

Introduction to Logic

PHIL 170

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Announcements

- ▶ read Chapter 10.
- ▶ Do the practice problems!
- ▶ Solutions for the translations available on the website.
- ▶ Lab is due on **Wednesday, Nov. 18 at 11.59pm.**

- ▶ **Tautology:** A formula of predicate logic is a tautology just in case it is true on every interpretation.
- ▶ **Contradictory Formula:** A formula of predicate logic is a contradictory just in case it is false on every interpretation.
- ▶ **Contingent Formula:** A formula of predicate logic is contingent just in case it is true on some interpretations, and false on others.

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Are the following formulas a tautology, contradictory or contingent?

1. $(\forall x)P(x) \vee \neg(\forall x)P(x)$
2. $(\forall x)(P(x) \vee \neg P(x))$
3. $(\forall x)P(x) \vee (\forall x)\neg P(x)$
4. $(P(a) \ \& \ \neg(\exists x)P(x))$
5. $(P(a) \ \& \ (\forall x)\neg P(x))$
6. $(P(a) \ \& \ (\exists x)\neg P(x))$
7. $(\exists x)(P(x) \ \& \ \neg(\exists y)P(y))$
8. $\neg(\exists x)(R(x, x) \ \& \ \neg(\exists y)R(x, y))$

Are the following formulas a tautology, contradictory or contingent?

1. $(\forall x)P(x) \vee \neg(\forall x)P(x)$: Tautology
2. $(\forall x)(P(x) \vee \neg P(x))$: Tautology
3. $(\forall x)P(x) \vee (\forall x)\neg P(x)$: Contingent
4. $(P(a) \ \& \ \neg(\exists x)P(x))$: Contradictory
5. $(P(a) \ \& \ (\forall x)\neg P(x))$: Contradictory
6. $(P(a) \ \& \ (\exists x)\neg P(x))$: Contingent
7. $(\exists x)(P(x) \ \& \ \neg(\exists y)P(y))$: Contradictory
8. $\neg(\exists x)(R(x, x) \ \& \ \neg(\exists y)R(x, y))$: Tautology

An argument of predicate logic is **quantificationally valid** just in case there is no interpretation that makes all the premises of the argument true and the conclusion false.

Which arguments are quantificationally valid?

$$\begin{array}{l} (\forall x)(H(x) \rightarrow M(x)) \\ 1. \quad H(s) \\ \hline \therefore M(s) \end{array}$$

$$\begin{array}{l} (\forall x)(H(x) \rightarrow M(x)) \\ 2. \quad \neg H(a) \\ \hline \therefore \neg M(a) \end{array}$$

$$\begin{array}{l} (\exists x)(S(x) \& C(x)) \\ 3. \quad (\exists x)(S(x) \& D(x)) \\ \hline \therefore (\exists x)(S(x) \& (D(x) \& C(x))) \end{array}$$

$$\begin{array}{l} (\forall x)(P(x) \rightarrow Q(x)) \\ 4. \quad (\forall x)(Q(x) \rightarrow R(x)) \\ \hline \therefore (\forall x)(P(x) \rightarrow R(x)) \end{array}$$

Truth-trees for predicate logic

$(\varphi \& \psi)$

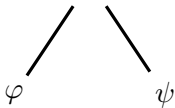
φ

ψ

$\neg(\varphi \& \psi)$



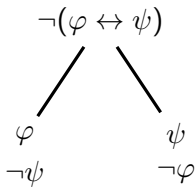
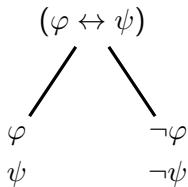
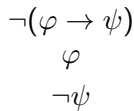
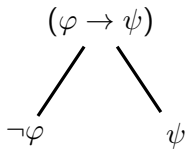
$(\varphi \vee \psi)$



$\neg(\varphi \vee \psi)$

$\neg\varphi$

$\neg\psi$



Decomposition Rules for Quantifiers

$$(\exists u)\varphi$$

$$(\forall u)\varphi$$

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$$\varphi[v/u]$$

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Provided v does not
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$$\varphi[v/u]$$

$$(\forall u)\varphi \quad t$$

$$\varphi[t/u]$$

Provided v does not
appear on the branch

Decomposition Rules for Quantifiers, continued

$$\neg(\exists u)\varphi$$

$$(\forall u)\neg\varphi$$

$$\neg(\forall u)\varphi$$

$$(\exists u)\neg\varphi$$

When is a branch completed?

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$$P(x, y)$$

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$P(x, y)$

Free variables: x, y , Bound variables: none

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$$(\forall x)P(x, y)$$

Free variables: y , Bound variables: x

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$$(\forall x)(R(x) \rightarrow (\exists y)P(x, y))$$

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$$(\forall x) (R(x) \rightarrow (\exists y) P(x, y))$$

Free variables: none, Bound variables: x, y

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$$(\forall x)Q(\textcolor{blue}{z})$$

Free variables: $\textcolor{blue}{z}$, Bound variables: none

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$$(\forall x) P(x) \& Q(x)$$

Free variables: second x , Bound variables: first x

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admissible term: any constant or variable that has a **free occurrence** in a formula on the branch.

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A truth-tree is **completed** once any formula on an open branch is either an atomic formula, the negation of an atomic formula, checked off, or a universally quantified formula that has been instantiated with every admissible term.

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Important Point

If the formula we are considering is in fact valid, then our procedure **will** eventually close the tree in a finite number of steps, confirming the validity.

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$$(\forall x)(\exists y)S(x, y)$$