

Introduction to Logic

PHIL 170

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Announcements

- ▶ **Final Exam:** Wed., Dec 16, 8:00am - 10:00am, LEF 2205
- ▶ See the review sheet with sample problems for the final exam (available on the course website).
- ▶ The **best** way to study for the final exam is to work on the sample problems. Answers will be made available on Friday afternoon.
- ▶ Extra office hours: Monday, Dec. 14 (I'll be in my office most of the day).

Probability/Inductive Logic

Arguments

I need to be at UMD by 11am.

$\therefore$ Lily needs to be at the bus-stop by 9am.

Arguments



I need to be at UMD by 11am.

$\therefore$ Lily needs to be at the bus-stop by 9am.

Ann brought her laptop to first three lectures.

$\therefore$ Ann will bring her laptop to today's lecture.

Arguments



I need to be at UMD by 11am.

∴ Lily needs to be at the bus-stop by 9am.

Ann brought her laptop to first three lectures.

∴ Ann will bring her laptop to today's lecture.

Ann will have salad or steak.

Ann will not have steak.

∴ Ann will have salad.

Arguments



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Every student in PHIL170 will get an A.

Ann is a student in PHIL170.

∴ Ann will get an A.

Arguments



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Arguments

$$\begin{array}{c} \text{X} \quad \frac{U}{\therefore L} \end{array}$$

$$\begin{array}{c} \text{X} \quad \frac{((L_1 \ \& \ L_2) \ \& \ L_3)}{\therefore L_4} \end{array}$$

$$\begin{array}{c} \checkmark \quad \frac{A \vee S}{\neg S} \\ \hline \therefore A \end{array}$$

$$\begin{array}{c} \checkmark \quad \frac{(\forall x)(S(x) \rightarrow A(x))}{S(a)} \\ \hline \therefore A(a) \end{array}$$

Arguments

$$\text{X} \quad \frac{U}{\therefore L}$$

$$\text{☺} \quad \frac{((L_1 \ \& \ L_2) \ \& \ L_3)}{\therefore L_4}$$

$$\text{✓} \quad \frac{A \vee S \quad \neg S}{\therefore A}$$

$$\text{✓} \quad \frac{(\forall x)(S(x) \rightarrow A(x)) \quad S(a)}{\therefore A(a)}$$

Probabilistic Truth-Tables

A	B	$\neg A$	$\neg B$	$A \vee B$	$A \& B$	$A \rightarrow B$	$A \vee \neg A$	...
T	T	F	F	T	T	T	T	...
T	F	F	T	T	F	F	T	...
F	T	T	F	T	F	T	T	...
F	F	T	T	F	F	T	T	...

Probabilistic Truth-Tables

	<i>A</i>	<i>B</i>	$\neg A$	$\neg B$	$A \vee B$	$A \& B$	$A \rightarrow B$	$A \vee \neg A$	...
p_1	T	T	F	F	T	T	T	T	...
p_2	T	F	F	T	T	F	F	T	...
p_3	F	T	T	F	T	F	T	T	...
p_4	F	F	T	T	F	F	T	T	...

where, $p_1 + p_2 + p_3 + p_4 = 1$

Probabilistic Truth-Tables

	<i>A</i>	<i>B</i>	$\neg A$	$\neg B$	$A \vee B$	$A \& B$	$A \rightarrow B$	$A \vee \neg A$	...
p_1	T	T	F	F	T	T	T	T	...
p_2	T	F	F	T	T	F	F	T	...
p_3	F	T	T	F	T	F	T	T	...
p_4	F	F	T	T	F	F	T	T	...

$$Pr(\varphi) = \sum \{p_i \mid i \text{ is a row that makes } \varphi \text{ true}\}$$

Probabilistic Truth-Tables

- ▶ $Pr(\neg A) = 1 - Pr(A)$
- ▶ $Pr(A \vee B) = Pr(A) + Pr(B) - Pr(A \& B)$
- ▶ If $A \rightarrow B$ is true, then $Pr(A) \leq Pr(B)$

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \& B$	$A \rightarrow B$	$A \vee \neg A$	...
$\frac{1}{10}$	T	T	F	F	T	T	T	T	...
$\frac{1}{20}$	T	F	F	T	T	F	F	T	...
$\frac{2}{5}$	F	T	T	F	T	F	T	T	...
$\frac{9}{20}$	F	F	T	T	F	F	T	T	...

$Pr(\varphi \mid \psi)$ is the probability of φ *give that ψ is true*

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \& B$	$A \rightarrow B$	$A \vee \neg A$	...
$\frac{1}{10}$	T	T	F	F	T	T	T	T	...
$\frac{1}{20}$	T	F	F	T	T	F	F	T	...
$\frac{2}{5}$	F	T	T	F	T	F	T	T	...
$\frac{9}{20}$	F	F	T	T	F	F	T	T	...

$Pr(B \mid A)$ is the probability of B give that A is true

Probabilistic Truth-Tables

	<i>A</i>	<i>B</i>	$\neg A$	$\neg B$	$A \vee B$	$A \& B$	$A \rightarrow B$	$A \vee \neg A$	...
$\frac{1}{10}$	T	T	F	F	T	T	T	T	...
$\frac{1}{20}$	T	F	F	T	T	F	F	T	...
$\frac{2}{5}$	F	T	T	F	T	F	T	T	...
$\frac{9}{20}$	F	F	T	T	F	F	T	T	...

$$\frac{\frac{1}{10}}{\frac{1}{10} + \frac{1}{20}} = \frac{\frac{2}{20}}{\frac{3}{20}} = \frac{2}{3}$$

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Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \& B$	$A \rightarrow B$	$A \vee \neg A$	...
$\frac{2}{3}$	T	T	F	F	T	T	T	T	...
$\frac{1}{3}$	T	F	F	T	T	F	F	T	...
$\frac{2}{5}$	F	T	T	F	T	F	T	T	...
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$$Pr(\textcolor{red}{B} \mid A) = \frac{2}{3}$$

Probabilistic Truth-Tables

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$\frac{1}{20}$	T	F	F	T	T	F	F	T	...
$\frac{2}{5}$	F	T	T	F	T	F	T	T	...
$\frac{9}{20}$	F	F	T	T	F	F	T	T	...

$$Pr(A) = \frac{3}{20}$$

$$Pr(A \mid A \vee B) = \frac{\frac{1}{10} + \frac{1}{20}}{\frac{1}{10} + \frac{2}{20} + \frac{2}{5}} = \frac{\frac{3}{20}}{\frac{11}{20}} = \frac{3}{11}$$

$$Pr(A \mid A \vee \neg A) = \frac{3}{20}$$

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \& B$	$A \rightarrow B$	$A \vee \neg A$	...
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$$Pr(A \mid A \vee \neg A) = \frac{3}{20}$$

Arguments

Argument 1

$$\frac{A \vee B}{A}$$

Argument 2

$$\frac{A \vee \neg A}{A}$$

Arguments

Argument 1

$$\frac{A \vee B}{A}$$

Argument 2

$$\frac{A \vee \neg A}{A}$$

- ▶ Argument 1 and Argument 2 are not *valid*.
- ▶ Intuitively, Argument 1 is *stronger* than Argument 2:
 $Pr(A \mid A \vee B) > Pr(A)$, but $Pr(A \mid A \vee \neg A) = Pr(A)$

$$\frac{P}{\therefore C}$$

When is an argument inductively strong?

1. C is **probable given** P : $Pr(C \mid P)$ is “high” (i.e., $Pr(C \mid P) > \frac{1}{2}$)
2. P is **positively relevant** to C : $Pr(C \mid P) > Pr(P)$
3. (The argument is not valid)

Differences between 1 & 2

A (deductively) valid argument: $E \rightarrow (P \ \& \ Q) \models E \rightarrow P$

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If E is a strong argument for $P \& Q$, then E is a strong argument for P .

If $Pr(P \& Q \mid E) > \frac{1}{2}$, then $Pr(P \mid E) > \frac{1}{2}$. In fact,

$$Pr(P \mid E) \geq Pr(P \& Q \mid E)$$

However, E may be positively relevant for $P \& Q$ without being positively relevant for P :

$Pr(P \& Q \mid E) > Pr(P \& Q)$ does not necessarily imply that $Pr(P \mid E) > Pr(P)$.

P	Q	E	$P \& Q$	$E \rightarrow (P \& Q)$	$E \rightarrow P$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

$$E \rightarrow (P \& Q) \models E \rightarrow P$$

	P	Q	E	$P \& Q$	$E \rightarrow (P \& Q)$	$E \rightarrow P$
p_1	T	T	T	T	T	T
p_2	T	T	F	T	T	T
p_3	T	F	T	F	F	T
p_4	T	F	F	F	T	T
p_5	F	T	T	F	F	F
p_6	F	T	F	F	T	T
p_7	F	F	T	F	F	F
p_8	F	F	F	F	T	T

$$Pr(P \mid E) \geq Pr(P \& Q \mid E)$$

	P	Q	E	$P \& Q$	$E \rightarrow (P \& Q)$	$E \rightarrow P$
$\frac{1}{52}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{1}{52}$	T	F	T	F	F	T
$\frac{2}{52}$	T	F	F	F	T	T
$\frac{12}{52}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{12}{52}$	F	F	T	F	F	F
$\frac{24}{52}$	F	F	F	F	T	T

$$Pr(P \& Q \mid E) = \frac{1}{26} > Pr(P \& Q) = \frac{1}{52}, \text{ but } Pr(P \mid E) = Pr(P) = \frac{2}{26}$$

Differences between 1 & 2

A (deductively) valid argument: $E \rightarrow (P \& Q) \models E \rightarrow P$

If E is a strong argument for $P \& Q$, then E is a strong argument for P .

If $Pr(P \& Q \mid E) > \frac{1}{2}$, then $Pr(P \mid E) > \frac{1}{2}$. In fact,

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Conjunction Fallacy

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

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Which is more probable?

1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

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Which is more probable?

1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

Typically a large percentage of people asked say 2 is more probable than 1.

A. Tversky and D. Kahneman. *Extensions versus intuitive reasoning: The conjunction fallacy in probability judgment*. Psychological Review 90 (4): 293 - 315, 1983.

Conjunction Fallacy

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Which is more probable?

1. Linda is a bank teller. P
2. Linda is a bank teller and is active in the feminist movement. $P \& Q$

$$Pr(P \mid E) \geq Pr(P \& Q \mid E)$$

But, E is positively relevant for $P \& Q$ (and less so than to P)

Non-Classical Logic

The set of parameters characterizing a logic can be divided in three subsets:

1. Choice of formal language
2. Choice of a semantics for the formal language
3. Choice of a definition of valid arguments in the language

Classical Logic “Parameters”

1. *Syntax*: if φ, ψ are sentences, then so are $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, and $\varphi \rightarrow \psi$
2. *Semantics* (truth-functionality): the truth-value of a sentence is a function of the truth-values of its components only
3. *Semantics* (bivalence): sentences are either true or false, with nothing in-between
4. *Consequence*: $\alpha_1 \dots \alpha_n / \beta$ is valid iff β is true in all models of $\alpha_1, \dots, \alpha_n$

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Domains to which classical logic is applicable must satisfy these four assumptions.

Truth-functionality without bivalence: “unknown”

Many-valued logic

E.g., “Is $2^{1257787} - 1$ prime?”

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P	$\neg P$
T	F
F	T
U	U

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P	Q	$P \& Q$
T	T	T
T	F	F
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T	T	T
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F	T	F
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U	F	F
U	T	U
F	U	F
T	U	U
U	U	U

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F
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U	T	T
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T	F
F	T
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U	T	U
F	U	F
T	U	U
U	U	U

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T	T	T
T	F	T
F	T	T
F	F	F
U	F	U
U	T	T
F	U	U
T	U	T
U	U	U

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T
U	F	U
U	T	T
F	U	T
T	U	U
U	U	U

Non-Truth-Functional Semantics

Intuitionistic logic

1. $\varphi \wedge \psi$ means “I have a proof of both φ and ψ ”
2. $\varphi \vee \psi$ means “I have a proof of φ or a proof of ψ ”
3. $\varphi \rightarrow \psi$ means “I have a construction that transforms a proof of φ into a proof of ψ ”
4. $\neg\varphi$ means “Any proof of φ leads to a contradiction”

Clearly, $\varphi \vee \neg\varphi$ is not valid.

Introducing Modal Logic

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Prosecutor: $G \rightarrow A$

Defense: $\neg(G \rightarrow A)$

Judge: $\neg(G \rightarrow A)$

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Defense: $\neg(G \rightarrow A)$

Judge: $\neg(G \rightarrow A) \Leftrightarrow G \wedge \neg A$, therefore G !

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Prosecutor: “If Eric is guilty then he had an accomplice.”

Defense: “I disagree!”

Judge: “I agree with the defense.”

Prosecutor: $\Box(G \rightarrow A)$ (It **must** be the case that ...)

Defense: $\neg\Box(G \rightarrow A)$

Judge: $\neg\Box(G \rightarrow A)$ (*What can the Judge conclude?*)

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Judge: “I agree with the defense.”

Prosecutor: $\Box(G \rightarrow A)$ (It **must** be the case that ...)

Defense: $\neg\Box(G \rightarrow A)$

Judge: $\neg\Box(G \rightarrow A)$ (*What can the Judge conclude?*)

Introducing Modal Logic

Gradually, the study of the modalities themselves became dominant, with the study of “conditionals” developing into a separate topic.

What is a modal?

A modality is any word or phrase that can be applied to a statement S to create a new statement that makes an assertion that *qualifies* the truth of S .

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John _____ happy.

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- ▶ is obliged to be

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- ▶ will be
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- ▶ ...

Modal Languages

Modal languages extend some logical language (e.g., propositional logic or first-order logic) with (at least) two new symbols ' $\Box$ ' and ' $\Diamond$ '.

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$\Box\varphi$: “it is *necessary* that φ is true”

$\Diamond\psi$: “it is *possible* that ψ is true”

Modal Languages

Modal languages extend some logical language (e.g., propositional logic or first-order logic) with (at least) two new symbols ' $\Box$ ' and ' $\Diamond$ '.

$\Box\varphi$: “it is *knowing* that φ is true”

$\Diamond\psi$: “it is *consistent with everything that is known* that φ is true”

Modal Languages

Modal languages extend some logical language (e.g., propositional logic or first-order logic) with (at least) two new symbols ' $\Box$ ' and ' $\Diamond$ '.

$\Box\varphi$: “it is *will always be* that φ is true”

$\Diamond\psi$: “it is *will sometimes be* that ψ is true”

Modal Languages

Modal languages extend some logical language (e.g., propositional logic or first-order logic) with (at least) two new symbols ' $\Box$ ' and ' $\Diamond$ '.

$\Box\varphi$: “it is *ought to be* that φ is true”

$\Diamond\psi$: “it is *permissible* that ψ is true”

Modal Languages

Modal languages extend some logical language (e.g., propositional logic or first-order logic) with (at least) two new symbols ' $\Box$ ' and ' $\Diamond$ '.

$\Box\varphi$: “it is _____ that φ is true”

$\Diamond\psi$: “it is _____ that ψ is true”

E.g., $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$, $\Box P \rightarrow \Box\Box P$, $\neg\Box P \rightarrow \Box\neg\Box P$,
 $(\exists x)\Box L(x)$ and $\Box(\exists x)L(x)$.

Types of Modal Logics

tense: henceforth, eventually, hitherto, previously, now, tomorrow, yesterday, since, until, inevitably, finally, ultimately, endlessly, it will have been, it is being,...

Types of Modal Logics

tense: henceforth, eventually, hitherto, previously, now, tomorrow, yesterday, since, until, inevitably, finally, ultimately, endlessly, it will have been, it is being,...

epistemic: it is known to *a* that, it is common knowledge that

Types of Modal Logics

tense: henceforth, eventually, hitherto, previously, now, tomorrow, yesterday, since, until, inevitably, finally, ultimately, endlessly, it will have been, it is being,...

epistemic: it is known to *a* that, it is common knowledge that

doxastic: it is believed that

Types of Modal Logics

tense: henceforth, eventually, hitherto, previously, now, tomorrow, yesterday, since, until, inevitably, finally, ultimately, endlessly, it will have been, it is being,...

epistemic: it is known to *a* that, it is common knowledge that

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geometric: it is locally the case that

metalogic: it is valid/satisfiable/provable/consistent that

Self-Reference

The Liar

This sentence is false.

Truth Predicate

' S ' is true if, and only if, S

► $T(S) \leftrightarrow S$

► $F(S) \leftrightarrow \neg S$

S If sentence S is true, then Santa Claus exists.

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1.	Sentence S is true.	Assumption
2.	If sentence S is true, then Santa Claus exists.	' S is true' $\leftrightarrow S$
3.	Santa Claus exists.	$\rightarrow E: 1, 2$
4.	If sentence S is true, then Santa Claus exists.	$\rightarrow I: 3$
5.	Sentence S is true.	' S is true' $\leftrightarrow S$

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Since deductions are *sound*, the above deduction shows that 'sentence S is true' is true.

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Since deductions are *sound*, the above deduction shows that ' S is true' is true.

By Modus Ponens, Santa Claus exists!

Logic is Fun!

- ▶ Meta-theory: PHIL370 Intermediate Logic (Staff), PHIL470 Incompleteness and Undecidability (Pacuit)
- ▶ Probability/Inductive Logic: PHIL408? Bayesian Epistemology (Lyon), PHIL308?/408? Philosophy, Politics, Economics/Game and Decision Theory (Pacuit)
- ▶ Non-Classical Logic: PHIL478? Philosophical Logic (Horty, Pacuit)
- ▶ Self-Reference/Philosophy of Logic: PHIL308T A Philosopher's Toolkit (Rey), PHIL470 Incompleteness and Undecidability (Pacuit)

Deduction for Predicate Logic

Conjunction Introduction (&I)

$p1.$ φ

$p2.$ ψ

$\vdots$

$c.$ $(\varphi \ \& \ \psi)$ $\&I: p1, p2$

Conjunction Elimination (&EL, &ER)

$p1.$	$(\varphi \ \& \ \psi)$	
	$\vdots$	
c.	φ	$\&EL : p1$

$p1.$	$(\varphi \ \& \ \psi)$	
	$\vdots$	
c.	ψ	$\&ER : p1$

Conditional Introduction ($\rightarrow$ I)

a1.	φ	Assumption
	$\vdots$	
p1.	ψ	Goal
c.	$(\varphi \rightarrow \psi)$	$\rightarrow$ I: p1

p1.	ψ
	$\vdots$
c.	$(\varphi \rightarrow \psi) \rightarrow$ I: p1

Conditional Elimination ($\rightarrow$ E)

$p1.$ φ

$p2.$ $(\varphi \rightarrow \psi)$

$\vdots$

$c.$ ψ $\rightarrow E: p1, p2$

Disjunction Introduction ($\vee$ IL, $\vee$ IR)

$p1.$	φ
$\vdots$	
c.	$(\varphi \vee \psi) \quad \vee\text{IR}: p1$

$p1.$	φ
$\vdots$	
c.	$(\psi \vee \varphi) \quad \vee\text{IL}: p1$

Disjunction Elimination ($\vee E$)

$p1.$	$(\varphi \vee \psi)$	Premise
$a1.$	φ	Assumption
	$\vdots$	
$p2.$	ρ	Goal
$a2.$	ψ	Assumption
	$\vdots$	
$p3.$	ρ	Goal
$c.$	ρ	$\vee E: p1, p2, p3$

Negation Introduction/Elimination ($\neg I$, $\neg E$)

a1.	φ	Assumption
	$\vdots$	
p1.	$\perp$	Goal
c.	$\neg\varphi$	$\neg I : p1$

a1.	$\neg\varphi$	Assumption
	$\vdots$	
p1.	$\perp$	Goal
c.	φ	$\neg E : p1$

Falsum Introduction/Elimination ($\perp I, \perp E$)

$p1.$	φ
$p2.$	$\neg\varphi$
	$\vdots$
c.	$\perp \quad \perp I: p1, p2$

$p1.$	$\perp$
	$\vdots$
c.	$\varphi \quad \perp E: p1$

Biconditional Introduction ($\leftrightarrow$ I)

a1.	φ	Assumption
	$\vdots$	
p1.	ψ	Goal
a2.	ψ	Assumption
	$\vdots$	
p2.	φ	Goal
c.	$(\varphi \leftrightarrow \psi)$	$\leftrightarrow$ I: p1, p2

p1.	φ
p2.	ψ
	$\vdots$
c.	$(\varphi \leftrightarrow \psi) \quad \leftrightarrow$ I: p1, p2

Biconditional Elimination ($\leftrightarrow$ E)

$p1.$	$(\varphi \leftrightarrow \psi)$	
$p2.$	φ	
	$\vdots$	
$c.$	ψ	$\leftrightarrow E : p1, p2$

$p1.$	$(\varphi \leftrightarrow \psi)$	
$p2.$	ψ	
	$\vdots$	
$c.$	φ	$\leftrightarrow E : p1, p2$

Universal Elimination/Introduction ($\forall E$, $\forall I$)

$p1. (\forall x)\varphi$

$\vdots$

c. $\varphi[\tau/x] \quad \forall E: p1$

$p1. \varphi[v/u]$

$\vdots$

c. $(\forall u)\varphi \quad \forall I: p1$

1. v is a variable
2. v does not occur in $(\forall u)\varphi$
3. v does not occur free in any assumption on which line $p1$ depends.

Existential Introduction/Elimination ($\exists I$, $\exists E$)

$p1.$	$\varphi[\tau/x]$
	$\vdots$
$c.$	$(\exists x)\varphi \quad \exists I: p1$

$p1.$	$(\exists u)\varphi$	
$a1.$	$\varphi[v/u]$	Assumption
	$\vdots$	
$p2.$	ψ	Goal
$c.$	ψ	$\exists E: p1, p2$

1. v is a variable,
2. v does not occur in φ
3. v does not occur in ψ
4. v does not occur free in any assumption on which line $p2$ depends (except in $a1$)

Truth-trees for predicate logic

$(\varphi \& \psi)$

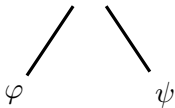
φ

ψ

$\neg(\varphi \& \psi)$



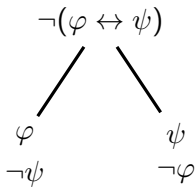
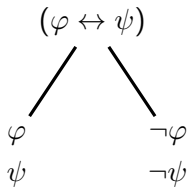
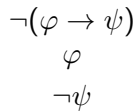
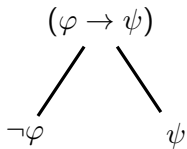
$(\varphi \vee \psi)$



$\neg(\varphi \vee \psi)$

$\neg\varphi$

$\neg\psi$



Decomposition Rules for Quantifiers

$$(\exists u)\varphi$$

$$\varphi[v/u]$$

$$(\forall u)\varphi$$

$$\varphi[t/u]$$

Provided v does not
appear on the branch

$$\neg(\exists u)\varphi$$

$$(\forall u)\neg\varphi$$

$$\neg(\forall u)\varphi$$

$$(\exists u)\neg\varphi$$

When is a branch completed?

For every decomposition rule, except the universal decomposition rule, when it is applied, check off the formula. *Universally quantified formulas are never checked off.*

admissible term: any constant or variable that has a **free occurrence** in a formula on the branch.

A truth-tree is **completed** once any formula on an open branch is either an atomic formula, the negation of an atomic formula, checked off, or a universally quantified formula that has been instantiated with every admissible term.