Introduction to Logic

PHIL 170

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Announcements

- read Chapter 11 (and Chapter 12).
- Do the practice problems!
- Schedule
  1. Wed., 12/2: Ch. 11, more deductions, course summary
  2. Mon., 12/7: Concluding remarks (general perspectives)
  3. Wed., 12/9: Concluding remarks (general perspectives)
- Lab - Truth-Trees and Lab - Derivations (12/2, 11.59pm)
- In-class quiz in sections: A deduction similar to the practice problems in Chapter 11.
- Chapter 11 Quize & Problem Set, due 12/8, 11.59pm
- **Final Exam**: Wed., Dec 16, 8:00am - 10:00am, LEF 2205
- See the review sheet with sample problems for the final exam (available on the course website).

Basic Concepts

- **Tautology**: A formula of predicate logic is a tautology just in case it is true on every interpretation.

- **Contradictory Formula**: A formula of predicate logic is a contradictory just in case it is false on every interpretation.

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An argument of predicate logic is **quantificationally valid** just in case there is no interpretation that makes all the premises of the argument true and the conclusion false.

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Deduction for Predicate Logic

Conjunction Introduction (&I)

*p*1.	φ	
*p*2.	ψ	
	$\vdots$	
c.	$(\varphi \,\&\, \psi)$	&I: *p*1, *p*2

Conjunction Elimination (&EL, &ER)

$p1.$	$(\varphi \,\&\, \psi)$	
	$\vdots$	
c.	φ	&EL : $p1$

$p1.$	$(\varphi \,\&\, \psi)$	
	$\vdots$	
c.	ψ	&ER : $p1$

Conditional Introduction $(\to I)$

$a1.$	φ	Assumption
	$\vdots$	
$p1.$	ψ	Goal
c.	$(\varphi \to \psi)$	$\to$I : $p1$

$p1.$	ψ	
	$\vdots$	
c.	$(\varphi \to \psi)$	$\to$I : $p1$

Conditional Elimination ($\rightarrow$ E)

$p1.$	φ	
$p2.$	$(\varphi \rightarrow \psi)$	
	$\vdots$	
c.	ψ	$\rightarrow$E : $p1$, $p2$

Disjunction Introduction ($\vee$IL, $\vee$IR)

$p1.$	φ	
	$\vdots$	
c.	$(\varphi \vee \psi)$	$\vee$IR : $p1$

$p1.$	φ	
	$\vdots$	
c.	$(\psi \vee \varphi)$	$\vee$IL: $p1$

Disjunction Elimination ($\vee$E)

$p1.$	$(\varphi \vee \psi)$	Premise
$a1.$	φ	Assumption
	$\vdots$	
$p2.$	ρ	Goal
$a2.$	ψ	Assumption
	$\vdots$	
$p3.$	ρ	Goal
$c.$	ρ	$\vee$E: $p1, p2, p3$

Negation Introduction/Elimination ($\neg$I, $\neg$E)

$a1.$	$\boxed{\varphi}$	Assumption
	$\vdots$	
$p1.$	$\bot$	Goal
c.	$\neg\varphi$	$\neg$I : $p1$

$a1.$	$\neg\varphi$	Assumption
	$\vdots$	
$p1.$	$\bot$	Goal
c.	φ	$\neg$E : $p1$

Falsum Introduction/Elimination ($\perp$I, $\perp$E)

$p1.$	φ	
$p2.$	$\neg\varphi$	
	$\vdots$	
c.	$\perp$	$\perp$I : $p1$, $p2$

$p1.$	$\perp$	
	$\vdots$	
c.	φ	$\perp$E : $p1$

Biconditional Introduction ($\leftrightarrow$I)

$a1.$	φ	Assumption
	$\vdots$	
$p1.$	ψ	Goal
$a2.$	ψ	Assumption
	$\vdots$	
$p2.$	φ	Goal
$c.$	$(\varphi \leftrightarrow \psi)$	$\leftrightarrow$I: $p1, p2$

$p1.$	φ	
$p2.$	ψ	
	$\vdots$	
c.	$(\varphi \leftrightarrow \psi)$	$\leftrightarrow$I: $p1, p2$

Biconditional Elimination ($\leftrightarrow$E)

$p1.$	$(\varphi \leftrightarrow \psi)$	
$p2.$	φ	
	$\vdots$	
c.	ψ	$\leftrightarrow$E : $p1$, $p2$

$p1.$	$(\varphi \leftrightarrow \psi)$	
$p2.$	ψ	
	$\vdots$	
c.	φ	$\leftrightarrow$E : $p1$, $p2$

Universal Elimination/Introduction ($\forall$E, $\forall$I)

$p1.$	$(\forall x)\varphi$	
	$\vdots$	
c.	$\varphi[\tau/x]$	$\forall\mathrm{E}: p1$

$p1.$	$\varphi[v/u]$	
	$\vdots$	
c.	$(\forall u)\varphi$	$\forall\mathrm{I}: p1$

1. v is a variable
2. v does not occur in $(\forall u)\varphi$
3. v does not occur free in any assumption on which line $p1$ depends.

Existential Introduction/Elimination (∃I, ∃E)

$p1.$	$\varphi[\tau/x]$	
	$\vdots$	
c.	$(\exists x)\varphi$	$\exists$I : $p1$

$p1.$	$(\exists u)\varphi$	
$a1.$	$\varphi[v/u]$	Assumption
	$\vdots$	
$p2.$	ψ	Goal
c.	ψ	$\exists$E : $p1, p2$

1. v is a variable,
2. v does not occur in φ
3. v does not occur in ψ
4. v does not occur free in any assumption on which line $p2$ depends (except in $a1$)

Example 1

	$\vdots$	
$n.$	$(\forall x)P(x) \vee (\exists x)\neg P(x)$	Goal

Example 2

1.	$(\exists x)F(x) \rightarrow (\forall x)F(x)$	Premise
	$\vdots$	
n.	$(\forall x)(\forall y)(F(x) \leftrightarrow F(y))$	Goal

Example 3

1.	$((\exists w)V(w)\ \&\ (\exists w)C(w)) \vee ((\forall w)\neg V(w)\ \&\ (\forall w)\neg C(w))$	Premise
	$\vdots$	
n.	$((\exists w)V(w) \leftrightarrow (\exists w)C(w))$	Goal

Truth-trees for predicate logic

$$(\varphi \,\&\, \psi)$$
$$\varphi$$
$$\psi$$

$$\neg(\varphi \,\&\, \psi)$$
$$\neg\varphi \qquad\qquad \neg\psi$$

$$(\varphi \vee \psi)$$
$$\varphi \qquad\qquad \psi$$

$$\neg(\varphi \vee \psi)$$
$$\neg\varphi$$
$$\neg\psi$$

$(\varphi \to \psi)$

$\neg\varphi \qquad \psi$

$\neg(\varphi \to \psi)$

φ

$\neg\psi$

$(\varphi \leftrightarrow \psi)$

$\varphi \qquad \neg\varphi$

$\psi \qquad \neg\psi$

$\neg(\varphi \leftrightarrow \psi)$

$\varphi \qquad \psi$

$\neg\psi \qquad \neg\varphi$

$\neg\neg\varphi$

φ

Decomposition Rules for Quantifiers

$$\begin{array}{c}(\exists u)\varphi \\ \varphi[v/u]\end{array}$$

$$\begin{array}{c}(\forall u)\varphi \\ \varphi[t/u]\end{array}$$

Provided v does not appear on the branch

$$\begin{array}{c}\neg(\exists u)\varphi \\ (\forall u)\neg\varphi\end{array}$$

$$\begin{array}{c}\neg(\forall u)\varphi \\ (\exists u)\neg\varphi\end{array}$$

When is a branch completed?

For every decomposition rule, except the universal decomposition rule, when it is applied, check off the formula. *Universally quantified formulas are never checked off*.

admissible term: any constant or variable that has a **free occurrence** in a formula on the branch.

A truth-tree is **completed** once any formula on an open branch is either an atomic formula, the negation of an atomic formula, checked off, or a universally quantified formula that has been instantiated with every admissible term.