

Introduction to Logic

PHIL 170

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Announcements

- ▶ read Chapter 11 (and Chapter 12).
- ▶ Do the practice problems!
- ▶ Schedule
 1. Wed., 12/2: Ch. 11, more deductions, course summary
 2. Mon., 12/7: Concluding remarks (general perspectives)
 3. Wed., 12/9: Concluding remarks (general perspectives)
- ▶ Lab - Truth-Trees and Lab - Derivations (12/2, 11.59pm)
- ▶ In-class quiz in sections: A deduction similar to the practice problems in Chapter 11.
- ▶ Chapter 11 Quiz & Problem Set, due 12/8, 11.59pm
- ▶ **Final Exam:** Wed., Dec 16, 8:00am - 10:00am, LEF 2205
- ▶ See the review sheet with sample problems for the final exam (available on the course website).

Basic Concepts

- ▶ **Tautology:** A formula of predicate logic is a tautology just in case it is true on every interpretation.
- ▶ **Contradictory Formula:** A formula of predicate logic is a contradictory just in case it is false on every interpretation.
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An argument of predicate logic is **quantificationally valid** just in case there is no interpretation that makes all the premises of the argument true and the conclusion false.

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Deduction for Predicate Logic

Conjunction Introduction (&I)

$p1.$ φ

$p2.$ ψ

\vdots

$c.$ $(\varphi \ \& \ \psi)$ $\&I: p1, p2$

Conjunction Elimination (&EL, &ER)

$p1.$	$(\varphi \ \& \ \psi)$	
	\vdots	
c.	φ	&EL: $p1$

$p1.$	$(\varphi \ \& \ \psi)$	
	\vdots	
c.	ψ	&ER: $p1$

Conditional Introduction (\rightarrow I)

a1.	φ	Assumption
	\vdots	
p1.	ψ	Goal
c.	$(\varphi \rightarrow \psi)$	\rightarrow I: p1

p1.	ψ
	\vdots
c.	$(\varphi \rightarrow \psi) \rightarrow$ I: p1

Conditional Elimination (\rightarrow E)

$p1.$ φ

$p2.$ $(\varphi \rightarrow \psi)$

\vdots

$c.$ ψ $\rightarrow E: p1, p2$

Disjunction Introduction (\vee IL, \vee IR)

$p1.$	φ
\vdots	
c.	$(\varphi \vee \psi) \quad \vee\text{IR}: p1$

$p1.$	φ
\vdots	
c.	$(\psi \vee \varphi) \quad \vee\text{IL}: p1$

Disjunction Elimination ($\vee E$)

$p1.$	$(\varphi \vee \psi)$	Premise
$a1.$	φ	Assumption
	\vdots	
$p2.$	ρ	Goal
$a2.$	ψ	Assumption
	\vdots	
$p3.$	ρ	Goal
$c.$	ρ	$\vee E: p1, p2, p3$

Negation Introduction/Elimination ($\neg I$, $\neg E$)

a1.	φ	Assumption
	\vdots	
p1.	\perp	Goal
c.	$\neg\varphi$	$\neg I : p1$

a1.	$\neg\varphi$	Assumption
	\vdots	
p1.	\perp	Goal
c.	φ	$\neg E : p1$

Falsum Introduction/Elimination ($\perp I, \perp E$)

$p1.$	φ
$p2.$	$\neg\varphi$
	\vdots
c.	$\perp \quad \perp I: p1, p2$

$p1.$	\perp
	\vdots
c.	$\varphi \quad \perp E: p1$

Biconditional Introduction (\leftrightarrow I)

a1.	φ	Assumption
	\vdots	
p1.	ψ	Goal
a2.	ψ	Assumption
	\vdots	
p2.	φ	Goal
c.	$(\varphi \leftrightarrow \psi)$	\leftrightarrow I: p1, p2

p1.	φ
p2.	ψ
	\vdots
c.	$(\varphi \leftrightarrow \psi) \quad \leftrightarrow$ I: p1, p2

Biconditional Elimination (\leftrightarrow E)

$p1.$	$(\varphi \leftrightarrow \psi)$	
$p2.$	φ	
	\vdots	
$c.$	ψ	$\leftrightarrow E : p1, p2$

$p1.$	$(\varphi \leftrightarrow \psi)$	
$p2.$	ψ	
	\vdots	
$c.$	φ	$\leftrightarrow E : p1, p2$

Universal Elimination/Introduction ($\forall E$, $\forall I$)

$p1. \quad (\forall x)\varphi$

\vdots

c. $\varphi[\tau/x] \quad \forall E: p1$

$p1. \quad \varphi[v/u]$

\vdots

c. $(\forall u)\varphi \quad \forall I: p1$

1. v is a variable
2. v does not occur in $(\forall u)\varphi$
3. v does not occur free in any assumption on which line $p1$ depends.

Existential Introduction/Elimination ($\exists I$, $\exists E$)

$p1.$	$\varphi[\tau/x]$
	\vdots
$c.$	$(\exists x)\varphi \quad \exists I: p1$

$p1.$	$(\exists u)\varphi$	
$a1.$	$\varphi[v/u]$	Assumption
	\vdots	
$p2.$	ψ	Goal
$c.$	ψ	$\exists E: p1, p2$

1. v is a variable,
2. v does not occur in φ
3. v does not occur in ψ
4. v does not occur free in any assumption on which line $p2$ depends (except in $a1$)

Example 1

⋮

$n. (\forall x)P(x) \vee (\exists x)\neg P(x)$ Goal

Example 2

1.	$(\exists x)F(x) \rightarrow (\forall x)F(x)$	Premise
	\vdots	
$n.$	$(\forall x)(\forall y)(F(x) \leftrightarrow F(y))$	Goal

Example 3

1. $((\exists w)V(w) \ \& \ (\exists w)C(w)) \vee ((\forall w)\neg V(w) \ \& \ (\forall w)\neg C(w))$ Premise

\vdots

$n.$ $((\exists w)V(w) \leftrightarrow (\exists w)C(w))$ Goal

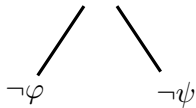
Truth-trees for predicate logic

$(\varphi \& \psi)$

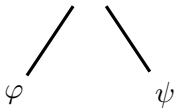
φ

ψ

$\neg(\varphi \& \psi)$



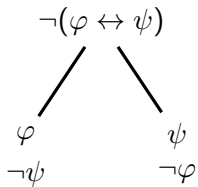
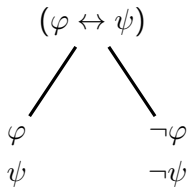
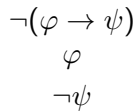
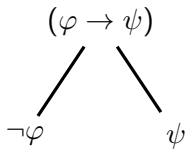
$(\varphi \vee \psi)$



$\neg(\varphi \vee \psi)$

$\neg\varphi$

$\neg\psi$



Decomposition Rules for Quantifiers

$$(\exists u)\varphi$$

$$\varphi[v/u]$$

$$(\forall u)\varphi$$

$$\varphi[t/u]$$

Provided v does not
appear on the branch

$$\neg(\exists u)\varphi$$

$$(\forall u)\neg\varphi$$

$$\neg(\forall u)\varphi$$

$$(\exists u)\neg\varphi$$

When is a branch completed?

For every decomposition rule, except the universal decomposition rule, when it is applied, check off the formula. *Universally quantified formulas are never checked off.*

admissible term: any constant or variable that has a **free occurrence** in a formula on the branch.

A truth-tree is **completed** once any formula on an open branch is either an atomic formula, the negation of an atomic formula, checked off, or a universally quantified formula that has been instantiated with every admissible term.