Introduction to Logic

PHIL 170

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November 16, 2015

Announcements

- read Chapter 10.
- Do the practice problems!
- Solutions for the translations available on the website.
- Lab is due on **Wednesday, Nov. 18 at 11.59pm**.

- **Tautology**: A formula of predicate logic is a tautology just in case it is true on every interpretation.
- **Contradictory Formula**: A formula of predicate logic is a contradictory just in case it is false on every interpretation.
- **Contingent Formula**: A formula of predicate logic is contingent just in case it is true on some interpretations, and false on others.

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Are the following formulas a tautology, contradictory or contingent?

1. $(\forall x)P(x) \vee \neg(\forall x)P(x)$
2. $(\forall x)(P(x) \vee \neg P(x))$
3. $(\forall x)P(x) \vee (\forall x)\neg P(x)$
4. $(P(a) \ \& \ \neg(\exists x)P(x))$
5. $(P(a) \ \& \ (\forall x)\neg P(x))$
6. $(P(a) \ \& \ (\exists x)\neg P(x))$
7. $(\exists x)(P(x) \ \& \ \neg(\exists y)P(y))$
8. $\neg(\exists x)(R(x,x) \ \& \ \neg(\exists y)R(x,y))$

Are the following formulas a tautology, contradictory or contingent?

1. $(\forall x)P(x) \vee \neg(\forall x)P(x)$: Tautology
2. $(\forall x)(P(x) \vee \neg P(x))$: Tautology
3. $(\forall x)P(x) \vee (\forall x)\neg P(x)$: Contingent
4. $(P(a) \,\&\, \neg(\exists x)P(x))$: Contradictory
5. $(P(a) \,\&\, (\forall x)\neg P(x))$: Contradictory
6. $(P(a) \,\&\, (\exists x)\neg P(x))$: Contingent
7. $(\exists x)(P(x) \,\&\, \neg(\exists y)P(y))$: Contradictory
8. $\neg(\exists x)(R(x,x) \,\&\, \neg(\exists y)R(x,y))$: Tautology

An argument of predicate logic is **quantificationally valid** just in case there is no interpretation that makes all the premises of the argument true and the conclusion false.

Which arguments are quantificationally valid?

1.
$$\frac{\begin{array}{l}(\forall x)(H(x) \rightarrow M(x)) \\ H(s)\end{array}}{\therefore M(s)}$$

2.
$$\frac{\begin{array}{l}(\forall x)(H(x) \rightarrow M(x)) \\ \neg H(a)\end{array}}{\therefore \neg M(a)}$$

3.
$$\frac{\begin{array}{l}(\exists x)(S(x) \,\&\, C(x)) \\ (\exists x)(S(x) \,\&\, D(x))\end{array}}{\therefore (\exists x)(S(x) \,\&\, (D(x) \,\&\, C(x)))}$$

4.
$$\frac{\begin{array}{l}(\forall x)(P(x) \rightarrow Q(x)) \\ (\forall x)(Q(x) \rightarrow R(x))\end{array}}{\therefore (\forall x)(P(x) \rightarrow R(x))}$$

Truth-trees for predicate logic

$$\begin{array}{c} (\varphi \,\&\, \psi) \\ \varphi \\ \psi \end{array}$$

$$\begin{array}{ccc} & \neg(\varphi \,\&\, \psi) & \\ \swarrow & & \searrow \\ \neg\varphi & & \neg\psi \end{array}$$

$$\begin{array}{ccc} & (\varphi \vee \psi) & \\ \swarrow & & \searrow \\ \varphi & & \psi \end{array}$$

$$\begin{array}{c} \neg(\varphi \vee \psi) \\ \neg\varphi \\ \neg\psi \end{array}$$

$$
\begin{array}{c}
(\varphi \rightarrow \psi) \\
\diagup \qquad \diagdown \\
\neg\varphi \qquad\quad \psi
\end{array}
\qquad\qquad
\begin{array}{c}
\neg(\varphi \rightarrow \psi) \\
\varphi \\
\neg\psi
\end{array}
$$

$$
\begin{array}{c}
(\varphi \leftrightarrow \psi) \\
\diagup \qquad \diagdown \\
\begin{array}{cc} \varphi & \neg\varphi \\ \psi & \neg\psi \end{array}
\end{array}
\qquad\qquad
\begin{array}{c}
\neg(\varphi \leftrightarrow \psi) \\
\diagup \qquad \diagdown \\
\begin{array}{cc} \varphi & \psi \\ \neg\psi & \neg\varphi \end{array}
\end{array}
$$

$$
\begin{array}{c}
\neg\neg\varphi \\
\varphi
\end{array}
$$

Decomposition Rules for Quantifiers

$(\exists u)\varphi$

$(\forall u)\varphi$

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$(\exists u)\varphi$

$\varphi[v/u]$

Provided v does not appear on the branch

$(\forall u)\varphi$

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$$(\exists u)\varphi$$

$$\varphi[v/u]$$

Provided v does not appear on the branch

$$(\forall u)\varphi \quad t$$

$$\varphi[t/u]$$

Decomposition Rules for Quantifiers, continued

$$\begin{array}{ccc} \neg(\exists u)\varphi & \qquad & \neg(\forall u)\varphi \\ (\forall u)\neg\varphi & \qquad & (\exists u)\neg\varphi \end{array}$$

When is a branch completed?

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$$P(x, y)$$

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$P(\boxed{x}, \boxed{y})$

Free variables: x, y, Bound variables: none

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$\boxed{(\forall x)}P(\boxed{x}, \boxed{y})$

Free variables: y, Bound variables: x

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$$(\forall x)(R(x) \to (\exists y)P(x, y))$$

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$$\boxed{(\forall x)}\ (\ R(\boxed{x}) \rightarrow \boxed{(\exists y)}\, P(\boxed{x}, \boxed{y})\ )$$

Free variables: none, Bound variables: x, y

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$$(\forall x)Q(z)$$

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$(\forall x)Q(\boxed{z})$

Free variables: z, Bound variables: none

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$$(\forall x)P(x) \mathbin{\&} Q(x)$$

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$\boxed{(\forall x)}\, P(\boxed{x})\ \&\ Q(\boxed{x})$

Free variables: second x, Bound variables: first x

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admissible term: any constant or variable that has a **free occurrence** in a formula on the branch.

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A truth-tree is **completed** once any formula on an open branch is either an atomic formula, the negation of an atomic formula, checked off, or a universally quantified formula that has been instantiated with every admissible term.

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Important Point

If the formula we are considering is in fact valid, then our procedure **will** eventually close the tree in a finite number of steps, confirming the validity.

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$(\forall x)(\exists y)S(x, y)$