## PHIL309P

# Philosophy, Politics and Economics 

Eric Pacuit<br>University of Maryland, College Park<br>pacuit.org<br>Politics cases maxan Phionme Nition ine Philosophy Game The May's Theorem Gaus Nash Condorcet's Paradox kneeted<br>Rational Choice Theory. ParetoHarsany<br>ArrowSocial Choice TheorySen<br>Rationality<br>Arrow's Theorem

## Announcements

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- Course website https://myelms.umd.edu/courses/1133211
- Problem set 1
- Online quiz 2
- Reading: Gaus, Ch 2; Reiss, Ch 3; Briggs SEP article.
- Weekly writing: Due Wednesday, 11.59pm.


## Decision Problems

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Arrows theorem

In many circumstances the decision maker doesn't get to choose outcomes directly, but rather chooses an instrument that affects what outcome actually occurs.

## Decision Problems

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In many circumstances the decision maker doesn't get to choose outcomes directly, but rather chooses an instrument that affects what outcome actually occurs.

Choice under

- certainty: highly confident about the relationship between actions and outcomes
- risk: clear sense of possibilities and their likelihoods
- uncertainty: the relationship between actions and outcomes is so imprecise that it is not possible to assign likelihoods


## Decision Problems

 Nash Consorcets Paradox LCO Rational Choice' Theory ParetoHarsanyi ArrowSocial Choice
Rationality

## Decision Problems

| $w_{1}$ |
| :--- |$w_{2} \quad \cdots \quad w_{n-1} \quad w_{n}$.

## Decision Problems



An act is a function $A: W \rightarrow O$

## Making an omelet

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States: \{the sixth egg is good, the sixth egg is rotten\}
Consequences: $\{$ six-egg omelet, no omelet and five good eggs destroyed, six-egg omelet and a cup to wash....\}

Acts: $\{$ break egg into bowl, break egg into a cup, throw egg away $\}$

## Strict Dominance

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$$
\forall w \in W, u(A(w))>u(B(w))
$$

## Weak Dominance

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Rationality


$$
\forall w \in W, u(A(w)) \geq u(B(w)) \text { and } \exists w \in W, u(A(w))>u(B(w))
$$

## MaxMin (Security)



$$
\min (\{u(A(w)) \mid w \in W\})
$$

## MaxMax



$$
\max (\{u(A(w)) \mid w \in W\})
$$

## Maximize (Subjective) Expected Utility



$$
\sum_{w \in W} P_{A}(w) * u(A(w))
$$

## Subjective Expected Utility


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Probability: Suppose that $W=\left\{w_{1}, \ldots, w_{n}\right\}$ is a finite set of states. A probability function on $W$ is a function $P: W \rightarrow[0,1]$ where $\sum_{w \in W} P(w)=1$ (i.e., $P\left(w_{1}\right)+P\left(w_{2}\right)+\cdots+P\left(w_{n}\right)=1$ ).

Suppose that $A$ is an act for a set of outcomes $O$ (i.e., $A: W \rightarrow O$ ). The expected utility of $A$ is:

$$
\sum_{w \in W} P(w) * u(A(w))
$$

## Cardinal Utility Theory

 Nash Condorcets Parrasox Theory ParetoHarsany
Rational Choice
ArrowSocial Choice Theory Sen

$$
u: X \rightarrow \mathbb{R}
$$

Which comparisons are meaningful?

1. $u(x)$ and $u(y)$ ? (ordinal utility)
2. $u(x)-u(y)$ and $u(a)-u(b)$ ?
3. $u(x)$ and $2 * u(z)$ ?

## Cardinal Utility Theory


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$x \succ y \succ z$ is represented by both $(3,2,1)$ and $(1000,999,1)$, so we cannot say $y$ whether is "closer" to $x$ than to $z$.

## Cardinal Utility Theory

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$x \succ y \succ z$ is represented by both $(3,2,1)$ and $(1000,999,1)$, so we cannot say $y$ whether is "closer" to $x$ than to $z$.

Key idea: Ordinal preferences over lotteries allows us to infer a cardinal scale (with some additional axioms).

John von Neumann and Oskar Morgenstern. The Theory of Games and Economic Behavior. Princeton University Press, 1944.

|  | Heads | Tails |
| :---: | :---: | :---: |
| $L_{1}$ | $\$ 1 \mathrm{M}$ | $\$ 1 \mathrm{M}$ |
| $L_{2}$ | $\$ 2 \mathrm{M}$ | $\$ 0$ |

Which of the two lotteries would you choose?

- $L_{1}\left(L_{1} \succ L_{2}\right)$
- $L_{2}\left(L_{2} \succ L_{1}\right)$
- I am indifferent between the two lotteries ( $L_{1} \sim L_{2}$ )



## Lotteries


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Suppose that $X=\left\{x_{1}, \ldots, x_{n}\right\}$ is a set of outcomes. A lottery over $X$ is a tuple $\left[p_{1}: x_{1}, \ldots, p_{n}: x_{n}\right]$ where $\sum_{i} p_{i}=1$.

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Let $\mathcal{L}$ be the set of lotteries. Suppose that $\succeq \subseteq \mathcal{L} \times \mathcal{L}$ is a preference ordering on $\mathcal{L}$.

## Axioms of Cardinal Utility

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Suppose that $X$ is a set of outcomes and consider lotteries over $X$ (i.e., probability distributions over $X$ )

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A compound lottery is $\alpha L+(1-\alpha) L^{\prime}$ meaning "play lottery $L$ with probability $\alpha$ and $L^{\prime}$ with probability $1-\alpha^{\prime \prime}$

## Axioms of Cardinal Utility

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A compound lottery is $\alpha L+(1-\alpha) L^{\prime}$ meaning "play lottery $L$ with probability $\alpha$ and $L^{\prime}$ with probability $1-\alpha^{\prime \prime}$

Running example: Suppose Ann prefers pizza ( $p$ ) over taco ( $t$ ) over yogurt ( $y$ ) ( $p \succ t \succ y$ ) and consider the different lotteries where the prizes are $p, t$ and $y$.

## Continuity

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Continuity: for all options $a, b$ and $c$ if $a \succeq b \succeq c$, there is some lottery $L$ with probability $p$ of getting $a$ and $(1-p)$ of getting $c$ such that the agent is indifferent between $L$ and $b$.

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$$
p \succ t \succ y
$$

## Suppose Ann has $t$.

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Consider the lottery $L=0.99$ get $y$ and 0.01 get $p$

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Consider the lottery $L^{\prime}=0.99$ get $p$ and 0.01 get $y$ Would Ann trade $t$ for $L^{\prime}$ ?

Continuity says that there is must be some lottery where Ann is indifferent between keeping $t$ and playing the lottery.

## Better Prizes

Better Prizes: suppose $L_{1}$ is a lottery over $(w, x)$ and $L_{2}$ is over $(y, z)$ suppose that $L_{1}$ and $L_{2}$ have the same probability over prizes. The lotteries each have an equal prize in one position they have unequal prizes in the other position then if $L_{1}$ is the lottery with the better prize then $L_{1} \succ L_{2}$; if neither lottery has a better prize then $L_{1} \approx L_{2}$.

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$$
p \succ t \succ y
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Lottery $1\left(L_{1}\right)$ is 0.6 chance for $p$ and 0.4 chance for $y$
Lottery $2\left(L_{2}\right)$ is 0.6 chance for $t$ and 0.4 chance for $y$

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Lottery $2\left(L_{2}\right)$ is 0.6 chance for $t$ and 0.4 chance for $y$

Since Ann prefers $p$ to $t$, this axiom says that Ann prefers $L_{1}$ to $L_{2}$

## Better Chances

Better Chances: Suppose $L_{1}$ and $L_{2}$ are two lotteries which have the same prizes, then if $L_{1}$ offers a better chance of the better prize, then $L_{1} \succ L_{2}$

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Lottery $1\left(L_{1}\right)$ is 0.7 chance for $p$ and 0.3 chance for $y$
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This axioms states that Ann must prefer $L_{1}$ to $L_{2}$

## Reduction of Compound Lotteries

Reduction of Compound Lotteries: If the prize of a lottery is another lottery, then this can be reduced to a simple lottery over prizes.

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Rational Choice Theory ParetoHarsany Arrow Rationality

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This eliminates utility from the thrill of gambling and so the only ultimate concern is the prizes.


## Von Neumann-Morgenstern Theorem

Suppose that $\mathcal{L}$ is the set of lotteries. A function $u: \mathcal{L} \rightarrow \Re$ is linear provided for all $L=\left[L_{1}: p_{1}, \ldots, L_{n}: p_{n}\right] \in \mathcal{L}$,

$$
u(L)=\sum_{i=1}^{n} p_{i} u\left(L_{i}\right)
$$

Von Neumann-Morgenstern Theorem. A binary relation $\succeq$ on $\mathcal{L}$ is transitive, complete, and satisfies Continuity, Better Prizes, Better Chances, Reduction of Compound Lotteries iff $\succeq$ is representable by a linear utility function $u: \mathcal{L} \rightarrow \Re$.

Moreover, $u^{\prime}: \mathcal{L} \rightarrow \Re$ represents $\succeq$ iff there exists real numbers $c>0$ and $d$ such that $u^{\prime}(\cdot)=c u(\cdot)+d$. (" $u$ is unique up to linear transformations.")

## Cardinal Utility Theory



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Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agent's ordinal utility function can be turned into cardinal utility function.

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- Issue with continuity: 1EUR $\succ 1$ cent $\succ$ death, but who would accept a lottery which is $p$ for 1EUR and $(1-p)$ for death??


## Cardinal Utility Theory

Von Neumann-Morgenstern Theorem. If an agent satisfies the previous axioms, then the agent's ordinal utility function can be turned into cardinal utility function.

- Utility is unique only up to linear transformations. So, it still does not make sense to add two different agents cardinal utility functions.
- Issue with continuity: 1EUR $\succ 1$ cent $\succ$ death, but who would accept a lottery which is $p$ for 1EUR and $(1-p)$ for death??
- Important issues about how to identify correct descriptions of the outcomes and options.


## Issue with Better Prizes

Suppose you have a kitten, which you plan to give away to either Ann or Bob. Ann and Bob both want the kitten very much. Both are deserving, and both would care for the kitten. You are sure that giving the kitten to $\operatorname{Ann}(x)$ is at least as good as giving the kitten to Bob ( $y$ ) (so $x \succeq y$ ). But you think that would be unfair to Bob. You decide to flip a fair coin: if the coin lands heads, you will give the kitten to Bob, and if it lands tails, you will give the kitten to Ann.
(J. Drier, "Morality and Decision Theory" in [HR])


- $x$ is the outcome "Ann gets the kitten"
- $y$ is the outcome "Bob gets the kitten"

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- $x$ is the outcome "Ann gets the kitten, in a fair way"
- $y$ is the outcome "Bob gets the kitten"

- $x$ is the outcome "Ann gets the kitten"
- $z$ is the outcome "Ann gets the outcome, fairly
- $y$ is the outcome "Bob gets the kitten, fairly"


If all the agent cares about is who gets the kitten, then $L_{1} \succeq L_{2}$
If all the agent cares about is being fair, then $L_{1} \preceq L_{2}$

## Comments on Expected Utility

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| Options | $1 / 2$ | $1 / 2$ |
| :---: | :---: | :---: |
| $L_{1}$ | $1 M$ | $1 M$ |
| $L_{2}$ | $3 M$ | $0 M$ |

## Comments on Expected Utility


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| :---: | :---: | :---: |
| $L_{1}$ | $1 M$ | $1 M$ |
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$$
\begin{aligned}
& \operatorname{EVM}\left(L_{1}\right)=1 / 2 \cdot 1+1 / 2 \cdot 1=1 \\
& \operatorname{EVM}\left(L_{1}\right)=1 / 2 \cdot 3+1 / 2 \cdot 0=1.5
\end{aligned}
$$

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What numbers should we use in place of monetary value?

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\end{aligned}
$$

What numbers should we use in place of monetary value? (moral) value? personal utility?




Risk neutral


Risk neutral Risk seeking


Risk neutral Risk seeking Risk averse

## Why maximize expected utility?

Law of Large Numbers: everyone who maximizes expected utility will almost certainly be better off in the long run. By performing a random experiment sufficiently many times, the probability that the average outcome differs from the expected outcome can be rendered arbitrarily small.

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## Allais Paradox

|  | Options | Red (1) | White (89) | Blue (10) |
| :---: | :---: | :---: | :---: | :---: |
| $S_{1}$ | $A$ | $1 M$ | $1 M$ | $1 M$ |
|  | $B$ | 0 | $1 M$ | $5 M$ |

## Allais Paradox

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Rationality

Options Red (1) White (89) Blue (10)

| $S_{2}$ | $C$ | $1 M$ | 0 | $1 M$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $D$ | 0 | 0 | $5 M$ |

## Allais Paradox

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## Allais Paradox



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|  | Options | Red (1) | White (89) | Blue (10) |
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| $S_{1}$ | $A$ | $1 M$ | $1 M$ | $1 M$ |
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| $S_{2}$ | $C$ | $1 M$ | 0 | $1 M$ |
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Options Red (1) White (89) Blue (10)

| $S_{1}$ | $A$ | $1 M$ | $1 M$ | $1 M$ |
| :---: | :---: | :---: | :---: | :---: |
|  | $B$ | 0 | $1 M$ | $5 M$ |
| $S_{2}$ | $C$ | $1 M$ | 0 | $1 M$ |
|  | $D$ | 0 | 0 | $5 M$ |

$$
A \succeq B \text { iff } C \succeq B
$$

## Allais Paradox

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(a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or

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We should not conclude either
(a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or
(b) those who choose $A$ in $S_{1}$ and $D$ is $S_{2}$ are irrational.

## Allais Paradox

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We should not conclude either
(a) The axioms of cardinal utility fail to adequately capture our understanding of rational choice, or
(b) those who choose $A$ in $S_{1}$ and $D$ is $S_{2}$ are irrational.

Rather, people's utility functions (their rankings over outcomes) are often far more complicated than the monetary bets would indicate....

## Ellsberg Paradox


 Arrowsocial Choice

|  | 30 |  | 60 |  |
| :---: | :---: | :---: | :---: | :---: |
| Lotteries | Blue | Yellow | Green |  |
| $L_{1}$ | $1 M$ | 0 | 0 |  |
| $L_{2}$ | 0 | $1 M$ | 0 |  |
| $L_{3}$ | $1 M$ | 0 | $1 M$ |  |
| $L_{4}$ | 0 | $1 M$ | $1 M$ |  |

## Ellsberg Paradox



## Ellsberg Paradox


 Arrowsocial Choice

|  | 30 |  |  | 60 |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Lotteries | Blue | Yellow | Green |  |  |
| $L_{1}$ | $1 M$ | 0 | 0 |  |  |
| $L_{2}$ | 0 |  | $1 M$ | 0 |  |
| $L_{3}$ | $1 M$ | 0 | $1 M$ |  |  |
| $L_{4}$ | 0 | $1 M$ | $1 M$ |  |  |

$$
L_{1} \succeq L_{2} \text { iff } L_{3} \succeq L_{4}
$$

D. Kahneman and A. Tversky. Prospect Theory: An Analysis of Decision under Risk. Econometrica, Vol. 47, No. 2., pgs. . 263-292, 1979.
N. Barberis. Thirty Years of Prospect Theory in Economics: A Review and Assessment. Journal of Economic Perspectives, 27:1, pgs. 171-196, 2013.

## Prospect Theory


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Consider a gamble

$$
\left(x_{-m} ; p_{-m} ; x_{-m+1} ; p_{-m+1} ; \ldots ; x_{0} ; p_{0} ; \ldots ; x_{n-1}, p_{n-1} ; x_{n}, p_{n}\right)
$$

where $x_{i}<x_{j}$ for $i<j$ and $x_{0}=0$
Expected Utility

$$
\sum_{i=-m}^{n} p_{i} U\left(W+x_{i}\right)
$$

where $W$ is current wealth and $U(\cdot)$ is an increasing and concave utility function.

## Prospect Theory

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Consider a gamble

$$
\left(x_{-m} ; p_{-m} ; x_{-m+1} ; p_{-m+1} ; \ldots ; x_{0} ; p_{0} ; \ldots ; x_{n-1}, p_{n-1} ; x_{n}, p_{n}\right)
$$

where $x_{i}<x_{j}$ for $i<j$ and $x_{0}=0$
Cumulative Prospect Theory

$$
\sum_{i=-m}^{n} \pi_{i} v\left(x_{i}\right)
$$

where $v(\cdot)$ is the "value function" is an increasing function with $v(0)=0$ and $\pi_{i}$ are "decision weights".
reference dependence: people derive utility from gains and loses, measured relative to some reference point, rather than from absolute levels of wealth.
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loss aversion: people are much more sensitive to losses-even small losses-than to gains of the same magnitude. Many people turn down a gamble ( $-\$ 100: \frac{1}{2}, \$ 110: \frac{1}{2}$ ), but this is very hard to explain in classical utility theory (Rabin, 2000)
diminishing sensitivity: people tend to be risk averse over moderate probability gains (they typically prefer a certain gain of $\$ 500$ to a 50 precent chance of $\$ 1,000$ ) and risk seeking over losses (they prefer a 50 precent chance of loosing $\$ 1000$ to loosing $\$ 500$ for sure)
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probability weighting: people tend to overweight the tails of a probability distribution (they tend to overweight extremely unlikely outcomes).

## Framing Matters

UMD plays Ohio State next year. Suppose that (miraculously) UMD wins the game. There are two headlines that could run in the Diamondback:

1. "The Terps Won!"
2. "The Buckeyes Lost!"

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1. "The Terps Won!"
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Do the two headlines have the same meaning?

## Framing Matters

UMD plays Ohio State next year. Suppose that (miraculously) UMD wins the game. There are two headlines that could run in the Diamondback:

1. "The Terps Won!"
2. "The Buckeyes Lost!"

Do the two headlines have the same meaning?
"The fact that logically equivalent statements evoke different reactions makes it impossible for Humans to be as reliably rational as Econs." (Kahneman, pg. 363)

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Would you accept a gamble that offers a $10 \%$ chance to win $\$ 95$ and a $90 \%$ chance to loose $\$ 5$ ?

Would you pay $\$ 5$ to participate in a lottery that offers a $10 \%$ chance to win $\$ 100$ and a $90 \%$ chance to win nothing?

Would you accept a gamble that offers a $10 \%$ chance to win $\$ 95$ and a $90 \%$ chance to loose $\$ 5$ ?

Would you pay $\$ 5$ to participate in a lottery that offers a $10 \%$ chance to win $\$ 100$ and a $90 \%$ chance to win nothing?

"A bad outcome is much more acceptable if it is framed as the cost of a lottery ticket that did not win than if it is imply described as losing a gamble. We should not be surprised: losses evokes stronger negative feelings than costs. " (Kahneman, pg. 364).

Suppose you are given $\$ 50$.

Situation 1: Choose one of the following:

1. You keep $\$ 20$.
2. There is a $40 \%$ chance that you keep $\$ 50$, otherwise you keep nothing.

Situation 2: Choose one of the following:

1. You loose $\$ 30$.
2. There is a $40 \%$ chance that you keep $\$ 50$, otherwise you keep nothing.


Logicophilia, a virulent virus, threatens 600 students at the University of Maryland
[Adapted from Tversky and Kahneman (1981)]

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1. You must choose between two prevention programs, resulting in:

A: 200 participants will be saved for sure.
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2. You must choose between two prevention programs, resulting in: A': 400 will not be saved, for sure.
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- Standard decision theory is extensional
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Also true of many formalisms of beliefs:

- "Believing" $A$ and $\vdash A \leftrightarrow B$ implies "Believing" $B$.
"The different choices in the two frames fit prospect theory, in which choices between gambles and sure things are resolved differently, depending on whether the outcomes are good or bad. Decision makers tend to prefer the sure thing over the gamble (they are risk averse) when the outcomes are good. They tend to reject the sure thing and accept the gamble (the are risk seeking) when both outcomes are negative. "
(Kahneman, pg. 368)


## Schelling's Example

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Pat Arrow Rationality

Suppose your tax depends on your income and how many kids you have.

- The "child deduction" might be, say, 1000 per child:

$$
\operatorname{Tax}(i, k)=\operatorname{Base}(i)-[\max (k, 3) \cdot 1000]
$$

Q1: Should the child deduction be larger for the rich than for the poor?

## Schelling's Example


 Arrow Racia Choice

Instead of taking the "standard" household to be childless, we could lower the base tax for everyone (e.g., by 3000), and add a surcharge for households with less than 3 kids (e.g., 1000/2000/3000).

We could also let the surcharge depend on income.

$$
\operatorname{Tax}(i, k)=\operatorname{LowerBase}(i)+[(3-k) \cdot \operatorname{Surcharge}(i)]
$$

Q2: Should the childless poor pay as large a surcharge as the childless rich?

## Schelling's Example

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## Schelling's Example

Q1: Should the child exemption be larger for the rich than for the poor?
Q2: Should the childless poor pay as large a surcharge as the childless rich?
If you answered "No" to both, then you are not endorsing a coherent policy
As Kahneman puts the point...
"The difference between the tax owed by a childless family and by a family with two children can be described as a reduction or as an increase. If you want the poor to receive at least the same benefit as the rich for having children, then you must want the poor to pay at least the same penalty as the rich for being childless. "

## ...And One More

 Marys Theorem Geus Nash Consorcets Paradot ECO ParetoHarsanyi ArrowSocial Choice TheorySen $\underset{\text { Arows theorem }}{\substack{\text { Rationality }}}$Adam and Beth drive equal distances in a year.
Adam switches from a $12-\mathrm{mpg}$ to $14-\mathrm{mpg}$ car. Beth switches from a $30-\mathrm{mpg}$ to $40-\mathrm{mpg}$ car.

Who will save more gas?

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Who will save more gas?

Adam: $\quad \frac{10,000}{12}=833 \quad \frac{10,000}{14}=714 \quad$ saving of 119 gallons
Beth: $\frac{10,000}{30}=333 \quad \frac{10,000}{40}=250$ saving of 83 gallons
"The message about the nature of framing is stark: framing should not be viewed as an intervention that masks or distorts an underlying preference. At least in this instance...there is no underlying preference that is masked or distorted by the frame. Our preferences are about framed problems, and our moral intuitions are about descriptions, not substance."

