Introduction to Logic PHIL 170

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Announcements

- read Chapter 10.
- Do the practice problems!
- Quiz is due Friday Nov. 13 at 11.59pm.
- Lab is due on Sunday, Nov. 15 at 11.59pm.
- Start the lab early. Do the practice quiz.

Basic Expressions: Predicates and Terms

Predicates: For each non-negative integer *n*, we have an infinite stock of *n*-place predicate letters, represented by upper case letters (possibly with subscripts).

Individual Constants: We have an infinite stock of individual constants, represented by lower case letters ranging from a to t (possibly with subscripts).

Recall: Predicates and Terms

Syntax of Predicates and Terms

- Every 0-place predicate letter is a formula.
- If n > 0, φ is an n-place predicate letter, and τ₁,...,τ_n are n individual constants, then φ(τ₁,...,τ_n) is a formula.

Formulas of Predicate Logic

- 1. Every 0-place predicate letter is a formula.
- If n > 0, φ is an n-place predicate letter, and τ₁,..., τ_n are n individual constants, then φ(τ₁,...,τ_n) is a formula.
- 3. If φ is a formula of predicate logic, then so is $\neg \varphi$.
- 4. If φ and ψ are formulae of predicate logic, then so are each of the following:
 - a. $(\varphi \& \psi)$
 - b. $(\varphi \lor \psi)$
 - c. $(\varphi \rightarrow \psi)$
 - d. $(\varphi \leftrightarrow \psi)$
- 5. An expression of predicate logic is a formula only if it can be constructed by one or more applications of the four three rules.

Interpretations

An interpretation includes each of the following:

- A domain of discourse.
- An assignment to each individual constant of an individual from the domain of discourse.
- ► An assignment of truth-values to 0-place predicate letters.
- ▶ For n > 0, an assignment of a set of ordered n-tuples of members of the domain of discourse to each n-place predicate letter.

Suppose that *I* is an interpretation.

- If φ is a 0-place predicate letter, then φ is true on I just in case $I(\varphi) = T$.
- If φ is of the form ψ(τ₁,...,τ_n) where ψ is an n-place predicate letter (with n > 0), and τ₁,...,τ_n are n terms, then φ is true on I just in case ⟨I(τ₁),...,I(τ_n)⟩ ∈ I(ψ).

Expression	Interpretation
j	John
m	Mary
b	Bob
L(x, y)	x likes y

Mary likes John.

Expression	Interpretation
j	John
т	Mary
b	Bob
L(x,y)	x likes y

Mary likes John. L(m, j)

Bob likes himself.

Expression	Interpretation
j	John
т	Mary
b	Bob
L(x, y)	x likes y

Mary likes John. L(m, j)

Bob likes himself. L(b, b)

Mary and John like each other:

Expression	Interpretation
j	John
т	Mary
b	Bob
L(x, y)	x likes y

Mary likes John. L(m, j)

Bob likes himself. L(b, b)

Mary and John like each other: L(m, j) & L(j, m)

Everyone likes Mary.

Expression	Interpretation
j	John
т	Mary
b	Bob
L(x,y)	x likes y

Mary likes John. L(m, j)

Bob likes himself. L(b, b)

Mary and John like each other: L(m, j) & L(j, m)Everyone likes Mary. L(j, m) & (L(m, m) & L(b, m))Someone likes Mary.

Expression	Interpretation
j	John
т	Mary
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L(x, y)	x likes y

Mary likes John. L(m, j)Bob likes himself. L(b, b)

Mary and John like each other: L(m,j) & L(j,m)Everyone likes Mary. L(j,m) & (L(m,m) & L(b,m))Someone likes Mary. $L(j,m) \lor (L(m,m) \lor L(b,m))$ Everyone likes someone.

Expression	Interpretation
j	John
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L(x, y)	x likes y

Mary likes John. L(m, j)

Bob likes himself. L(b, b)

Mary and John like each other: L(m,j) & L(j,m)

Everyone likes Mary. L(j,m) & (L(m,m) & L(b,m))

Someone likes Mary. $L(j, m) \vee (L(m, m) \vee L(b, m))$

Everyone likes someone.

Expression	Interpretation
P(x)	x is enrolled in PHIL 170
A(x)	x's grade is an A in PHIL 170
B(x)	x's grade is a B in PHIL 170
N(x,y)	x is sitting next to y

Jessica got an A in PHIL 170.

Expression	Interpretation
j	Jessica
P(x)	x is enrolled in PHIL 170
A(x)	x's grade is an A in PHIL 170
B(x)	x's grade is a B in PHIL 170
N(x,y)	x is sitting next to y

Jessica got an A in PHIL 170. A(j)

Everyone got an A or B in PHIL 170.

Someone got an A in PHIL 170.

Domain of discourse: $\mathbb{N} = \{0,1,2,3,\ldots\}$

Expression	Interpretation
Z(x)	$x \text{ is } 0 \ (x = 0)$
G(x,y)	x is greater than to $y (x > y)$
A(x,y,z)	z = x + y

1 > 0.

Domain of discourse: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

Expression	Interpretation
е	0
0	1
Z(x)	$x \text{ is } 0 \ (x = 0)$
G(x,y)	x is greater than to $y (x > y)$
A(x,y,z)	z = x + y

1 > 0. G(o, e)

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1 > 0. G(o, e)

Some number is greater than 0.

Every number (except 0) is greater than 0.

Every number is the sum of two numbers.

Variables: lower case letters (*u* through *z*, possibly with subscripts) are going to be used as individual variables.

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Quantifier symbols: \forall indicates we are talking about **all** or **every** individual under consideration; \exists indicates **some** or **at least one** of the individuals under consideration.

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Quantifier: Combine quantifier symbol with an individual constant (and parentheses to make it easier to read). E.g., $(\forall x), (\forall y), (\exists x), (\exists y)$.

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Quantifier: Combine quantifier symbol with an individual constant (and parentheses to make it easier to read). E.g., $(\forall x), (\forall y), (\exists x), (\exists y)$.

- $(\forall x)(P(x) \rightarrow Q(x))$
- $(\forall x)(P(y) \rightarrow Q(y))$
- $\blacktriangleright (\forall x) P(x) \lor \neg (\forall x) P(x)$
- $\blacktriangleright (\forall x)(\forall y)R(x,y)$
- ► $(\forall x)(\exists y)R(x,y)$

Expression	Interpretation
j	John
т	Mary
b	Bob
L(x,y)	x likes y

Mary likes John. L(m, j)

Bob likes himself. L(b, b)

Mary and John like each other: L(m,j) & L(j,m)

Everyone likes Mary. L(j, m) & (L(m, m) & L(b, m))

Someone likes Mary. $L(j, m) \vee (L(m, m) \vee L(b, m))$

Everyone likes someone.

Expression	Interpretation
j	John
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Everyone likes Mary. $L(j, m) \& (L(m, m) \& L(b, m)) \quad (\forall x)L(x, m)$

Someone likes Mary. $L(j, m) \vee (L(m, m) \vee L(b, m))$

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Everyone likes someone.

 $(L(j,j) \lor L(j,m) \lor L(j,b)) & (L(m,j) \lor L(m,m) \lor L(m,b)) & (L(b,j) \lor L(b,m) \lor L(b,b)) \quad (\forall x) (\exists y) L(x,y)$

Domain of discourse: All students at UMD
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Expression	Interpretation
j	Jessica
P(x)	x is enrolled in PHIL 170
A(x)	x's grade is an A in PHIL 170
B(x)	x's grade is a B in PHIL 170
N(x,y)	x is sitting next to y

Jessica got an A in PHIL 170. A(j)

Everyone got an A or B in PHIL 170.

Someone got an A in PHIL 170.

Expression	Interpretation
j	Jessica
P(x)	x is enrolled in PHIL 170
A(x)	x's grade is an A in PHIL 170
B(x)	x's grade is a B in PHIL 170
N(x,y)	x is sitting next to y

Jessica got an A in PHIL 170. A(j)

Everyone got an A or B in PHIL 170. $(\forall x)(A(x) \lor B(x))$

Someone got an A in PHIL 170.

Expression	Interpretation
j	Jessica
P(x)	x is enrolled in PHIL 170
A(x)	x's grade is an A in PHIL 170
B(x)	x's grade is a B in PHIL 170
N(x,y)	x is sitting next to y

Jessica got an A in PHIL 170. A(j)

Everyone got an A or B in PHIL 170.

$$(\forall x)$$
 $(A(x) \lor B(x))$

Someone got an A in PHIL 170.

Expression	Interpretation
j	Jessica
P(x)	x is enrolled in PHIL 170
A(x)	x's grade is an A in PHIL 170
B(x)	x's grade is a B in PHIL 170
N(x,y)	x is sitting next to y

Jessica got an A in PHIL 170. A(j)

Everyone got an A or B in PHIL 170.

Someone got an A in PHIL 170.

 $(\forall x)$ $(\neg P(x) \lor (A(x) \lor B(x)))$

Expression	Interpretation
j	Jessica
P(x)	x is enrolled in PHIL 170
A(x)	x's grade is an A in PHIL 170
B(x)	x's grade is a B in PHIL 170
N(x,y)	x is sitting next to y

Jessica got an A in PHIL 170. A(j)

Everyone got an A or B in PHIL 170. $(\forall x)(P(x) \rightarrow (A(x) \lor B(x)))$ Someone got an A in PHIL 170.

Expression	Interpretation
j	Jessica
P(x)	x is enrolled in PHIL 170
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B(x)	x's grade is a B in PHIL 170
N(x,y)	x is sitting next to y

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Everyone got an A or B in PHIL 170. $(\forall x)(P(x) \rightarrow (A(x) \lor B(x)))$ Someone got an A in PHIL 170. $(\exists x)(P(x) \& A(x))$

Expression	Interpretation
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P(x)	x is enrolled in PHIL 170
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B(x)	x's grade is a B in PHIL 170
N(x,y)	x is sitting next to y

Jessica got an A in PHIL 170. A(j)

Everyone got an A or B in PHIL 170. $(\forall x)(P(x) \rightarrow (A(x) \lor B(x)))$ Someone got an A in PHIL 170. $(\exists x)(P(x) \& A(x))$

Everyone is sitting next to someone that got an A in PHIL 170. $(\forall x)(\exists y)(P(x) \& P(y) \& N(x, y) \& A(y))$ Domain of discourse: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

Expression	Interpretation
е	0
о	1
Z(x)	$x \text{ is } 0 \ (x = 0)$
G(x,y)	x is greater than to $y (x > y)$
A(x,y,z)	z = x + y

1 > 0. G(o, e)

Some number is greater than 0.

Every number (except 0) is greater than 0.

Every number is the sum of two numbers.

Domain of discourse: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

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Some number is greater than 0. $(\exists x)G(x, e)$

Every number (except 0) is greater than 0. $(\forall x)(\neg Z(x) \rightarrow G(x, e))$ Every number is the sum of two numbers. Domain of discourse: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

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Some number is greater than 0. $(\exists x)G(x, e)$ Every number (except 0) is greater than 0. $(\forall x)(\neg Z(x) \rightarrow G(x, e))$ Every number is the sum of two numbers. $(\forall x)(\exists y)(\exists z)A(y, z, x)$

Basic Expressions of Predicate Logic

- ▶ For each non-negative integer n, infinitely many n-place predicate letters, A, B, C, and so on (possibly with subscripts), are basic expressions of predicate logic.
- The falsum (⊥) is included as a special 0-place predicate (which, as in sentential logic, appears only in derivations).

Terms:

- Infinitely many **individual constants**, represented by lower case letters ranging from *a* to *t* (possibly with subscripts), are basic expressions of predicate logic.
- Infinitely many **individual variables**, represented by lower case letters ranging from *u* to *z* (possibly with subscripts), are basic expressions of predicate logic.

Basic Expressions of Predicate Logic, continued

- ► The symbols for the logical connectives, &, V, →, ↔ and ¬ are basic expressions of predicate logic.
- ► The universal ∀ and existential ∃ quantifier symbols are basic expressions of predicate logic.
- The parentheses, (and), and the comma (,), which are used as punctuation, are basic expressions of predicate logic.

Formulas of Predicate Logic

- 1. Every 0-place predicate letter is a formula. If n > 0, φ is an *n*-place predicate letter, and τ_1, \ldots, τ_n are *n* terms, then $\varphi(\tau_1, \ldots, \tau_n)$ is a formula.
- 2. If φ is a formula of predicate logic, then so is $\neg \varphi$.
- 3. If φ and ψ are formulae of predicate logic, then so are each of the following:

 $(\varphi \And \psi)$, $(\varphi \lor \psi)$, $(\varphi \to \psi)$, $(\varphi \leftrightarrow \psi)$

- 4. If φ is a formula and u an individual variable, then the following are formulas:
 - a. $(\forall u)\varphi$
 - **b**. $(\exists u)\varphi$
- 5. An expression of predicate logic is a formula only if it can be constructed by one or more applications of the above four rules.

Formulas of Predicate Logic

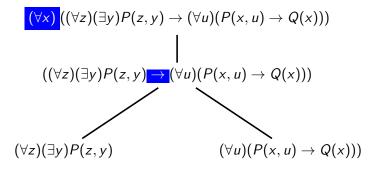
- 1. Every 0-place predicate letter is a formula. If n > 0, φ is an *n*-place predicate letter, and τ_1, \ldots, τ_n are *n* terms, then $\varphi(\tau_1, \ldots, \tau_n)$ is a formula.
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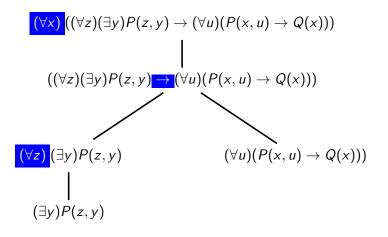
$$(\varphi \And \psi)$$
, $(\varphi \lor \psi)$, $(\varphi \to \psi)$, $(\varphi \leftrightarrow \psi)$

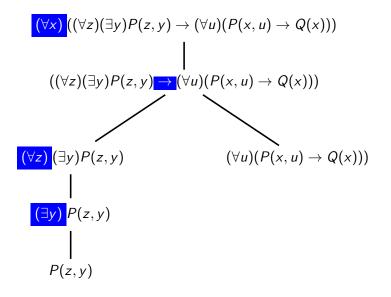
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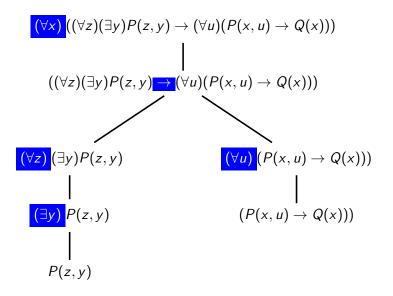
$(\forall x)((\forall z)(\exists y)P(z,y) \rightarrow (\forall u)(P(x,u) \rightarrow Q(x)))$

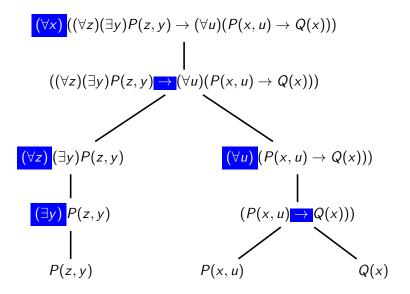
$\begin{array}{c} (\forall x) \\ ((\forall z)(\exists y)P(z,y) \to (\forall u)(P(x,u) \to Q(x))) \\ \\ \\ ((\forall z)(\exists y)P(z,y) \to (\forall u)(P(x,u) \to Q(x))) \end{array}$











H(x)	x is happy.
S(x)	x is a student.

All students are happy.

H(x)	x is happy.
S(x)	x is a student.

All students are happy.

- ? $(\forall x)(S(x) \rightarrow H(x))$
- ? $(\forall x)(S(x) \& H(x))$

H(x)	x is happy.
S(x)	x is a student.

All students are happy.

✓
$$(\forall x)(S(x) \rightarrow H(x))$$

★ $(\forall x)(S(x) \& H(x))$ (Everyone is a student and happy.

H(x)	x is happy.
S(x)	x is a student.

All students are happy.

✓
$$(\forall x)(S(x) \rightarrow H(x))$$

★ $(\forall x)(S(x) \& H(x))$ (Everyone is a student and happy.

?
$$(\exists x)(S(x) \to H(x))$$

? $(\exists x)(S(x) \& H(x))$

H(x)	x is happy.
S(x)	x is a student.

All students are happy.

✓ $(\forall x)(S(x) \rightarrow H(x))$ × $(\forall x)(S(x) \& H(x))$ (Everyone is a student and happy.)

- × $(\exists x)(S(x) \rightarrow H(x))$ (There is someone that is either not a student or happy.)
- $\checkmark (\exists x)(S(x) \& H(x))$

every A is B	$(\forall x)(A(x) \rightarrow B(x))$
some A is B	$(\exists x)(A(x) \& B(x))$
no A is B	$\neg(\exists x)(A(x) \& B(x))$
some A is not B	$(\exists x)(A(x) \& \neg B(x))$
every A is a non- B	$(\forall x)(A(x) \rightarrow \neg B(x))$
not every A is B	$ eg(\forall x)(A(x) o B(x))$

All logicians are mathematicians.

Some logicians are mathematicians.

No logician is a mathematician.

Some logicians are not mathematicians.

Every logician is not a mathematician.

L(x)	x is a logician
M(x)	x is a mathematician

Some logicians are mathematicians. $(\exists x)(L(x) \& M(x))$

No logician is a mathematician.

Some logicians are not mathematicians.

Every logician is not a mathematician.

L(x)	x is a logician
M(x)	x is a mathematician

Some logicians are mathematicians. $(\exists x)(L(x) \& M(x))$

No logician is a mathematician. $\neg(\exists x)(L(x) \& M(x))$

Some logicians are not mathematicians.

Every logician is not a mathematician.

L(x)	x is a logician
M(x)	x is a mathematician

Some logicians are mathematicians. $(\exists x)(L(x) \& M(x))$

No logician is a mathematician. $\neg(\exists x)(L(x) \& M(x))$

Some logicians are not mathematicians. $(\exists x)(L(x) \& \neg M(x))$

Every logician is not a mathematician.

L(x)	x is a logician
M(x)	x is a mathematician

Some logicians are mathematicians. $(\exists x)(L(x) \& M(x))$

No logician is a mathematician. $\neg(\exists x)(L(x) \& M(x))$

Some logicians are not mathematicians. $(\exists x)(L(x) \& \neg M(x))$

Every logician is not a mathematician. $(\forall x)(L(x) \rightarrow \neg M(x))$

L(x)	x is a logician
M(x)	x is a mathematician

Some logicians are mathematicians. $(\exists x)(L(x) \& M(x))$

No logician is a mathematician. $\neg(\exists x)(L(x) \& M(x))$

Some logicians are not mathematicians. $(\exists x)(L(x) \& \neg M(x))$

Every logician is not a mathematician. $(\forall x)(L(x) \rightarrow \neg M(x))$

$$\neg(\forall x)(L(x) \rightarrow M(x))$$

L(x)	x is a logician
M(x)	x is a mathematician