we have identified a paradox of grading systems, which is not just a mirror of the well-known differences that crop up in aggregating votes under ranking systems. Unlike these systems, for which there is no accepted way of reconciling which candidate to choose when, for example, the Hare, Borda and Condorcet winners differ, AV provides a solution when the aG and SG winners differ.

Theorem. When there are two grades, the AG and SG winners are identical.

## Manipulating an election outcome

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It has long been noted that a voter can achieve a preferred election outcome by misrepresenting his or her actual preferences.

## Manipulating an election outcome

 Ms.amicher Nationan chowe Trico $\underset{\text { Arows theovem }}{\text { Rationality }}$It has long been noted that a voter can achieve a preferred election outcome by misrepresenting his or her actual preferences.
C.L. Dodgson refers to a voters tendency to
"adopt a principle of voting which makes it a game of skill than a real test of the wishes of the elector."

## Manipulating an election outcome

It has long been noted that a voter can achieve a preferred election outcome by misrepresenting his or her actual preferences.
C.L. Dodgson refers to a voters tendency to
"adopt a principle of voting which makes it a game of skill than a real test of the wishes of the elector."
and that in his opinion
"it would be better for elections to be decided according to the wishes of the majority than of those who happen to be more skilled at the game."

## Manipulating an election outcome

"If we assume society discourages the concentration of power, then at least two methods of manipulation are always available, no matter what method of voting is used: First, those in control of procedures can manipulate the agenda (by, for example, restricting alternatives or by arranging the order in which they are brought up). Second, those not in control can still manipulate by false revelation of values."
(p. 137)
W. Riker. Liberalism Against Populism. Waveland Press, 1988.

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## Literature

 Nashtional Choice Theory Pareto Harsanyi ArrowsationaliceA. Taylor. Social Choice and the Mathematics of Manipulation. Cambridge University Press, 2005.
W. Poundstone. Gaming the Vote. Hill and Wang Publishers, 2008.

## Manipulation: setting the agenda

 M, Gme han Economics Nash Consorcet's Paradox ECORational Choice Theory ParetoHarsanyi ArrowSocial Choice
Rationality

| \# voters | 1 | 1 | 1 |
| :---: | :---: | :---: | :---: |
|  | B | A | C |
|  | D | B | A |
|  | C | D | B |
|  | A | C | D |

## Manipulation: setting the agenda

 uns nemene wein Arrowsocial Rnalice

| \# voters | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- |
|  | B | A | C |
|  | D | B | A |
|  | C | D | B |
|  | A | C | D |



A C D

## Manipulation: setting the agenda

 waven rame thery ArrowSocial Choice
Rationality


## Manipulation: setting the agenda

Politics cass hasum tum
 ArrowSocial Choice TheorySen


## Manipulation: setting the agenda

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 ArrowSocial Choice
Rationality


## Manipulation: setting the agenda



 ArrowSocial Choice
Rationality


## Manipulation: misrepresenting preferences

 mars sheorem GeusNash Condorceets Paradox ECOMN
EOMOMICS Nash Consorcet's Paradox ECO Con ParetoHarsanyi
Rational Choice Theory $\underset{\text { Rrrows theorem }}{\text { Ratity }}$

| \# voters | 3 | 3 | 1 |
| :--- | :---: | :---: | :---: |
|  | A | B | C |
|  | B | A | A |
|  | C | C | B |

Borda Winner: $A$

## Manipulation: misrepresenting preferences

 Nash Consorcets Paradox LCL
Rational Choice Theory ParetoHarsanyi $\underset{\text { Rrrows theorem }}{\text { Ratity }}$

\# voters |  | 3 | 3 |
| :---: | :---: | :---: |
|  | A | B |
| C |  |  |
|  | B | A |
| A |  |  |
|  | C | C |
|  | B |  |

Borda Winner: $A$

## Manipulation: misrepresenting preferences


 ArrowSocial Choice
Rationality

| \# voters | 3 | 3 | 1 | \# voters | 3 | 3 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C |  | A | B | C |
|  | B | A | A |  | B | C | A |
|  | C | C | B |  | C | A | B |

Borda Winner: $A$

## Manipulation: misrepresenting preferences

 ways rame thery Naschemances inge ECOMOMiCS ArrowSocial ChoiceRationality

| \# voters | 3 | 3 | 1 | \# voters | 3 | 3 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C |  | A | B | C |
|  | B | A | A |  | B | C | A |
|  | C | C | B |  | C | A | B |
| Borda Winner: $A$ |  |  |  | Borda Winner: $B$ |  |  |  |

## Manipulation: misrepresenting preferences

 waveneme weormeconomics ArrowSocial Choice
Rationality

| \# voters | 3 | 3 | 1 | \# voters | 3 | 3 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | B | C |  | A | B | C |
|  | B | A | A |  | B | C | A |
|  | C | C | B |  | C | A | B |
| Borda Winner: $A$ |  |  |  | Borda Winner: $B$ |  |  |  |

Borda: "My procedure is only meant for honest men!"

