Introduction to Logic
PHIL 170

Eric Pacuit

University of Maryland, College Park
pacuit.org
epacuit@umd.edu

December 9, 2015

Announcements

- **Final Exam**: Wed., Dec 16, 8:00am - 10:00am, LEF 2205
- See the review sheet with sample problems for the final exam (available on the course website).
- The **best** way to study for the final exam is to work on the sample problems. Answers will be made available on Friday afternoon.
- Extra office hours: Monday, Dec. 14 (I'll be in my office most of the day).

Probability/Inductive Logic

Arguments

I need to be at UMD by 11am.

$\therefore$ Lily needs to be at the bus-stop by 9am.

Arguments

✗ I need to be at UMD by 11am.
$\therefore$ Lily needs to be at the bus-stop by 9am.

Ann brought here laptop to first three lectures.
$\therefore$ Ann will bring her laptop to today's lecture.

Arguments

✗ I need to be at UMD by 11am.

$\therefore$ Lily needs to be at the bus-stop by 9am.

Ann brought here laptop to first three lectures.

$\therefore$ Ann will bring her laptop to today's lecture.

Ann will have salad or steak.

Ann will not have steak.

$\therefore$ Ann will have salad.

Arguments

✗ I need to be at UMD by 11am.

$\therefore$ Lily needs to be at the bus-stop by 9am.

Ann brought here laptop to first three lectures.

$\therefore$ Ann will bring her laptop to today's lecture.

Ann will have salad or steak.

Ann will not have steak.

$\therefore$ Ann will have salad.

Every student in PHIL170 will get an A.

Ann is a student in PHIL170.

$\therefore$ Ann will get an A.

Arguments

✗ I need to be at UMD by 11am.
∴ Lily needs to be at the bus-stop by 9am.

✗ Ann brought here laptop to first three lectures.
∴ Ann will bring her laptop to today's lecture.

✓ Ann will have salad or steak.
Ann will not have steak.
∴ Ann will have salad.

✓ Every student in PHIL170 will get an A.
Ann is a student in PHIL170.
∴ Ann will get an A.

Arguments

✗ $\dfrac{U}{\therefore L}$

✗ $\dfrac{((L_1 \mathbin{\&} L_2) \mathbin{\&} L_3)}{\therefore L_4}$

✓ $\dfrac{\begin{array}{l} A \vee S \\ \neg S \end{array}}{\therefore A}$

✓ $\dfrac{\begin{array}{l} (\forall x)(S(x) \rightarrow A(x)) \\ S(a) \end{array}}{\therefore A(a)}$

Arguments

✗
$$\frac{U}{\therefore L}$$

☺
$$\frac{((L_1 \;\&\; L_2) \;\&\; L_3)}{\therefore L_4}$$

✓
$$\begin{array}{l} A \vee S \\ \neg S \\ \hline \therefore A \end{array}$$

✓
$$\begin{array}{l} (\forall x)(S(x) \rightarrow A(x)) \\ S(a) \\ \hline \therefore A(a) \end{array}$$

Probabilistic Truth-Tables

A	B	$\neg A$	$\neg B$	$A \vee B$	$A \,\&\, B$	$A \rightarrow B$	$A \vee \neg A$	$\cdots$
T	T	F	F	T	T	T	T	$\cdots$
T	F	F	T	T	F	F	T	$\cdots$
F	T	T	F	T	F	T	T	$\cdots$
F	F	T	T	F	T	T	T	$\cdots$

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \,\&\, B$	$A \to B$	$A \vee \neg A$	$\cdots$
p_1	T	T	F	F	T	T	T	T	$\cdots$
p_2	T	F	F	T	T	F	F	T	$\cdots$
p_3	F	T	T	F	T	F	T	T	$\cdots$
p_4	F	F	T	T	F	F	T	T	$\cdots$

where, $p_1 + p_2 + p_3 + p_4 = 1$

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \,\&\, B$	$A \to B$	$A \vee \neg A$	$\cdots$
p_1	T	T	F	F	T	T	T	T	$\cdots$
p_2	T	F	F	T	T	F	F	T	$\cdots$
p_3	F	T	T	F	T	F	T	T	$\cdots$
p_4	F	F	T	T	F	F	T	T	$\cdots$

$$Pr(\varphi) = \sum \{p_i \mid \ i \text{ is a row that makes } \varphi \text{ true}\}$$

Probabilistic Truth-Tables

- $Pr(\neg A) = 1 - Pr(A)$
- $Pr(A \vee B) = Pr(A) + Pr(B) - Pr(A \,\&\, B)$
- If $A \rightarrow B$ is true, then $Pr(A) \leq Pr(B)$

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \& B$	$A \to B$	$A \vee \neg A$	$\cdots$
$\frac{1}{10}$	T	T	F	F	T	T	T	T	$\cdots$
$\frac{1}{20}$	T	F	F	T	T	F	F	T	$\cdots$
$\frac{2}{5}$	F	T	T	F	T	F	T	T	$\cdots$
$\frac{9}{20}$	F	F	T	T	F	F	T	T	$\cdots$

$Pr(\varphi \mid \psi)$ is the probability of φ *give that* ψ *is true*

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \,\&\, B$	$A \to B$	$A \vee \neg A$	$\cdots$
$\frac{1}{10}$	T	T	F	F	T	T	T	T	$\cdots$
$\frac{1}{20}$	T	F	F	T	T	F	F	T	$\cdots$
$\frac{2}{5}$	F	T	T	F	T	F	T	T	$\cdots$
$\frac{9}{20}$	F	F	T	T	F	F	T	T	$\cdots$

$Pr(B \mid A)$ is the probability of B *give that* A *is true*

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \,\&\, B$	$A \rightarrow B$	$A \vee \neg A$	$\cdots$
$\frac{1}{10}$	T	T	F	F	T	T	T	T	$\cdots$
$\frac{1}{20}$	T	F	F	T	T	F	F	T	$\cdots$
$\frac{2}{5}$	F	T	T	F	T	F	T	T	$\cdots$
$\frac{9}{20}$	F	F	T	T	F	F	T	T	$\cdots$

$$\frac{\frac{1}{10}}{\frac{1}{10}+\frac{1}{20}} = \frac{\frac{2}{20}}{\frac{3}{20}} = \frac{2}{3} \qquad \frac{\frac{1}{20}}{\frac{1}{10}+\frac{1}{20}} = \frac{\frac{1}{20}}{\frac{3}{20}} = \frac{1}{3}$$

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \,\&\, B$	$A \to B$	$A \vee \neg A$	$\cdots$
$\frac{2}{3}$	T	T	F	F	T	T	T	T	$\cdots$
$\frac{1}{3}$	T	F	F	T	T	F	F	T	$\cdots$
$\frac{2}{5}$	F	T	T	F	T	F	T	T	$\cdots$
$\frac{9}{20}$	F	F	T	T	F	F	T	T	$\cdots$

$$Pr(B \mid A) = \frac{2}{3}$$

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \mathbin{\&} B$	$A \rightarrow B$	$A \vee \neg A$	$\cdots$
$\frac{1}{10}$	T	T	F	F	T	T	T	T	$\cdots$
$\frac{1}{20}$	T	F	F	T	T	F	F	T	$\cdots$
$\frac{2}{5}$	F	T	T	F	T	F	T	T	$\cdots$
$\frac{9}{20}$	F	F	T	T	F	F	T	T	$\cdots$

$$Pr(A) = \frac{3}{20}$$

$$Pr(A \mid A \vee B) = \frac{\frac{1}{10} + \frac{1}{20}}{\frac{1}{10} + \frac{2}{20} + \frac{2}{5}} = \frac{\frac{3}{20}}{\frac{11}{20}} = \frac{3}{11} \qquad Pr(A \mid A \vee \neg A) = \frac{3}{20}$$

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \,\&\, B$	$A \to B$	$A \vee \neg A$	$\cdots$
$\frac{1}{10}$	T	T	F	F	T	T	T	T	$\cdots$
$\frac{1}{20}$	T	F	F	T	T	F	F	T	$\cdots$
$\frac{2}{5}$	F	T	T	F	T	F	T	T	$\cdots$
$\frac{9}{20}$	F	F	T	T	F	F	T	T	$\cdots$

$$Pr(A) = \frac{3}{20}$$

$$Pr(A \mid A \vee B) = \frac{\frac{1}{10} + \frac{1}{20}}{\frac{1}{10} + \frac{2}{20} + \frac{2}{5}} = \frac{\frac{3}{20}}{\frac{11}{20}} = \frac{3}{11} \qquad Pr(A \mid A \vee \neg A) = \frac{3}{20}$$

Probabilistic Truth-Tables

	A	B	$\neg A$	$\neg B$	$A \vee B$	$A \,\&\, B$	$A \to B$	$A \vee \neg A$	$\cdots$
$\frac{1}{10}$	T	T	F	F	T	T	T	T	$\cdots$
$\frac{1}{20}$	T	F	F	T	T	F	F	T	$\cdots$
$\frac{2}{5}$	F	T	T	F	T	F	T	T	$\cdots$
$\frac{9}{20}$	F	F	T	T	F	F	T	T	$\cdots$

$$Pr(A) = \frac{3}{20}$$

$$Pr(A \mid A \vee B) = \frac{\frac{1}{10} + \frac{1}{20}}{\frac{1}{10} + \frac{2}{20} + \frac{2}{5}} = \frac{\frac{3}{20}}{\frac{11}{20}} = \frac{3}{11} \qquad Pr(A \mid A \vee \neg A) = \frac{3}{20}$$

Arguments

Argument 1	Argument 2
$\dfrac{A \vee B}{A}$	$\dfrac{A \vee \neg A}{A}$

Arguments

Argument 1	Argument 2
$\dfrac{A \vee B}{A}$	$\dfrac{A \vee \neg A}{A}$

- Argument 1 and Argument 2 are not *valid*.
- Intuitively, Argument 1 is *stronger* than Argument 2:

  $Pr(A \mid A \vee B) > Pr(A)$, but $Pr(A \mid A \vee \neg A) = Pr(A)$

$$\frac{P}{\therefore C}$$

When is an argument inductively strong?

1. C is **probable given** P: $Pr(C \mid P)$ is "high" (i.e., $Pr(C \mid P) > \frac{1}{2}$)
2. P is **positively relevant** to C: $Pr(C \mid P) > Pr(P)$
3. (The argument is not valid)

Differences between 1 & 2

A (deductively) valid argument: $E \rightarrow (P \,\&\, Q) \models E \rightarrow P$

Differences between 1 & 2

A (deductively) valid argument: $E \rightarrow (P \,\&\, Q) \models E \rightarrow P$

If E is a strong argument for $P \,\&\, Q$, then E is a strong argument for P.

If $Pr(P \,\&\, Q \mid E) > \frac{1}{2}$, then $Pr(P \mid E) > \frac{1}{2}$. In fact,

$$Pr(P \mid E) \geq Pr(P \,\&\, Q \mid E)$$

However, E may be positively relevant for $P \,\&\, Q$ without being positively relevant for P:

$Pr(P \,\&\, Q \mid E) > P(P \,\&\, Q)$ does not necessarily imply that $Pr(P \mid E) > Pr(P)$.

P	Q	E	$P \,\&\, Q$	$E \to (P \,\&\, Q)$	$E \to P$
T	T	T	T	T	T
T	T	F	T	T	T
T	F	T	F	F	T
T	F	F	F	T	T
F	T	T	F	F	F
F	T	F	F	T	T
F	F	T	F	F	F
F	F	F	F	T	T

$$E \to (P \,\&\, Q) \models E \to P$$

	P	Q	E	$P \& Q$	$E \rightarrow (P \& Q)$	$E \rightarrow P$
p_1	T	T	T	T	T	T
p_2	T	T	F	T	T	T
p_3	T	F	T	F	F	T
p_4	T	F	F	F	T	T
p_5	F	T	T	F	F	F
p_6	F	T	F	F	T	T
p_7	F	F	T	F	F	F
p_8	F	F	F	F	T	T

$$Pr(P \mid E) \geq Pr(P \& Q \mid E)$$

	P	Q	E	$P \,\&\, Q$	$E \to (P \,\&\, Q)$	$E \to P$
$\frac{1}{52}$	T	T	T	T	T	T
0	T	T	F	T	T	T
$\frac{1}{52}$	T	F	T	F	F	T
$\frac{2}{52}$	T	F	F	F	T	T
$\frac{12}{52}$	F	T	T	F	F	F
0	F	T	F	F	T	T
$\frac{12}{52}$	F	F	T	F	F	F
$\frac{24}{52}$	F	F	F	F	T	T

$$Pr(P \,\&\, Q \mid E) = \frac{1}{26} > Pr(P \,\&\, Q) = \frac{1}{52}, \text{ but } Pr(P \mid E) = Pr(P) = \frac{2}{26}$$

Differences between 1 & 2

A (deductively) valid argument: $E \to (P \,\&\, Q) \models E \to P$

If E is a strong argument for $P \,\&\, Q$, then E is a strong argument for P.

If $Pr(P \,\&\, Q \mid E) > \frac{1}{2}$, then $Pr(P \mid E) > \frac{1}{2}$. In fact,

$$Pr(P \mid E) \geq Pr(P \,\&\, Q \mid E)$$

However, E may be positively relevant for $P \,\&\, Q$ without being positively relevant for P:

$Pr(P \,\&\, Q \mid E) > P(P \,\&\, Q)$ does not necessarily imply that $Pr(P \mid E) > Pr(P)$.

Conjunction Fallacy

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Conjunction Fallacy

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

Conjunction Fallacy

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

Conjunction Fallacy

Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1. Linda is a bank teller.
2. Linda is a bank teller and is active in the feminist movement.

Typically a large percentage of people asked say 2 is more probable than 1.

A. Tversky and D. Kahneman. *Extensions versus intuitive reasoning: The conjunction fallacy in probability judgment*. Psychological Review 90 (4): 293 - 315, 1983.

Conjunction Fallacy

E Linda is 31 years old, single, outspoken, and very bright. She majored in philosophy. As a student, she was deeply concerned with issues of discrimination and social justice, and also participated in anti-nuclear demonstrations.

Which is more probable?

1. Linda is a bank teller. P
2. Linda is a bank teller and is active in the feminist movement. $P \& Q$

$Pr(P \mid E) \geq Pr(P \& Q \mid E)$

But, E is positively relevant for $P \& Q$ (and less so than to P)

Non-Classical Logic

The set of parameters characterizing a logic can be divided in three subsets:

1. Choice of formal language
2. Choice of a semantics for the formal language
3. Choice of a definition of valid arguments in the language

Classical Logic "Parameters"

1. *Syntax*: if φ, ψ are sentences, then so are $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, and $\varphi \rightarrow \psi$
2. *Semantics* (truth-functionality): the truth-value of a sentence is a function of the truth-values of its components only
3. *Semantics* (bivalence): sentences are either true or false, with nothing in-between
4. *Consequence*: $\alpha_1 \ldots \alpha_n/\beta$ is valid iff β is true in all models of $\alpha_1, \ldots, \alpha_n$

Classical Logic "Parameters"

1. *Syntax*: if φ, ψ are sentences, then so are $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, and $\varphi \rightarrow \psi$
2. *Semantics* (truth-functionality): the truth-value of a sentence is a function of the truth-values of its components only
3. *Semantics* (bivalence): sentences are either true or false, with nothing in-between
4. *Consequence*: $\alpha_1 \ldots \alpha_n/\beta$ is valid iff β is true in all models of $\alpha_1, \ldots, \alpha_n$

Domains to which classical logic is applicable must satisfy these four assumptions.

Truth-functionality without bivalence: "unknown"

Many-valued logic

E.g., "Is $2^{1257787} - 1$ prime?"

Truth-functionality without bivalence: "unknown"

Many-valued logic

E.g., "Is $2^{1257787} - 1$ prime?"

P	$\neg P$
T	F
F	T
U	U

Truth-functionality without bivalence: "unknown"

Many-valued logic

E.g., "Is $2^{1257787} - 1$ prime?"

P	$\neg P$
T	F
F	T
U	U

P	Q	$P \mathbin{\&} Q$
T	T	T
T	F	F
F	T	F
F	F	F
U	F	F
U	T	U
F	U	F
T	U	U
U	U	U

Truth-functionality without bivalence: "unknown"

Many-valued logic

E.g., "Is $2^{1257787} - 1$ prime?"

P	$\neg P$
T	F
F	T
U	U

P	Q	$P \,\&\, Q$
T	T	T
T	F	F
F	T	F
F	F	F
U	F	F
U	T	U
F	U	F
T	U	U
U	U	U

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F
U	F	U
U	T	T
F	U	U
T	U	T
U	U	U

Truth-functionality without bivalence: "unknown"

Many-valued logic

E.g., "Is $2^{1257787} - 1$ prime?"

P	$\neg P$
T	F
F	T
U	U

P	Q	$P \& Q$
T	T	T
T	F	F
F	T	F
F	F	F
U	F	F
U	T	U
F	U	F
T	U	U
U	U	U

P	Q	$P \vee Q$
T	T	T
T	F	T
F	T	T
F	F	F
U	F	U
U	T	T
F	U	U
T	U	T
U	U	U

P	Q	$P \rightarrow Q$
T	T	T
T	F	F
F	T	T
F	F	T
U	F	U
U	T	T
F	U	T
T	U	U
U	U	U

Non-Truth-Functional Semantics

Intuitionistic logic

1. $\varphi \wedge \psi$ means "I have a proof of both φ and ψ"
2. $\varphi \vee \psi$ means "I have a proof of φ or a proof of ψ"
3. $\varphi \to \psi$ means "I have a construction that transforms a proof of φ into a proof of ψ"
4. $\neg\varphi$ means "Any proof of φ leads to a contradiction"

Clearly, $\varphi \vee \neg\varphi$ is not valid.

Introducing Modal Logic

Introducing Modal Logic

Prosecutor: "If Eric is guilty then he had an accomplice."

Introducing Modal Logic

Prosecutor: "If Eric is guilty then he had an accomplice."
Defense: "I disagree!"

Introducing Modal Logic

Prosecutor: "If Eric is guilty then he had an accomplice."
Defense: "I disagree!"
Judge: "I agree with the defense."

Introducing Modal Logic

Prosecutor: "If Eric is guilty then he had an accomplice."
Defense: "I disagree!"
Judge: "I agree with the defense."

Prosecutor: $G \to A$
Defense: $\neg(G \to A)$
Judge: $\neg(G \to A)$

Introducing Modal Logic

Prosecutor: "If Eric is guilty then he had an accomplice."
Defense: "I disagree!"
Judge: "I agree with the defense."

Prosecutor: $G \rightarrow A$
Defense: $\neg(G \rightarrow A)$
Judge: $\neg(G \rightarrow A)$ $\Leftrightarrow G \wedge \neg A$, therefore G!

Introducing Modal Logic

Prosecutor: "If Eric is guilty then he had an accomplice."
Defense: "I disagree!"
Judge: "I agree with the defense."

Prosecutor: $\Box(G \to A)$ (It must be the case that ...)
Defense: $\neg\Box(G \to A)$
Judge: $\neg\Box(G \to A)$ (*What can the Judge conclude?*)

Introducing Modal Logic

Prosecutor: "If Eric is guilty then he had an accomplice."
Defense: "I disagree!"
Judge: "I agree with the defense."

Prosecutor: $\Box(G \to A)$ (It must be the case that ...)
Defense: $\neg\Box(G \to A)$
Judge: $\neg\Box(G \to A)$ (*What can the Judge conclude?*)

Introducing Modal Logic

Gradually, the study of the modalities themselves became dominant, with the study of "conditionals" developing into a separate topic.

What is a modal?

A modality is any word or phrase that can be applied to a statement S to create a new statement that makes an assertion that *qualifies* the truth of S.

What is a modal?

A modality is any word or phrase that can be applied to a statement S to create a new statement that makes an assertion that *qualifies* the truth of S.

John _______ happy.

What is a modal?

A modality is any word or phrase that can be applied to a statement S to create a new statement that makes an assertion that *qualifies* the truth of S.

John _______ happy.

- is necessarily
- is possibly

What is a modal?

A modality is any word or phrase that can be applied to a statement S to create a new statement that makes an assertion that *qualifies* the truth of S.

John _______ happy.

- is necessarily
- is possibly
- is known/believed/certain (by Ann) to be

What is a modal?

A modality is any word or phrase that can be applied to a statement S to create a new statement that makes an assertion that *qualifies* the truth of S.

John _______ happy.

- is necessarily
- is possibly
- is known/believed/certain (by Ann) to be
- is permitted to be
- is obliged to be

What is a modal?

A modality is any word or phrase that can be applied to a statement S to create a new statement that makes an assertion that *qualifies* the truth of S.

John _______ happy.

- is necessarily
- is possibly
- is known/believed/certain (by Ann) to be
- is permitted to be
- is obliged to be
- is now
- will be

What is a modal?

A modality is any word or phrase that can be applied to a statement S to create a new statement that makes an assertion that *qualifies* the truth of S.

John _______ happy.

- is necessarily
- is possibly
- is known/believed/certain (by Ann) to be
- is permitted to be
- is obliged to be
- is now
- will be
- can do something to ensure that he is

What is a modal?

A modality is any word or phrase that can be applied to a statement S to create a new statement that makes an assertion that *qualifies* the truth of S.

John _______ happy.

- is necessarily
- is possibly
- is known/believed/certain (by Ann) to be
- is permitted to be
- is obliged to be
- is now
- will be
- can do something to ensure that he is
- ...

Modal Languages

Modal languages extend some logical language (e.g., propositional logic or first-order logic) with (at least) two new symbols '$\Box$' and '$\Diamond$'.

Modal Languages

Modal languages extend some logical language (e.g., propositional logic or first-order logic) with (at least) two new symbols '$\Box$' and '$\Diamond$'.

$\Box\varphi$: "it is *necessary* that φ is true"

$\Diamond\psi$: "it is *possible* that φ is true"

Modal Languages

Modal languages extend some logical language (e.g., propositional logic or first-order logic) with (at least) two new symbols '$\Box$' and '$\Diamond$'.

$\Box\varphi$: "it is *knowing* that φ is true"

$\Diamond\psi$: "it is *consistent with everything that is known* that φ is true"

Modal Languages

Modal languages extend some logical language (e.g., propositional logic or first-order logic) with (at least) two new symbols '$\Box$' and '$\Diamond$'.

$\Box\varphi$: "it is *will always be* that φ is true"

$\Diamond\psi$: "it is *will sometimes be* that φ is true"

Modal Languages

Modal languages extend some logical language (e.g., propositional logic or first-order logic) with (at least) two new symbols '$\Box$' and '$\Diamond$'.

$\Box\varphi$: "it is *ought to be* that φ is true"

$\Diamond\psi$: "it is *permissible* that φ is true"

Modal Languages

Modal languages extend some logical language (e.g., propositional logic or first-order logic) with (at least) two new symbols '$\Box$' and '$\Diamond$'.

$\Box\varphi$: "it is ______ that φ is true"

$\Diamond\psi$: "it is ______ that φ is true"

E.g., $\Box(\varphi \rightarrow \psi) \rightarrow (\Box\varphi \rightarrow \Box\psi)$, $\Box P \rightarrow \Box\Box P$, $\neg\Box P \rightarrow \Box\neg\Box P$, $(\exists x)\Box L(x)$ and $\Box(\exists x)L(x)$.

Types of Modal Logics

tense: henceforth, eventually, hitherto, previously, now, tomorrow, yesterday, since, until, inevitably, finally, ultimately, endlessly, it will have been, it is being,...

Types of Modal Logics

tense: henceforth, eventually, hitherto, previously, now, tomorrow, yesterday, since, until, inevitably, finally, ultimately, endlessly, it will have been, it is being,...

epistemic: it is known to *a* that, it is common knowledge that

Types of Modal Logics

tense: henceforth, eventually, hitherto, previously, now, tomorrow, yesterday, since, until, inevitably, finally, ultimately, endlessly, it will have been, it is being,...

epistemic: it is known to *a* that, it is common knowledge that

doxastic: it is believed that

Types of Modal Logics

tense: henceforth, eventually, hitherto, previously, now, tomorrow, yesterday, since, until, inevitably, finally, ultimately, endlessly, it will have been, it is being,...

epistemic: it is known to a that, it is common knowledge that

doxastic: it is believed that

deontic: it is obligatory/forbidden/permitted/unlawful that

Types of Modal Logics

tense: henceforth, eventually, hitherto, previously, now, tomorrow, yesterday, since, until, inevitably, finally, ultimately, endlessly, it will have been, it is being,...

epistemic: it is known to *a* that, it is common knowledge that

doxastic: it is believed that

deontic: it is obligatory/forbidden/permitted/unlawful that

dynamic: after the program/computation/action finishes, the program enables, throughout the computation

Types of Modal Logics

tense: henceforth, eventually, hitherto, previously, now, tomorrow, yesterday, since, until, inevitably, finally, ultimately, endlessly, it will have been, it is being,...

epistemic: it is known to *a* that, it is common knowledge that

doxastic: it is believed that

deontic: it is obligatory/forbidden/permitted/unlawful that

dynamic: after the program/computation/action finishes, the program enables, throughout the computation

geometric: it is locally the case that

Types of Modal Logics

tense: henceforth, eventually, hitherto, previously, now, tomorrow, yesterday, since, until, inevitably, finally, ultimately, endlessly, it will have been, it is being,...

epistemic: it is known to *a* that, it is common knowledge that

doxastic: it is believed that

deontic: it is obligatory/forbidden/permitted/unlawful that

dynamic: after the program/computation/action finishes, the program enables, throughout the computation

geometric: it is locally the case that

metalogic: it is valid/satisfiable/provable/consistent that

Self-Reference

The Liar

This sentence is false.

Truth Predicate

'S' is true if, and only if, S

- $T(S) \leftrightarrow S$
- $F(S) \leftrightarrow \neg S$

S If sentence S is true, then Santa Claus exists.

S If sentence S is true, then Santa Claus exists.

1.	Sentence S is true.	Assumption
2.	If sentence S is true, then Santa Claus exists.	'S is true' $\leftrightarrow S$
3.	Santa Claus exists.	$\rightarrow$E : 1, 2
4.	If sentence S is true, then Santa Claus exists.	$\rightarrow$I : 3
5.	Sentence S is true.	'S is true' $\leftrightarrow S$

S If sentence S is true, then Santa Claus exists.

1.	Sentence S is true.	Assumption
2.	If sentence S is true, then Santa Claus exists.	'S is true' $\leftrightarrow S$
3.	Santa Claus exists.	$\to$E : 1, 2
4.	If sentence S is true, then Santa Claus exists.	$\to$I : 3
5.	Sentence S is true.	'S is true' $\leftrightarrow S$

Since deductions are *sound*, the above deduction shows that 'sentence S is true' is true.

S If sentence S is true, then Santa Claus exists.

1.	Sentence S is true.	Assumption
2.	If sentence S is true, then Santa Claus exists.	'S is true' $\leftrightarrow S$
3.	Santa Claus exists.	$\rightarrow$E : 1, 2
4.	If sentence S is true, then Santa Claus exists.	$\rightarrow$I : 3
5.	Sentence S is true.	'S is true' $\leftrightarrow S$

Since deductions are *sound*, the above deduction shows that 'sentence S is true' is true.

By Modus Ponens, Santa Claus exists!

Logic is Fun!

- Meta-theory: PHIL370 Intermediate Logic (Staff), PHIL470 Incompleteness and Undecidability (Pacuit)
- Probability/Inductive Logic: PHIL408? Bayesian Epistemology (Lyon), PHIL308?/408? Philosophy, Politics, Economics/Game and Decision Theory (Pacuit)
- Non-Classical Logic: PHIL478? Philosophical Logic (Horty, Pacuit)
- Self-Reference/Philosophy of Logic: PHIL308T A Philosopher's Toolkit (Rey), PHIL470 Incompleteness and Undecidability (Pacuit)

Deduction for Predicate Logic

Conjunction Introduction (&I)

$p1.$	φ	
$p2.$	ψ	
	$\vdots$	
c.	$(\varphi \mathbin{\&} \psi)$	&I: $p1$, $p2$

Conjunction Elimination (&EL, &ER)

$p1.$	$(\varphi \mathbin{\&} \psi)$	
	$\vdots$	
c.	φ	&EL : $p1$

$p1.$	$(\varphi \mathbin{\&} \psi)$	
	$\vdots$	
c.	ψ	&ER : $p1$

Conditional Introduction ($\rightarrow$ I)

*a*1.	φ	Assumption
	$\vdots$	
*p*1.	ψ	Goal
c.	$(\varphi \rightarrow \psi)$	$\rightarrow$I : *p*1

*p*1.	ψ	
	$\vdots$	
c.	$(\varphi \rightarrow \psi)$	$\rightarrow$I : *p*1

Conditional Elimination ($\rightarrow$ E)

*p*1.	φ	
*p*2.	$(\varphi \rightarrow \psi)$	
	$\vdots$	
c.	ψ	$\rightarrow$E : *p*1, *p*2

Disjunction Introduction ($\vee$IL, $\vee$IR)

$p1.$	φ	
	$\vdots$	
c.	$(\varphi \vee \psi)$	$\vee$IR : $p1$

$p1.$	φ	
	$\vdots$	
c.	$(\psi \vee \varphi)$	$\vee$IL: $p1$

Disjunction Elimination ($\vee$E)

$p1.$	$(\varphi \vee \psi)$	Premise
$a1.$	φ	Assumption
	$\vdots$	
$p2.$	ρ	Goal
$a2.$	ψ	Assumption
	$\vdots$	
$p3.$	ρ	Goal
$c.$	ρ	$\vee$E : $p1, p2, p3$

Negation Introduction/Elimination ($\neg$I, $\neg$E)

$a1.$	φ	Assumption
	$\vdots$	
$p1.$	$\bot$	Goal
c.	$\neg\varphi$	$\neg$I : $p1$

$a1.$	$\neg\varphi$	Assumption
	$\vdots$	
$p1.$	$\bot$	Goal
c.	φ	$\neg$E : $p1$

Falsum Introduction/Elimination ($\bot$I, $\bot$E)

$p1.$	φ	
$p2.$	$\neg\varphi$	
	$\vdots$	
c.	$\bot$	$\bot$I : $p1, p2$

$p1.$	$\bot$	
	$\vdots$	
c.	φ	$\bot$E : $p1$

Biconditional Introduction ($\leftrightarrow$I)

*a*1.	φ	Assumption
	$\vdots$	
*p*1.	ψ	Goal
*a*2.	ψ	Assumption
	$\vdots$	
*p*2.	φ	Goal
c.	$(\varphi \leftrightarrow \psi)$	$\leftrightarrow$I: *p*1, *p*2

*p*1.	φ	
*p*2.	ψ	
	$\vdots$	
c.	$(\varphi \leftrightarrow \psi)$	$\leftrightarrow$I: *p*1, *p*2

Biconditional Elimination ($\leftrightarrow$E)

*p*1.	$(\varphi \leftrightarrow \psi)$	
*p*2.	φ	
	$\vdots$	
c.	ψ	$\leftrightarrow$E : *p*1, *p*2

*p*1.	$(\varphi \leftrightarrow \psi)$	
*p*2.	ψ	
	$\vdots$	
c.	φ	$\leftrightarrow$E : *p*1, *p*2

Universal Elimination/Introduction ($\forall$E, $\forall$I)

$p1.$	$(\forall x)\varphi$	
	$\vdots$	
c.	$\varphi[\tau/x]$	$\forall\text{E}: p1$

$p1.$	$\varphi[v/u]$	
	$\vdots$	
c.	$(\forall u)\varphi$	$\forall\text{I}: p1$

1. v is a variable
2. v does not occur in $(\forall u)\varphi$
3. v does not occur free in any assumption on which line $p1$ depends.

Existential Introduction/Elimination ($\exists$I, $\exists$E)

$p1.$	$\varphi[\tau/x]$	
	$\vdots$	
c.	$(\exists x)\varphi$	$\exists\text{I}: p1$

$p1.$	$(\exists u)\varphi$	
$a1.$	$\varphi[v/u]$	Assumption
	$\vdots$	
$p2.$	ψ	Goal
c.	ψ	$\exists\text{E}: p1, p2$

1. v is a variable,
2. v does not occur in φ
3. v does not occur in ψ
4. v does not occur free in any assumption on which line $p2$ depends (except in $a1$)

Truth-trees for predicate logic

$(\varphi \,\&\, \psi)$

φ

ψ

$\neg(\varphi \,\&\, \psi)$

$\neg\varphi \qquad \neg\psi$

$(\varphi \vee \psi)$

$\varphi \qquad \psi$

$\neg(\varphi \vee \psi)$

$\neg\varphi$

$\neg\psi$

$$(\varphi \to \psi)$$

$\neg\varphi$ | ψ

$$\neg(\varphi \to \psi)$$
$$\varphi$$
$$\neg\psi$$

$$(\varphi \leftrightarrow \psi)$$

φ, ψ | $\neg\varphi$, $\neg\psi$

$$\neg(\varphi \leftrightarrow \psi)$$

φ, $\neg\psi$ | ψ, $\neg\varphi$

$$\neg\neg\varphi$$
$$\varphi$$

Decomposition Rules for Quantifiers

$$\frac{(\exists u)\varphi}{\varphi[v/u]}$$

Provided v does not appear on the branch

$$\frac{(\forall u)\varphi}{\varphi[t/u]}$$

$$\frac{\neg(\exists u)\varphi}{(\forall u)\neg\varphi}$$

$$\frac{\neg(\forall u)\varphi}{(\exists u)\neg\varphi}$$

When is a branch completed?

For every decomposition rule, except the universal decomposition rule, when it is applied, check off the formula. *Universally quantified formulas are never checked off*.

admissible term: any constant or variable that has a **free occurrence** in a formula on the branch.

A truth-tree is **completed** once any formula on an open branch is either an atomic formula, the negation of an atomic formula, checked off, or a universally quantified formula that has been instantiated with every admissible term.