## PHIL309P

# Philosophy, Politics and Economics 

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## Announcements

- Course website

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https://myelms.umd.edu/courses/1133211
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- Reading
- Gaus, Ch. 5
- EP, Voting Methods (Stanford Encyclopedia of Philosophy)
- C. List, Social Choice Theory (Stanford Encyclopedia of Philosophy)
- M. Morreau, Arrow's Theorem (Stanford Encyclopedia of Philosophy)


## Voting Situations

 Neshemenerem Economics Nashlouna chice Theory| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| $\boldsymbol{q}_{\text {worst }}$ | D | D | A | A |

- 21 voters and 4 candidates: Ann (A), Bob (B), Charles (C) and Dora (D)


## Voting Situations

 wavs neme thern Economics Nastional Choice Theory ParetoHarsany| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | D | B | C | D |
|  | D | A | A |  |

- 21 voters and 4 candidates: Ann (A), Bob (B), Charles (C) and Dora (D)
- Each voter ranks the candidates from best (at the top of the list) to worst (at the bottom of the list) resulting in the 4 voting blocks given in the above table


## Voting Situations

 Nash conarcets Rational Choice Theory ParetoHarsanyi

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | D | B | C | D |
|  | D | A | A |  |

Who should win the election?

## Which candidate should be chosen?

 Nash Consorcet's Paradot ECO OPM Rational Choice Theory ArrowSocial Choice
Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | D | D | A | A |

## Which candidate should be chosen?

Politicsass numm tume

 ArrowSocial Choice
Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
|  | B | C | D | B |
|  | C | B | C | D |
|  | D | D | A | A |

- Candidate $A$ : More people (8) rank $A$ first than any other candidate


## Which candidate should be chosen?

 nsan shime theoryems ArrowSocial Choice
Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\overbrace{\text { worst }}$ | B | C | D | B |
|  | C | B | C | D |
|  |  | A | A |  |

- Candidate $A$ : More people rank $A$ first than any other candidate
- Candidate $A$ should not win: more than half rank $A$ last


## Which candidate should be chosen?




| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
|  | D | D | A | A |

- Candidate $A$ : More people rank $A$ first than any other candidate
- Candidate $D$ should not win


## Which candidate should be chosen?


 ArrowSocial Choice
Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| Worst | C | B | C | D |
|  | D | D | A | A |

- Candidate $A$ : More people rank $A$ first than any other candidate
- Candidate $D$ should not win: everyone ranks $B$ higher than $D$


## Which candidate should be chosen?

 Nash Consorcets Paradox LCL
Rational Choice Theory ParetoHarsanyi Arrow Sociationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
| D | A | A |  |  |

- Which of $B$ or $C$ should win?


## Which candidate should be chosen?



Marquis de Condorcet (1743-1794)


Jean-Charles de Borda (1733-1799)

## Which candidate should be chosen?

 Nasheman choie Troay Arrow Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
| D | A | A |  |  |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)


## Which candidate should be chosen?

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| $\uparrow_{\text {worst }}$ | C | B | C | D |
|  | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)


## Which candidate should be chosen?

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
|  | C | B | C | D |
| worst | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)


## Which candidate should be chosen?

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
|  | B | C | D | B |
|  | C | B | C | D |
|  | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)


## Which candidate should be chosen?

 Nash benaxe fisyet

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
| D | D | A | A |  |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)


## Which candidate should be chosen?

 Nash Consorcets Paradox
Rational Choice Theory ParetoHarsany

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
|  | B | C | D | B |
|  | C | B | C | D |
| worst | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- B gets 13 (vs. A)


## Which candidate should be chosen?

 Nash Condorcets Paradox Rational Choice Theory ParetoHarsany

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
|  | B | C | D | B |
|  | C | B | C | D |
| worst | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- B gets 13 (vs. A) + 10 (vs. C)


## Which candidate should be chosen?

 Nash Condorcets Paradox Rational Choice Theory ParetoHarsany Arrow Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best   <br> $\uparrow$ A A <br>  B C <br>  C B <br> worst D D C | D |  |  |  |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- B gets 13 (vs. $A)+10($ vs. $C)+21$ (vs. $D)=44$ points


## Which candidate should be chosen?


 Nash Consorcet's Paradox ECO
Rational Choice Theory ParetoHarsanyi ArrowSocial Choice
Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\overbrace{\text { worst }}$ | B | C | D | B |
|  | C | B | C | D |
|  | D | A | A |  |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- $C$ get $13($ vs. $A)+11($ vs. $B)+14($ vs. $D)=38$ points


## Which candidate should be chosen?


 Nash Consorcet's Paradox ECO
Rational Choice Theory ParetoHarsanyi

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best <br> $\uparrow$ | A | A | B | C |
|  | B | C | D | B |
| worst | C | B | C | D |
|  | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- $C$ get $13($ vs. $A)+11($ vs. $B)+14($ vs. $D)=38$ points


## Which candidate should be chosen?

 Nashh Consorcets Paradox
Rational Choice Theory ParetoHarsany

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
|  | C | B | C | D |
| worst | D | D | A | A |

- Candidate $C$ should win: $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ should win: Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)
- C get 13 (vs. $A)+11($ vs. $B)+14($ vs. $D)=38$ points


## Which candidate should be chosen?

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
|  | D | A | A |  |

- Candidate $A$ should not win: more than half rank $A$ last
- Candidate $D$ should not win: everyone ranks $B$ higher than $D$
- Candidate $C$ : $C$ beats every other candidate in head-to-head elections ( $C$ is the Condorcet winner)
- Candidate $B$ : Taking into account the entire ordering, $B$ has the most "support" ( $B$ is the Borda winner)


## Which candidate should be chosen?

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | D | B | C | D |
|  |  | A | A |  |

- Conclusion: there are many ways to answer the above question!
- Candidate B: Taking into account the entire ordering, $B$ has the most "support" (B is the Borda winner)


# The Condorcet Paradox 

## Recall Condorcet's Idea

 Game Theory Downsmars Theorem Guss
Nash Consorests Paratox ECOMOMICS Nash consorcets Rational Choice Theory ParetoHarsanyi

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
| D | A | A |  |  |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections.


## Recall Condorcet's Idea


 Arrow Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
|  | D | A | A |  |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections.


## Recall Condorcet's Idea

 waven same therams Nast Rana Arrow Rationality| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
| w | D | A | A |  |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections.


## Recall Condorcet's Idea


 Arrow Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
|  | D | D | A | A |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections.


## Recall Condorcet's Idea


 Arrow Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| $\overbrace{\text { worst }}$ | D | B | C | D |
|  |  | D | A | A |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections. $B$ is ranked second


## Recall Condorcet's Idea


 Arrow Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst | C | B | C | D |
| w | A | A |  |  |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections. $B$ is ranked second


## Recall Condorcet's Idea


 ArrowSocial Choice
Rationality

| \# voters | 3 | 5 | 7 | 6 |
| :---: | :---: | :---: | :---: | :---: |
| best | A | A | B | C |
| $\uparrow$ | B | C | D | B |
| worst $^{\text {wors }}$ | D | B | C | D |
|  | D | A | A |  |

- Candidate $C$ should win since $C$ beats every other candidate in head-to-head elections. $B$ is ranked second, $D$ is ranked third, and $A$ is ranked last.

$$
C>_{M} B>_{M} D>_{M} A
$$

## The Majority Relation

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Rationality
arrows theocrem
Suppose that $X$ and $Y$ are candidates and $P_{i}$ represents voter $i$ 's strict preference.
$\mathbf{N}(X P Y)=\left|\left\{i \mid X P_{i} Y\right\}\right|$
"the number of voters that rank $X$ strictly above $Y$ "

## The Majority Relation

Suppose that $X$ and $Y$ are candidates and $P_{i}$ represents voter $i$ 's strict preference.
$\mathbf{N}(X P Y)=\left|\left\{i \mid X P_{i} Y\right\}\right|$
"the number of voters that rank $X$ strictly above $Y^{\prime \prime}$
$X \geq_{M} Y$ iff $\mathbf{N}(X P Y) \geq \mathbf{N}(Y P X)$
"a majority prefers candidate $X$ over candidate $Y^{\prime \prime}$

## The Majority Relation

Suppose that $X$ and $Y$ are candidates and $P_{i}$ represents voter $i$ 's strict preference.
$\mathbf{N}(X P Y)=\left|\left\{i \mid X P_{i} Y\right\}\right|$
"the number of voters that rank $X$ strictly above $Y$ "
$X \geq_{M} Y$ iff $\mathbf{N}(X P Y) \geq \mathbf{N}(Y P X)$
"a majority prefers candidate $X$ over candidate $Y$ "
$X$ is a Condorcet winner if $X$ beats every other candidate in an head-to-head election: there is no candidate $Y$ such that $Y>_{M} X$

## The Majority Relation

Suppose that $X$ and $Y$ are candidates and $P_{i}$ represents voter $i$ 's strict preference.
$\mathbf{N}(X P Y)=\left|\left\{i \mid X P_{i} Y\right\}\right|$
"the number of voters that rank $X$ strictly above $Y$ "
$X \geq_{M} Y$ iff $\mathbf{N}(X P Y) \geq \mathbf{N}(Y P X)$
"a majority prefers candidate $X$ over candidate $Y$ "
$X$ is a Condorcet winner if $X$ beats every other candidate in an head-to-head election: there is no candidate $Y$ such that $Y>_{M} X$
$X$ is a Condorcet loser if $X$ loses to every other candidate in an head-to-head elections: there is no candidate $Y$ such that, $X>_{M} Y$

## The Problem





ArrowSocial Choice
Rationality
Voter 1 Voter 2 Voter 3
A C B
B
A
C
$\begin{array}{lll}C & B & A\end{array}$

## The Problem

 was same wherneconomics Arrow Rationality
Voter 1 Voter 2 Voter 3

| A | C | B |
| :---: | :---: | :---: |
| B | A | C |
| C | B | A |

- Does the group prefer $A$ over $B$ ?


## The Problem

 Mays sheomem Gexusory Downs Nash Condorret's Paradox ECO ROMOS ArrowSocial Choice TheorySenVoter 1 Voter 2 Voter 3

| A | C | B |
| :---: | :---: | :---: |
| B | A | C |
| C | B | A |

- Does the group prefer $A$ over $B$ ? Yes


## The Problem

| Voter 1 | Voter 2 | Voter 3 |
| :---: | :---: | :---: |
| A | C | B |
| B | A | C |
| C | B | A |

- Does the group prefer $A$ over $B$ ? Yes
- Does the group prefer $B$ over C? Yes


## The Problem

Voter 1 Voter 2 Voter 3

| A | C | B |
| :---: | :---: | :---: |
| B | A | C |
| C | B | A |

- Does the group prefer $A$ over $B$ ? Yes
- Does the group prefer $B$ over C? Yes
- Does the group prefer $A$ over C? No


## The Problem

 Nash Consorcets Paradox ECO ParetoHarsanyi ArrowSocial Choice TheorySen $\underset{\text { Rrows theorem }}{\text { Ration }}$
Voter 1 Voter 2 Voter 3

| A | C | B |
| :--- | :--- | :--- |
| B | A | C |
| C | B | A |

The majority relation $>_{M}$ is not transitive!
There is a Condorcet cycle: $A>_{M} B>_{M} C>_{M} A$

## How bad is this?

- Final decisions are extremely sensitive to institutional features such as who can set the agenda, arbitrary time limits place on deliberation, who is permitted to make motions, etc.


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- Final decisions are extremely sensitive to institutional features such as who can set the agenda, arbitrary time limits place on deliberation, who is permitted to make motions, etc.
- Is there empirical evidence that Condorcet cycles have shown up in real elections?
W. Riker. Liberalism against Populism. Waveland Press, 1982.
G. Mackie. Democracy Defended. Cambridge University Press, 2003.


## How bad is this?

- Final decisions are extremely sensitive to institutional features such as who can set the agenda, arbitrary time limits place on deliberation, who is permitted to make motions, etc.
- Is there empirical evidence that Condorcet cycles have shown up in real elections?
W. Riker. Liberalism against Populism. Waveland Press, 1982.
G. Mackie. Democracy Defended. Cambridge University Press, 2003.
- How likely is a Condorcet cycle?

Should we select a Condorcet winner (when one exists)?

## Is the Condorcet winner the "best" choice?

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 ArrowSocial Choice
Rationality

| \# voters | 47 | 47 | 3 | 3 |
| :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | C |
|  | C | C | A | B |
|  | B | A | B | A |

$C$ is the Condorcet winner

## Is the Condorcet winner the "best" choice?

| \# voters | 47 | 47 | 3 | 3 |
| :---: | :---: | :---: | :---: | :---: |
|  | A | B | C | C |
|  | C | C | A | B |
|  | B | A | B | A |

$C$ is the Condorcet winner; however, it seems that supporters of the main rivals $A$ and $B$ would rather see $C$ win than their candidate's principal opponent, but this does not mean that there is "positive support" for $C$.

## Condorcet's Other Paradox

 Mars theorem Gews Nash Consorcet's Paradot ECO OPM Rational Choice TheoryArrowSocial Choice
Rationality

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |

## Condorcet's Other Paradox

等| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | A | A | B | B | C | C |
| 1 | B | C | A | C | A | B |
| 0 | C | B | C | A | B | A |

$$
\begin{aligned}
& B S(A)=2 \times 31+1 \times 39+0 \times 11=101 \\
& B S(B)=2 \times 39+1 \times 31+0 \times 11=109 \\
& B S(C)=2 \times 11+1 \times 11+0 \times 59=33
\end{aligned}
$$

$B>_{B C} A>_{B C} C$

## Condorcet's Other Paradox

 Nash Consorcet's Paradot ECO OPM Rational Choice Theory

ArrowSocial Choice
Rationality

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$

## Condorcet's Other Paradox

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 Nash Condorcet's Paradox ECO
Rational Choice Theory ParetoHarsanyi

Arrowsocial Choice

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
| B | C | A | C | A | B |  |
| C | B | C | A | B | A |  |

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$

## Condorcet's Other Paradox

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ArrowSocial Choice
Rationality

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$

## Condorcet's Other Paradox

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{2}$ | A | A | B | B | C | C |
| $s_{1}$ | B | C | A | C | A | B |
| $s_{0}$ | C | B | C | A | B | A |

Condorcet's Other Paradox: No scoring rule will work...

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$

## Condorcet's Other Paradox

 Nash Consorcets maredo
Rational Choice Theory ParetoHarsany
ArrowSocial Choice TheorySen

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{2}$ | A | A | B | B | C | C |
| $s_{1}$ | B | C | A | C | A | B |
| $s_{0}$ | C | B | C | A | B | A |

Condorcet's Other Paradox: No scoring rule will work...
Score $(A)=s_{2} \times 31+s_{1} \times 39+s_{0} \times 11$
Score $(B)=s_{2} \times 39+s_{1} \times 31+s_{0} \times 11$
$B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C$

## Condorcet's Other Paradox

 Nash Consorcets Parresox
Rational Choice Theory ParetoHarsany
ArrowSocial Choice TheorySen

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{2}$ | A | A | B | B | C | C |
| $s_{1}$ | B | C | A | C | A | B |
| $s_{0}$ | C | B | C | A | B | A |

Condorcet's Other Paradox: No scoring rule will work...
$\operatorname{Score}(A)=s_{2} \times 31+s_{1} \times 39+s_{0} \times 11$
Score $(B)=s_{2} \times 39+s_{1} \times 31+s_{0} \times 11$
$\operatorname{Score}(A)>\operatorname{Score}(B) \Rightarrow 31 s_{2}+39 s_{1}>39 s_{2}+31 s_{1} \Rightarrow s_{1}>s_{2}$
$B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C$

## Condorcet's Other Paradox

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| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $s_{2}$ | A | A | B | B | C | C |
| $s_{1}$ | B | C | A | C | A | B |
| $s_{0}$ | C | B | C | A | B | A |

Theorem (Fishburn 1974). For all $m \geq 3$, there is some voting situation with a Condorcet winner such that every scoring rule will have at least $m-2$ candidates with a greater score than the Condorcet winner.
P. Fishburn. Paradoxes of Voting. The American Political Science Review, 68:2, pgs. 537-546, 1974.




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| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A | Whane hrow Economics

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$$
\begin{array}{ccccccc}
\text { \# voters } & 30 & 1 & 29 & 10 & 10 & 1 \\
\hline 2 & \mathrm{~A} & \mathrm{~A} & \mathrm{~B} & \mathrm{~B} & \mathrm{C} & \mathrm{C} \\
1 & \mathrm{~B} & \mathrm{C} & \mathrm{~A} & \mathrm{C} & \mathrm{~A} & \mathrm{~B} \\
0 & \mathrm{C} & \mathrm{~B} & \mathrm{C} & \mathrm{~A} & \mathrm{~B} & \mathrm{~A} \\
B S(A)=2 \times 31+1 \times 39+0 \times 11=101 \\
B S(B)=2 \times 39+1 \times 31+0 \times 11=109 \\
B S(C)=2 \times 11+1 \times 11+0 \times 59=33
\end{array}
$$

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Rationality
Arrows theorem

$$
\begin{array}{lcccccc}
\text { \# voters } & 30 & 1 & 29 & 10 & 10 & 1 \\
\hline & \text { A } & \text { A } & \text { B } & \text { B } & \text { C } & \text { C } \\
& \text { B } & \text { C } & \text { A } & \text { C } & \text { A } & \text { B } \\
& \text { C } & \text { B } & \text { C } & \text { A } & \text { B } & \text { A }
\end{array}
$$

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$





| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
| B | C | A | C | A | B |  |
| C | B | C | A | B | A |  |

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$


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$$
\begin{array}{lcccccc}
\text { \# voters } & 30 & 1 & 29 & 10 & 10 & 1 \\
\hline & \mathrm{~A} & \mathrm{~A} & \mathrm{~B} & \mathrm{~B} & \mathrm{C} & \mathrm{C} \\
& \mathrm{~B} & \mathrm{C} & \mathrm{~A} & \mathrm{C} & \mathrm{~A} & \mathrm{~B} \\
& \mathrm{C} & \mathrm{~B} & \mathrm{C} & \mathrm{~A} & \mathrm{~B} & \mathrm{~A}
\end{array}
$$

$$
B>_{B C} A>_{B C} C \quad A>_{M} B>_{M} C
$$

## Condorcet Triples



 ArrowSocial Choice
Rationality

| $G_{1}$ | $G_{2}$ | $G_{3}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | C |  |  | $G_{1}$ | $G_{2}$ |
| A | $G_{3}$ |  |  |  |  |  |
| B | C | A | B |  |  |  |
| C | A | B |  |  | C | B |
| A | A | C |  |  |  |  |

If $G_{1}=G_{2}=G_{3}$, then this group of voters "cancel out" each other's votes

## Saari's argument


 ArrowSocial Choice

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| ---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |

## Saari's argument



 ArrowSocial Choice
Rationality

| \# voters | 30 | 1 | 29 | 10 | 10 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |
| 10 | 10 | 10 |  |  |  |  |
| A | B | C |  |  |  |  |
| B | C | A |  |  |  |  |
| C | A | B |  |  |  |  |
|  |  |  |  |  |  |  |

## Saari's argument

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Rational Choice Theory Pareto Harsanyi ArrowSocial Choice
Rationality

| \# voters | 20 | 1 | 29 | 0 | 0 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A | A | B | B | C | C |
|  | B | C | A | C | A | B |
|  | C | B | C | A | B | A |
|  |  |  |  |  |  |  |
| 10 | 10 | 10 |  |  | 1 | 1 |
| A | B | C |  |  | A | C |
| B | C | A |  | C | B | A |
| C | A | B |  | B | A | C |

## Saari's argument

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Rationality

| \# voters | 20 | 0 | 28 | 0 | 0 | 0 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | A |  | B |  |  |  |  |
|  | B |  | A |  |  |  |  |
|  | C |  | C |  |  |  |  |
| 10 | 10 | 10 |  | 1 | 1 | 1 |  |
| A | B | C |  |  | A | C | B |
| B | C | A |  | C | B | A |  |
| C | A | B |  | B | A | C |  |

## There are many different voting methods

Many different electoral methods: Plurality, Borda Count, Antiplurality/Veto, and k-approval; Plurality with Runoff; Single Transferable Vote (STV)/Hare; Approval Voting; Cup Rule/Voting Trees; Copeland; Banks; Slater Rule; Schwartz Rule; the Condorcet rule; Maximin/Simpson, Kemeny; Ranked Pairs/Tideman; Bucklin Method; Dodgson Method; Young's Method; Majority Judgment; Cumulative Voting; Range/Score Voting; ...

## Choosing how to choose

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Arrow Social Choice ParetoHarsany $\underset{\substack{\text { Rrows theorem }}}{\substack{\text { Rity } \\ \text { and }}}$

Pragmatic considerations: Is the procedure easy to use? Is it legal? The importance of ease of use should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.

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## Voting Methods

Positional Scoring Rules: Given the rankings of the candidates provided by the voters, each candidate is assigned a score. The candidate(s) with the highest score is(are) declared the winner(s).

Examples: Borda, Plurality

Generalized Scoring Rules: Voters assign scores, or "grades", to the candidates. The candidate(s) with the "best" aggregate score is(are) declared the winner(s).

Examples: Approval Voting, Majority Judgement, Range Voting

## Voting Methods

Staged Procedures: The winner(s) is(are) determined in stages. At each stage, one or more candidates are eliminated. The candidate or candidates that are never eliminated are declared the winner(s).

Examples: Plurality with Runoff, Hare, Coombs

Condorcet Consistent Methods: Voting methods that guarantee that the Condorcet winner is elected.

Examples: Copeland, Dodgson, Young

# Voting Methods Tutorial 

