Introduction to Logic

PHIL 170

Eric Pacuit

University of Maryland, College Park

pacuit.org

epacuit@umd.edu

November 11, 2015

Announcements

- read Chapter 10.
- Do the practice problems!
- Quiz is due **Friday Nov. 13 at 11.59pm**.
- Lab is due on **Monday, Nov. 16 at 11.59pm**.
- In-class quiz in Sections. Translations (see the examples on ELMS)

Quantifiers

Variables: lower case letters (u through z, possibly with subscripts) are going to be used as individual variables.

Quantifiers

Variables: lower case letters (u through z, possibly with subscripts) are going to be used as individual variables.

Quantifier symbols: $\forall$ indicates we are talking about **all** or **every** individual under consideration; $\exists$ indicates **some** or **at least one** of the individuals under consideration.

Quantifiers

Variables: lower case letters (u through z, possibly with subscripts) are going to be used as individual variables.

Quantifier symbols: $\forall$ indicates we are talking about **all** or **every** individual under consideration; $\exists$ indicates **some** or **at least one** of the individuals under consideration.

Quantifier: Combine quantifier symbol with an individual constant (and parentheses to make it easier to read). E.g., $(\forall x), (\forall y), (\exists x), (\exists y)$.

Quantifiers

Variables: lower case letters (u through z, possibly with subscripts) are going to be used as individual variables.

Quantifier symbols: $\forall$ indicates we are talking about **all** or **every** individual under consideration; $\exists$ indicates **some** or **at least one** of the individuals under consideration.

Quantifier: Combine quantifier symbol with an individual constant (and parentheses to make it easier to read). E.g., $(\forall x), (\forall y), (\exists x), (\exists y)$.

- $(\forall x)(P(x) \rightarrow Q(x))$
- $(\forall x)(P(y) \rightarrow Q(y))$
- $(\forall x)P(x) \vee \neg(\forall x)P(x)$
- $(\forall x)(\forall y)R(x, y)$
- $(\forall x)(\exists y)R(x, y)$

Domain of discourse: $\{\textbf{John}, \textbf{Mary}, \textbf{Bob}\}$

Expression	Interpretation
j	**John**
m	**Mary**
b	**Bob**
$L(x, y)$	x likes y

Mary likes John. $L(m, j)$

Bob likes himself. $L(b, b)$

Mary and John like each other: $L(m, j)$ & $L(j, m)$

Everyone likes Mary. $L(j, m)$ & $(L(m, m)$ & $L(b, m))$

Someone likes Mary. $L(j, m) \vee (L(m, m) \vee L(b, m))$

Everyone likes someone.
$(L(j, j) \vee L(j, m) \vee L(j, b))$ & $(L(m, j) \vee L(m, m) \vee L(m, b))$ &
$(L(b, j) \vee L(b, m) \vee L(b, b))$

Domain of discourse: $\{\textbf{John}, \textbf{Mary}, \textbf{Bob}\}$

Expression	Interpretation
j	**John**
m	**Mary**
b	**Bob**
$L(x, y)$	x likes y

Mary likes John. $L(m, j)$

Bob likes himself. $L(b, b)$

Mary and John like each other: $L(m, j)$ & $L(j, m)$

Everyone likes Mary. $L(j, m)$ & $(L(m, m)$ & $L(b, m))$ $(\forall x)L(x, m)$

Someone likes Mary. $L(j, m) \vee (L(m, m) \vee L(b, m))$

Everyone likes someone.
$(L(j, j) \vee L(j, m) \vee L(j, b))$ & $(L(m, j) \vee L(m, m) \vee L(m, b))$ &
$(L(b, j) \vee L(b, m) \vee L(b, b))$

Domain of discourse: $\{\textbf{John}, \textbf{Mary}, \textbf{Bob}\}$

Expression	Interpretation
j	**John**
m	**Mary**
b	**Bob**
$L(x, y)$	x likes y

Mary likes John. $L(m, j)$

Bob likes himself. $L(b, b)$

Mary and John like each other: $L(m, j)$ & $L(j, m)$

Everyone likes Mary. $L(j, m)$ & $(L(m, m)$ & $L(b, m))$ $(\forall x)L(x, m)$

Someone likes Mary. $L(j, m) \vee (L(m, m) \vee L(b, m))$ $(\exists x)L(x, m)$

Everyone likes someone.
$(L(j, j) \vee L(j, m) \vee L(j, b))$ & $(L(m, j) \vee L(m, m) \vee L(m, b))$ &
$(L(b, j) \vee L(b, m) \vee L(b, b))$

Domain of discourse: $\{\textbf{John}, \textbf{Mary}, \textbf{Bob}\}$

Expression	Interpretation
j	**John**
m	**Mary**
b	**Bob**
$L(x, y)$	x likes y

Mary likes John. $L(m, j)$

Bob likes himself. $L(b, b)$

Mary and John like each other: $L(m, j) \ \& \ L(j, m)$

Everyone likes Mary. $L(j, m) \ \& \ (L(m, m) \ \& \ L(b, m))$ $(\forall x)L(x, m)$

Someone likes Mary. $L(j, m) \vee (L(m, m) \vee L(b, m))$ $(\exists x)L(x, m)$

Everyone likes someone.
$(L(j, j) \vee L(j, m) \vee L(j, b)) \ \& \ (L(m, j) \vee L(m, m) \vee L(m, b)) \ \&$
$(L(b, j) \vee L(b, m) \vee L(b, b))$ $(\forall x)(\exists y)L(x, y)$

every **A** is **B**	$(\forall x)(A(x) \to B(x))$
some **A** is **B**	$(\exists x)(A(x)\ \&\ B(x))$
no **A** is **B**	$\neg(\exists x)(A(x)\ \&\ B(x))$
some **A** is not **B**	$(\exists x)(A(x)\ \&\ \neg B(x))$
every **A** is a non-**B**	$(\forall x)(A(x) \to \neg B(x))$
not every **A** is **B**	$\neg(\forall x)(A(x) \to B(x))$

All logicians are mathematicians.

Some logicians are mathematicians.

$L(x)$	x is a logician
$M(x)$	x is a mathematician

No logician is a mathematician.

Some logicians are not mathematicians.

Every logician is not a mathematician.

Not every logician is a mathematician.

All logicians are mathematicians.

$(\forall x)(L(x) \rightarrow M(x))$

Some logicians are mathematicians.

$(\exists x)(L(x) \,\&\, M(x))$

$L(x)$	x is a logician
$M(x)$	x is a mathematician

No logician is a mathematician.

Some logicians are not mathematicians.

Every logician is not a mathematician.

Not every logician is a mathematician.

All logicians are mathematicians.

$(\forall x)(L(x) \to M(x))$

Some logicians are mathematicians.

$(\exists x)(L(x) \& M(x))$

$L(x)$	x is a logician
$M(x)$	x is a mathematician

No logician is a mathematician.

$\neg(\exists x)(L(x) \& M(x))$

Some logicians are not mathematicians.

Every logician is not a mathematician.

Not every logician is a mathematician.

All logicians are mathematicians.

$(\forall x)(L(x) \rightarrow M(x))$

$L(x)$	x is a logician
$M(x)$	x is a mathematician

Some logicians are mathematicians.

$(\exists x)(L(x) \,\&\, M(x))$

No logician is a mathematician.

$\neg(\exists x)(L(x) \,\&\, M(x))$

Some logicians are not mathematicians.

$(\exists x)(L(x) \,\&\, \neg M(x))$

Every logician is not a mathematician.

Not every logician is a mathematician.

All logicians are mathematicians.

$(\forall x)(L(x) \rightarrow M(x))$

Some logicians are mathematicians.

$(\exists x)(L(x) \ \& \ M(x))$

No logician is a mathematician.

$\neg(\exists x)(L(x) \ \& \ M(x))$

Some logicians are not mathematicians.

$(\exists x)(L(x) \ \& \ \neg M(x))$

Every logician is not a mathematician.

$(\forall x)(L(x) \rightarrow \neg M(x))$

Not every logician is a mathematician.

$L(x)$	x is a logician
$M(x)$	x is a mathematician

All logicians are mathematicians.

$(\forall x)(L(x) \rightarrow M(x))$

Some logicians are mathematicians.

$(\exists x)(L(x) \,\&\, M(x))$

$L(x)$	x is a logician
$M(x)$	x is a mathematician

No logician is a mathematician.

$\neg(\exists x)(L(x) \,\&\, M(x))$

Some logicians are not mathematicians.

$(\exists x)(L(x) \,\&\, \neg M(x))$

Every logician is not a mathematician.

$(\forall x)(L(x) \rightarrow \neg M(x))$

Not every logician is a mathematician.

$\neg(\forall x)(L(x) \rightarrow M(x))$

Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$$\boxed{\begin{array}{l} I(a) = \mathbf{a} \\ I(b) = \mathbf{b} \\ I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\} \\ I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\} \end{array}}$$

$$(\forall x)(P(x) \;\&\; (\forall y)Q(x, y))$$

Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$$\boxed{\begin{array}{l} I(a) = \mathbf{a} \\ I(b) = \mathbf{b} \\ I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\} \\ I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\} \end{array}}$$

$$(\forall x)(P(x) \mathbin{\&} (\forall y)Q(x, y))$$

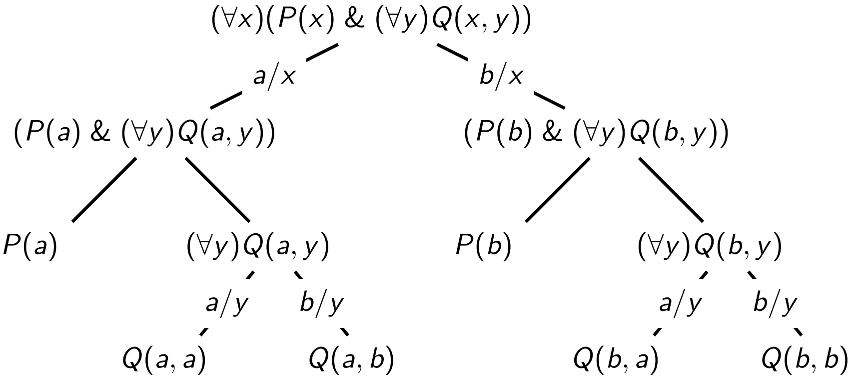
$$(P(x) \mathbin{\&} (\forall y)Q(x, y))$$

$$P(x) \qquad\qquad (\forall y)Q(x, y)$$

$$Q(x, y)$$

Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$$I(a) = \mathbf{a}$$
$$I(b) = \mathbf{b}$$
$$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$$
$$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$$



Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$I(a) = \mathbf{a}$

$I(b) = \mathbf{b}$

$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$

$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$

- $(\forall x)(P(x) \,\&\, (\forall y)Q(x, y))$
  - $(P(a) \,\&\, (\forall y)Q(a, y))$
    - $P(a)$: T
    - $(\forall y)Q(a, y)$
      - $Q(a, a)$
      - $Q(a, b)$
  - $(P(b) \,\&\, (\forall y)Q(b, y))$
    - $P(b)$
    - $(\forall y)Q(b, y)$
      - $Q(b, a)$
      - $Q(b, b)$

Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$I(a) = \mathbf{a}$

$I(b) = \mathbf{b}$

$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$

$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$

$(\forall x)(P(x) \,\&\, (\forall y)Q(x, y))$

$(P(a) \,\&\, (\forall y)Q(a, y))$ $\qquad$ $(P(b) \,\&\, (\forall y)Q(b, y))$

$P(a)$: T $\qquad$ $(\forall y)Q(a, y)$ $\qquad$ $P(b)$ $\qquad$ $(\forall y)Q(b, y)$

$Q(a, a)$: F $\qquad$ $Q(a, b)$ $\qquad$ $Q(b, a)$ $\qquad$ $Q(b, b)$

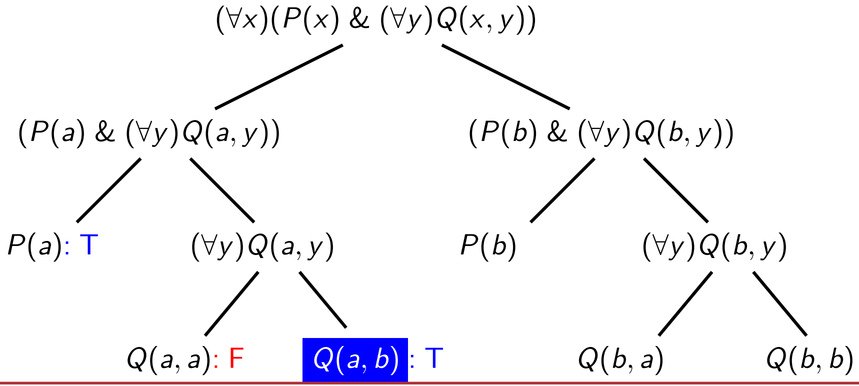
Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$I(a) = \mathbf{a}$

$I(b) = \mathbf{b}$

$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$

$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$



Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$I(a) = \mathbf{a}$

$I(b) = \mathbf{b}$

$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$

$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$

$(\forall x)(P(x) \,\&\, (\forall y)Q(x, y))$

$(P(a) \,\&\, (\forall y)Q(a, y))$ $\qquad$ $(P(b) \,\&\, (\forall y)Q(b, y))$

$P(a)$: T $\qquad$ $(\forall y)Q(a, y)$ $\qquad$ $P(b)$: T $\qquad$ $(\forall y)Q(b, y)$

$Q(a, a)$: F $\qquad$ $Q(a, b)$: T $\qquad$ $Q(b, a)$ $\qquad$ $Q(b, b)$

Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$I(a) = \mathbf{a}$

$I(b) = \mathbf{b}$

$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$

$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$

- $(\forall x)(P(x) \,\&\, (\forall y)Q(x, y))$
  - $(P(a) \,\&\, (\forall y)Q(a, y))$
    - $P(a)$: T
    - $(\forall y)Q(a, y)$
      - $Q(a, a)$: F
      - $Q(a, b)$: T
  - $(P(b) \,\&\, (\forall y)Q(b, y))$
    - $P(b)$: T
    - $(\forall y)Q(b, y)$
      - $Q(b, a)$: T
      - $Q(b, b)$

Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$I(a) = \mathbf{a}$

$I(b) = \mathbf{b}$

$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$

$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$

- $(\forall x)(P(x) \mathbin{\&} (\forall y)Q(x, y))$
  - $(P(a) \mathbin{\&} (\forall y)Q(a, y))$
    - $P(a)$: T
    - $(\forall y)Q(a, y)$
      - $Q(a, a)$: F
      - $Q(a, b)$: T
  - $(P(b) \mathbin{\&} (\forall y)Q(b, y))$
    - $P(b)$: T
    - $(\forall y)Q(b, y)$
      - $Q(b, a)$: T
      - $Q(b, b)$: T

Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$$
\boxed{
\begin{array}{l}
I(a) = \mathbf{a} \\
I(b) = \mathbf{b} \\
I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\} \\
I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}
\end{array}
}
$$

$$(\forall x)(P(x) \;\&\; (\forall y)Q(x, y))$$

$$(P(a) \;\&\; (\forall y)Q(a, y)) \qquad (P(b) \;\&\; (\forall y)Q(b, y))$$

$$P(a)\text{: T} \qquad \boxed{(\forall y)}\, Q(a, y)\text{: F} \qquad P(b)\text{: T} \qquad \boxed{(\forall y)}\, Q(b, y)\text{: T}$$

$$\boxed{Q(a, a)}\text{: F} \qquad \boxed{Q(a, b)}\text{: T} \qquad \boxed{Q(b, a)}\text{: T} \qquad \boxed{Q(b, b)}\text{: T}$$

Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$I(a) = \mathbf{a}$

$I(b) = \mathbf{b}$

$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$

$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$

$$(\forall x)(P(x) \mathbin{\&} (\forall y)Q(x, y))$$

$(P(a) \boxed{\&} (\forall y)Q(a, y))$: F

$(P(b) \boxed{\&} (\forall y)Q(b, y))$: T

$P(a)$: T

$\boxed{(\forall y)}\, Q(a, y)$: F

$P(b)$: T

$\boxed{(\forall y)}\, Q(b, y)$: T

$Q(a, a)$: F

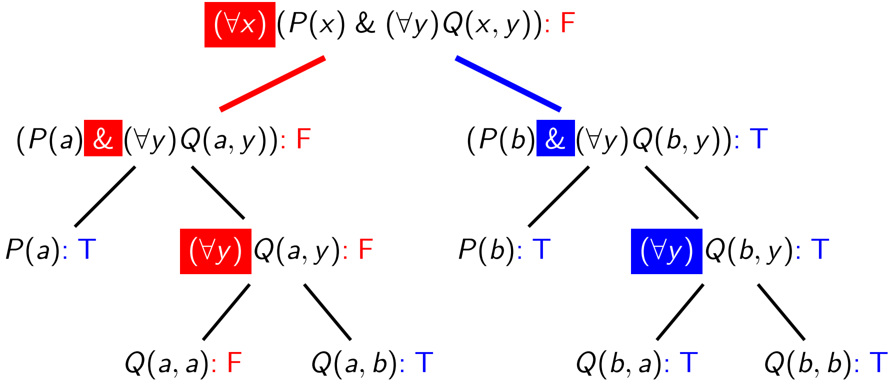
$Q(a, b)$: T

$Q(b, a)$: T

$Q(b, b)$: T

Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$$\boxed{\begin{array}{l} I(a) = \mathbf{a} \\ I(b) = \mathbf{b} \\ I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\} \\ I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\} \end{array}}$$



Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$I(a) = \mathbf{a}$

$I(b) = \mathbf{b}$

$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$

$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$

$(\exists x)(P(x) \,\&\, (\forall y)Q(x, y))$

$(P(a) \,\&\, (\forall y)Q(a, y))$: F $(P(b) \,\&\, (\forall y)Q(b, y))$: T

$P(a)$: T $(\forall y)\, Q(a, y)$: F $P(b)$: T $(\forall y)\, Q(b, y)$: T

$Q(a, a)$: F $Q(a, b)$: T $Q(b, a)$: T $Q(b, b)$: T

Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$$\boxed{\begin{array}{l} I(a) = \mathbf{a} \\ I(b) = \mathbf{b} \\ I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\} \\ I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\} \end{array}}$$

$\boxed{(\exists x)}(P(x)\ \&\ (\forall y)Q(x,y))$: T

$(P(a) \boxed{\&} (\forall y)Q(a,y))$: F

$(P(b) \boxed{\&} (\forall y)Q(b,y))$: T

$P(a)$: T

$\boxed{(\forall y)}\, Q(a,y)$: F

$P(b)$: T

$\boxed{(\forall y)}\, Q(b,y)$: T

$Q(a,a)$: F

$Q(a,b)$: T

$Q(b,a)$: T

$Q(b,b)$: T

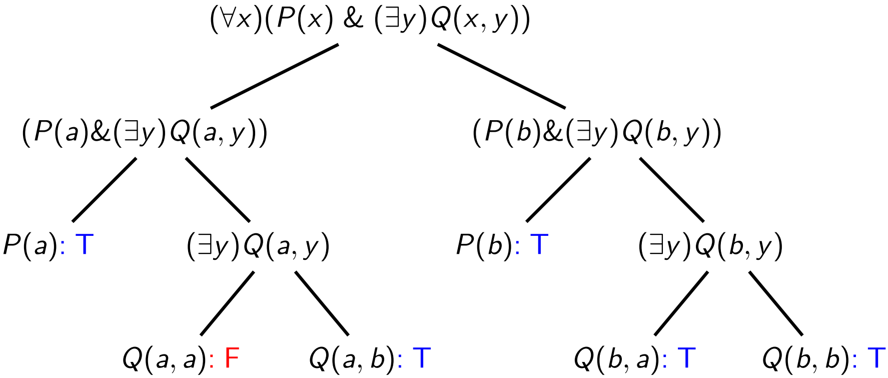
Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$I(a) = \mathbf{a}$

$I(b) = \mathbf{b}$

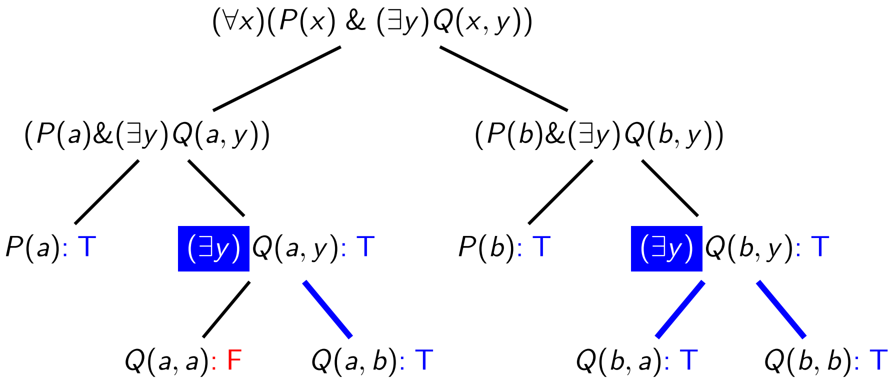
$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$

$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$



Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$$\boxed{\begin{array}{l} I(a) = \mathbf{a} \\ I(b) = \mathbf{b} \\ I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\} \\ I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\} \end{array}}$$



Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$$I(a) = \mathbf{a}$$
$$I(b) = \mathbf{b}$$
$$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$$
$$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$$

$$(\forall x)(P(x) \mathbin{\&} (\exists y)Q(x, y))$$

$$(P(a) \mathbin{\&} (\exists y)Q(a, y)): \text{T} \qquad (P(b) \mathbin{\&} (\exists y)Q(b, y)): \text{T}$$

$$P(a): \text{T} \qquad (\exists y)\, Q(a, y): \text{T} \qquad P(b): \text{T} \qquad (\exists y)\, Q(b, y): \text{T}$$

$$Q(a, a): \text{F} \qquad Q(a, b): \text{T} \qquad Q(b, a): \text{T} \qquad Q(b, b): \text{T}$$

Domain of discourse: $\{\mathbf{a}, \mathbf{b}\}$

$$I(a) = \mathbf{a}$$
$$I(b) = \mathbf{b}$$
$$I(P) = \{\langle \mathbf{a} \rangle, \langle \mathbf{b} \rangle\}$$
$$I(Q) = \{\langle \mathbf{a}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{b} \rangle, \langle \mathbf{b}, \mathbf{a} \rangle\}$$

- $\boxed{(\forall x)}(P(x) \,\&\, (\exists y)Q(x, y))$: T
  - $(P(a) \boxed{\&} (\exists y)Q(a, y))$: T
    - $P(a)$: T
    - $\boxed{(\exists y)}Q(a, y)$: T
      - $Q(a, a)$: F
      - $Q(a, b)$: T
  - $(P(b) \boxed{\&} (\exists y)Q(b, y))$: T
    - $P(b)$: T
    - $\boxed{(\exists y)}Q(b, y)$: T
      - $Q(b, a)$: T
      - $Q(b, b)$: T

Interpretations

An **interpretation** I for a domain of discourse assigns:

- an element of the domain of discourse to each individual constant;
- a truth value (i.e., T or F) to each 0-place predicate;
- a set of n-place tuples to each n-place predicate (for $n > 0$); and
- an element of the domain of discourse to each individual variable (called an **assignment** of values to variables)

Interpretations

An **interpretation** I for a domain of discourse assigns:

- an element of the domain of discourse to each individual constant;
- a truth value (i.e., T or F) to each 0-place predicate;
- a set of n-place tuples to each n-place predicate (for $n > 0$); and
- an element of the domain of discourse to each individual variable (called an **assignment** of values to variables)

If I is an interpretation, then $I[a/u]$ is the interpretation that is just like I except the variable u is assigned the element a.

Truth/Falsity

- If φ is of the form $\psi(\tau_1, \ldots, \tau_n)$ where ψ is an n-place predicate letter (with $n > 0$), and $\tau_1, \ldots, \tau_n$ are n terms, then φ is **true on** I just in case $\langle I(\tau_1), \ldots, I(\tau_n)\rangle$ is in $I(\psi)$, and false otherwise.
- If φ is of the form $(\forall u)\psi$, then φ is **true on** I just in case, for each member a of the domain of discourse, ψ is true on $I[a/u]$, and false otherwise.
- If φ is of the form $(\exists u)\psi$, then φ is **true on** I just in case, there is at least one member a of the domain of discourse such that ψ is true on $I[a/u]$, and false otherwise.

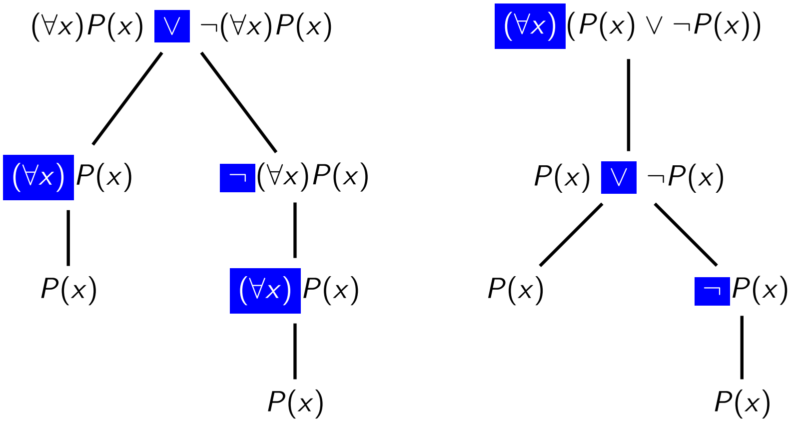
Read the example on pg. 144.

- **Tautology**: A formula of predicate logic is a tautology just in case it is true on every interpretation.
- **Contradictory Formula**: A formula of predicate logic is a contradictory just in case it is false on every interpretation.
- **Contingent Formula**: A formula of predicate logic is contingent just in case it is true on some interpretations, and false on others.

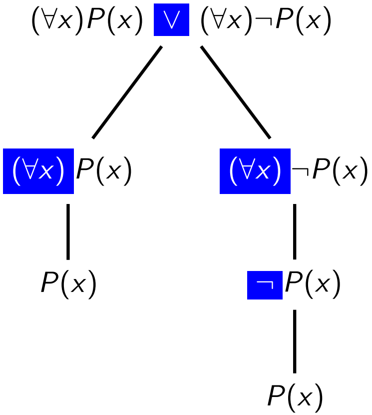
Are the following formulas a tautology, contradictory or contingent?

1. $(\forall x)P(x) \vee \neg(\forall x)P(x)$
2. $(\forall x)(P(x) \vee \neg P(x))$
3. $(\forall x)P(x) \vee (\forall x)\neg P(x)$

Tautologies



Contingent Formula



An argument of predicate logic is **quantificationally valid** just in case there is no interpretation that makes all the premises of the argument true and the conclusion false.

Which arguments are quantificationally valid?

1. $$\begin{array}{l} (\forall x)(H(x) \rightarrow M(x)) \\ H(s) \\ \hline \therefore M(s) \end{array}$$

2. $$\begin{array}{l} (\forall x)(H(x) \rightarrow M(x)) \\ \neg H(a) \\ \hline \therefore \neg M(a) \end{array}$$

3. $$\begin{array}{l} (\exists x)(S(x) \,\&\, C(x)) \\ (\exists x)(S(x) \,\&\, D(x)) \\ \hline \therefore (\exists x)(S(x) \,\&\, (D(x) \,\&\, C(x))) \end{array}$$

4. $$\begin{array}{l} (\forall x)(P(x) \rightarrow Q(x)) \\ (\forall x)(Q(x) \rightarrow R(x)) \\ \hline \therefore (\forall x)(P(x) \rightarrow R(x)) \end{array}$$

Truth-trees for predicate logic

$$\begin{array}{c} (\varphi \,\&\, \psi) \\ \varphi \\ \psi \end{array}$$

$$\begin{array}{ccc} & \neg(\varphi \,\&\, \psi) & \\ \neg\varphi & & \neg\psi \end{array}$$

$$\begin{array}{ccc} & (\varphi \vee \psi) & \\ \varphi & & \psi \end{array}$$

$$\begin{array}{c} \neg(\varphi \vee \psi) \\ \neg\varphi \\ \neg\psi \end{array}$$

$$\begin{array}{ccc} & (\varphi \to \psi) & \\ \swarrow & & \searrow \\ \neg\varphi & & \psi \end{array}$$

$$\begin{array}{c} \neg(\varphi \to \psi) \\ \varphi \\ \neg\psi \end{array}$$

$$\begin{array}{ccc} & (\varphi \leftrightarrow \psi) & \\ \swarrow & & \searrow \\ \varphi & & \neg\varphi \\ \psi & & \neg\psi \end{array}$$

$$\begin{array}{ccc} & \neg(\varphi \leftrightarrow \psi) & \\ \swarrow & & \searrow \\ \varphi & & \psi \\ \neg\psi & & \neg\varphi \end{array}$$

$$\begin{array}{c} \neg\neg\varphi \\ \varphi \end{array}$$

Decomposition Rules for Quantifiers

$(\exists u)\varphi$ $\qquad\qquad$ $(\forall u)\varphi$

Decomposition Rules for Quantifiers

$(\exists u)\varphi$

$\varphi[v/u]$

Provided v does not appear on the branch

$(\forall u)\varphi$

Decomposition Rules for Quantifiers

$(\exists u)\varphi$

$\varphi[v/u]$

Provided v does not appear on the branch

$(\forall u)\varphi$

Decomposition Rules for Quantifiers

$$\frac{(\exists u)\varphi}{\varphi[v/u]}$$

Provided v does not appear on the branch

$$\frac{(\forall u)\varphi \quad t}{\varphi[t/u]}$$

Decomposition Rules for Quantifiers, continued

$$\neg(\exists u)\varphi \qquad\qquad \neg(\forall u)\varphi$$

$$(\forall u)\neg\varphi \qquad\qquad (\exists u)\neg\varphi$$

When is a branch completed?

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$$P(x, y)$$

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$P(\boxed{x}, \boxed{y})$

Free variables: x, y, Bound variables: none

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$$(\forall x)P(x, y)$$

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$\boxed{(\forall x)}P(\boxed{x}, \boxed{y})$

Free variables: y, Bound variables: x

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$$(\forall x)(R(x) \rightarrow (\exists y)P(x, y))$$

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$$\boxed{(\forall x)}\ (\ R(\boxed{x}) \to \boxed{(\exists y)}\, P(\boxed{x}, \boxed{y})\ )$$

Free variables: none, Bound variables: x, y

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$$(\forall x)Q(z)$$

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$(\forall x)Q(\boxed{z})$

Free variables: z, Bound variables: none

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$$(\forall x)P(x) \,\&\, Q(x)$$

An occurrence of a variable u in a formula is **bound** just in case that occurrence is in the scope of a quantifier that has u as its variable of quantification. An occurrence of a variable is **free** just in case it is not bound.

$\boxed{(\forall x)}\, P(\boxed{x})\ \&\ Q(\boxed{x})$

Free variables: first x, Bound variables: second x

Universally quantified formulas are never checked off.

admissible term: any constant or variable that has a **free occurrence** in a formula on the branch.

Universally quantified formulas are never checked off.

admissible term: any constant or variable that has a **free occurrence** in a formula on the branch.

A truth-tree is **completed** once any formula on an open branch is either an atomic formula, the negation of an atomic formula, checked off, or a universally quantified formula that has been instantiated with every admissible term.

Which arguments are quantificationally valid?

1. $$\begin{array}{l} (\forall x)(H(x) \rightarrow M(x)) \\ H(s) \\ \hline \therefore M(s) \end{array}$$

2. $$\begin{array}{l} (\forall x)(H(x) \rightarrow M(x)) \\ \neg H(a) \\ \hline \therefore \neg M(a) \end{array}$$

3. $$\begin{array}{l} (\exists x)(S(x) \mathbin{\&} C(x)) \\ (\exists x)(S(x) \mathbin{\&} D(x)) \\ \hline \therefore (\exists x)(S(x) \mathbin{\&} (D(x) \mathbin{\&} C(x))) \end{array}$$

4. $$\begin{array}{l} (\forall x)(P(x) \rightarrow Q(x)) \\ (\forall x)(Q(x) \rightarrow R(x)) \\ \hline \therefore (\forall x)(P(x) \rightarrow R(x)) \end{array}$$

Important Point

If the formula we are considering is in fact valid, then our procedure **will** eventually close the tree in a finite number of steps, confirming the validity.

Important Point

If the formula we are considering is in fact valid, then our procedure **will** eventually close the tree in a finite number of steps, confirming the validity. However, if the formula is not valid, our procedure will sometimes indicate a counterexample after finitely many steps, *but it could also be the case that our procedure will never terminate, thus yielding no effective answer at all.*

Important Point

If the formula we are considering is in fact valid, then our procedure **will** eventually close the tree in a finite number of steps, confirming the validity. However, if the formula is not valid, our procedure will sometimes indicate a counterexample after finitely many steps, *but it could also be the case that our procedure will never terminate, thus yielding no effective answer at all.*

$(\forall x)(\exists y)S(x, y)$