Introduction to Logic
PHIL 170

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Announcements

- read Chapter 10.
- Do the practice problems!
- Quiz is due **Friday Nov. 13 at 11.59pm**.
- Lab is due on **Sunday, Nov. 15 at 11.59pm**.
- Start the lab early. Do the practice quiz.

Recall: Predicates and Terms

Basic Expressions: Predicates and Terms

Predicates: For each non-negative integer n, we have an infinite stock of n-place predicate letters, represented by upper case letters (possibly with subscripts).

Individual Constants: We have an infinite stock of individual constants, represented by lower case letters ranging from a to t (possibly with subscripts).

Recall: Predicates and Terms

Syntax of Predicates and Terms

- Every 0-place predicate letter is a formula.
- If $n > 0$, φ is an n-place predicate letter, and $\tau_1, \ldots, \tau_n$ are n individual constants, then $\varphi(\tau_1, \ldots, \tau_n)$ is a formula.

Formulas of Predicate Logic

1. Every 0-place predicate letter is a formula.
2. If $n > 0$, φ is an n-place predicate letter, and $\tau_1, \ldots, \tau_n$ are n individual constants, then $\varphi(\tau_1, \ldots, \tau_n)$ is a formula.
3. If φ is a formula of predicate logic, then so is $\neg\varphi$.
4. If φ and ψ are formulae of predicate logic, then so are each of the following:
    a. $(\varphi \,\&\, \psi)$
    b. $(\varphi \vee \psi)$
    c. $(\varphi \rightarrow \psi)$
    d. $(\varphi \leftrightarrow \psi)$
5. An expression of predicate logic is a formula only if it can be constructed by one or more applications of the four three rules.

Interpretations

An **interpretation** includes each of the following:

- A domain of discourse.
- An assignment to each individual constant of an individual from the domain of discourse.
- An assignment of truth-values to 0-place predicate letters.
- For $n > 0$, an assignment of a set of ordered n-tuples of members of the domain of discourse to each n-place predicate letter.

Definition of Truth

Suppose that I is an interpretation.

- If φ is a 0-place predicate letter, then φ is true on I just in case $I(\varphi) = \mathsf{T}$.
- If φ is of the form $\psi(\tau_1, \ldots, \tau_n)$ where ψ is an n-place predicate letter (with $n > 0$), and $\tau_1, \ldots, \tau_n$ are n terms, then φ is true on I just in case $\langle I(\tau_1), \ldots, I(\tau_n)\rangle \in I(\psi)$.

Domain of discourse: $\{\textbf{John}, \textbf{Mary}, \textbf{Bob}\}$

Expression	Interpretation
j	**John**
m	**Mary**
b	**Bob**
$L(x, y)$	x likes y

Mary likes John.

Domain of discourse: $\{$**John**, **Mary**, **Bob**$\}$

Expression	Interpretation
j	**John**
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Mary likes John. $L(m, j)$

Bob likes himself.

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Mary and John like each other:

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$L(x, y)$	x likes y

Mary likes John. $L(m, j)$

Bob likes himself. $L(b, b)$

Mary and John like each other: $L(m, j)$ & $L(j, m)$

Everyone likes Mary.

Domain of discourse: $\{\textbf{John}, \textbf{Mary}, \textbf{Bob}\}$

Expression	Interpretation
j	**John**
m	**Mary**
b	**Bob**
$L(x, y)$	x likes y

Mary likes John. $L(m, j)$

Bob likes himself. $L(b, b)$

Mary and John like each other: $L(m, j) \& L(j, m)$

Everyone likes Mary. $L(j, m) \& (L(m, m) \& L(b, m))$

Someone likes Mary.

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Someone likes Mary. $L(j, m) \vee (L(m, m) \vee L(b, m))$

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$(L(j, j) \vee L(j, m) \vee L(j, b)) \ \& \ (L(m, j) \vee L(m, m) \vee L(m, b)) \ \&$
$(L(b, j) \vee L(b, m) \vee L(b, b))$

Domain of discourse: All students at UMD

Expression	Interpretation
$P(x)$	x is enrolled in PHIL 170
$A(x)$	x's grade is an A in PHIL 170
$B(x)$	x's grade is a B in PHIL 170
$N(x, y)$	x is sitting next to y

Jessica got an A in PHIL 170.

Domain of discourse: All students at UMD

Expression	Interpretation
j	**Jessica**
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$N(x, y)$	x is sitting next to y

Jessica got an A in PHIL 170. $A(j)$

Everyone got an A or B in PHIL 170.

Someone got an A in PHIL 170.

Everyone is sitting next to someone that got an A in PHIL 170.

Domain of discourse: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

Expression	Interpretation
$Z(x)$	x is 0 ($x = 0$)
$G(x, y)$	x is greater than to y ($x > y$)
$A(x, y, z)$	$z = x + y$

$1 > 0$.

Domain of discourse: $\mathbb{N} = \{0, 1, 2, 3, \dots\}$

Expression	Interpretation
e	0
o	1
$Z(x)$	x is 0 ($x = 0$)
$G(x, y)$	x is greater than to y ($x > y$)
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$1 > 0$. $\quad G(o, e)$

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$1 > 0$. $\quad G(o, e)$

Some number is greater than 0.

Every number (except 0) is greater than 0.

Every number is the sum of two numbers.

Quantifiers

Variables: lower case letters (u through z, possibly with subscripts) are going to be used as individual variables.

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Quantifier: Combine quantifier symbol with an individual constant (and parentheses to make it easier to read). E.g., $(\forall x), (\forall y), (\exists x), (\exists y)$.

- $(\forall x)(P(x) \rightarrow Q(x))$
- $(\forall x)(P(y) \rightarrow Q(y))$
- $(\forall x)P(x) \vee \neg(\forall x)P(x)$
- $(\forall x)(\forall y)R(x, y)$
- $(\forall x)(\exists y)R(x, y)$

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Expression	Interpretation
j	**John**
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$L(x, y)$	x likes y

Mary likes John. $L(m, j)$

Bob likes himself. $L(b, b)$

Mary and John like each other: $L(m, j) \ \& \ L(j, m)$

Everyone likes Mary. $L(j, m) \ \& \ (L(m, m) \ \& \ L(b, m))$

Someone likes Mary. $L(j, m) \vee (L(m, m) \vee L(b, m))$

Everyone likes someone.
$(L(j, j) \vee L(j, m) \vee L(j, b)) \ \& \ (L(m, j) \vee L(m, m) \vee L(m, b)) \ \&$
$(L(b, j) \vee L(b, m) \vee L(b, b))$

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Everyone likes Mary. $L(j, m) \ \& \ (L(m, m) \ \& \ L(b, m))$ $(\forall x)L(x, m)$

Someone likes Mary. $L(j, m) \vee (L(m, m) \vee L(b, m))$

Everyone likes someone.
$(L(j, j) \vee L(j, m) \vee L(j, b)) \ \& \ (L(m, j) \vee L(m, m) \vee L(m, b)) \ \&$
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Everyone likes Mary. $L(j, m)$ & $(L(m, m)$ & $L(b, m))$ $(\forall x)L(x, m)$

Someone likes Mary. $L(j, m) \vee (L(m, m) \vee L(b, m))$ $(\exists x)L(x, m)$

Everyone likes someone.
$(L(j, j) \vee L(j, m) \vee L(j, b))$ & $(L(m, j) \vee L(m, m) \vee L(m, b))$ &
$(L(b, j) \vee L(b, m) \vee L(b, b))$

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Everyone likes someone.
$(L(j, j) \vee L(j, m) \vee L(j, b))\ \&\ (L(m, j) \vee L(m, m) \vee L(m, b))\ \&$
$(L(b, j) \vee L(b, m) \vee L(b, b))$ $(\forall x)(\exists y)L(x, y)$

Domain of discourse: All students at UMD

Expression	Interpretation
j	**Jessica**
$P(x)$	x is enrolled in PHIL 170
$A(x)$	x's grade is an A in PHIL 170
$B(x)$	x's grade is a B in PHIL 170
$N(x, y)$	x is sitting next to y

Jessica got an A in PHIL 170. $A(j)$

Everyone got an A or B in PHIL 170.

Someone got an A in PHIL 170.

Everyone is sitting next to someone that got an A in PHIL 170.

Domain of discourse: All students at UMD

Expression	Interpretation
j	**Jessica**
$P(x)$	x is enrolled in PHIL 170
$A(x)$	x's grade is an A in PHIL 170
$B(x)$	x's grade is a B in PHIL 170
$N(x, y)$	x is sitting next to y

Jessica got an A in PHIL 170. $A(j)$

Everyone got an A or B in PHIL 170. $(\forall x)(A(x) \vee B(x))$

Someone got an A in PHIL 170.

Everyone is sitting next to someone that got an A in PHIL 170.

Domain of discourse: All students at UMD

Expression	Interpretation
j	**Jessica**
$P(x)$	x is enrolled in PHIL 170
$A(x)$	x's grade is an A in PHIL 170
$B(x)$	x's grade is a B in PHIL 170
$N(x, y)$	x is sitting next to y

Jessica got an A in PHIL 170. $A(j)$

Everyone got an A or B in PHIL 170. $(\forall x)(A(x) \vee B(x))$

Someone got an A in PHIL 170.

Everyone is sitting next to someone that got an A in PHIL 170.

Domain of discourse: All students at UMD

Expression	Interpretation
j	**Jessica**
$P(x)$	x is enrolled in PHIL 170
$A(x)$	x's grade is an A in PHIL 170
$B(x)$	x's grade is a B in PHIL 170
$N(x, y)$	x is sitting next to y

Jessica got an A in PHIL 170. $A(j)$

Everyone got an A or B in PHIL 170. $(\forall x)(\neg P(x) \vee (A(x) \vee B(x)))$

Someone got an A in PHIL 170.

Everyone is sitting next to someone that got an A in PHIL 170.

Domain of discourse: All students at UMD

Expression	Interpretation
j	**Jessica**
$P(x)$	x is enrolled in PHIL 170
$A(x)$	x's grade is an A in PHIL 170
$B(x)$	x's grade is a B in PHIL 170
$N(x, y)$	x is sitting next to y

Jessica got an A in PHIL 170. $A(j)$

Everyone got an A or B in PHIL 170. $(\forall x)(P(x) \rightarrow (A(x) \vee B(x)))$

Someone got an A in PHIL 170.

Everyone is sitting next to someone that got an A in PHIL 170.

Domain of discourse: All students at UMD

Expression	Interpretation
j	**Jessica**
$P(x)$	x is enrolled in PHIL 170
$A(x)$	x's grade is an A in PHIL 170
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$N(x, y)$	x is sitting next to y

Jessica got an A in PHIL 170. $A(j)$

Everyone got an A or B in PHIL 170. $(\forall x)(P(x) \rightarrow (A(x) \vee B(x)))$

Someone got an A in PHIL 170. $(\exists x)(P(x)\ \&\ A(x))$

Everyone is sitting next to someone that got an A in PHIL 170.

Domain of discourse: All students at UMD

Expression	Interpretation
j	**Jessica**
$P(x)$	x is enrolled in PHIL 170
$A(x)$	x's grade is an A in PHIL 170
$B(x)$	x's grade is a B in PHIL 170
$N(x, y)$	x is sitting next to y

Jessica got an A in PHIL 170. $A(j)$

Everyone got an A or B in PHIL 170. $(\forall x)(P(x) \to (A(x) \vee B(x)))$

Someone got an A in PHIL 170. $(\exists x)(P(x) \mathbin{\&} A(x))$

Everyone is sitting next to someone that got an A in PHIL 170.
$(\forall x)(\exists y)(P(x) \mathbin{\&} P(y) \mathbin{\&} N(x, y) \mathbin{\&} A(y))$

Domain of discourse: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

Expression	Interpretation
e	0
o	1
$Z(x)$	x is 0 $(x = 0)$
$G(x, y)$	x is greater than to y $(x > y)$
$A(x, y, z)$	$z = x + y$

$1 > 0$. $\quad G(o, e)$

Some number is greater than 0.

Every number (except 0) is greater than 0.

Every number is the sum of two numbers.

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e	0
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Some number is greater than 0. $\quad (\exists x)G(x, e)$

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$A(x, y, z)$	$z = x + y$

$1 > 0.$ $G(o, e)$

Some number is greater than 0. $(\exists x)G(x, e)$

Every number (except 0) is greater than 0. $(\forall x)(\neg Z(x) \to G(x, e))$

Every number is the sum of two numbers.

Domain of discourse: $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$

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e	0
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$1 > 0$. $\quad G(o, e)$

Some number is greater than 0. $\quad (\exists x)G(x, e)$

Every number (except 0) is greater than 0. $\quad (\forall x)(\neg Z(x) \to G(x, e))$

Every number is the sum of two numbers. $\quad (\forall x)(\exists y)(\exists z)A(y, z, x)$

Basic Expressions of Predicate Logic

- For each non-negative integer n, infinitely many n-place **predicate letters**, A, B, C, and so on (possibly with subscripts), are basic expressions of predicate logic.

- The **falsum** ($\bot$) is included as a special 0-place predicate (which, as in sentential logic, appears only in derivations).

- **Terms**:
  - Infinitely many **individual constants**, represented by lower case letters ranging from a to t (possibly with subscripts), are basic expressions of predicate logic.
  - Infinitely many **individual variables**, represented by lower case letters ranging from u to z (possibly with subscripts), are basic expressions of predicate logic.

Basic Expressions of Predicate Logic, continued

- The symbols for the **logical connectives**, $\&, \vee, \rightarrow, \leftrightarrow$ and $\neg$ are basic expressions of predicate logic.
- The **universal** $\forall$ and **existential** $\exists$ quantifier symbols are basic expressions of predicate logic.
- The parentheses, (and), and the comma (,), which are used as punctuation, are basic expressions of predicate logic.

Formulas of Predicate Logic

1. Every 0-place predicate letter is a formula. If $n > 0$, φ is an n-place predicate letter, and $\tau_1, \ldots, \tau_n$ are n terms, then $\varphi(\tau_1, \ldots, \tau_n)$ is a formula.
2. If φ is a formula of predicate logic, then so is $\neg\varphi$.
3. If φ and ψ are formulae of predicate logic, then so are each of the following:

   $(\varphi \mathbin{\&} \psi)$, $(\varphi \vee \psi)$, $(\varphi \rightarrow \psi)$, $(\varphi \leftrightarrow \psi)$

4. If φ is a formula and u an individual variable, then the following are formulas:
   a. $(\forall u)\varphi$
   b. $(\exists u)\varphi$
5. An expression of predicate logic is a formula only if it can be constructed by one or more applications of the above four rules.

Formulas of Predicate Logic

1. Every 0-place predicate letter is a formula. If $n > 0$, φ is an n-place predicate letter, and $\tau_1, \ldots, \tau_n$ are n **terms**, then $\varphi(\tau_1, \ldots, \tau_n)$ is a formula.
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4. If φ is a formula and u an individual variable, then the following are formulas:
   a. $(\forall u)\varphi$
   b. $(\exists u)\varphi$
5. An expression of predicate logic is a formula only if it can be constructed by one or more applications of the above four rules.

$$(\forall x)((\forall z)(\exists y)P(z,y) \to (\forall u)(P(x,u) \to Q(x)))$$

$$\boxed{(\forall x)}((\forall z)(\exists y)P(z,y) \to (\forall u)(P(x,u) \to Q(x)))$$

$$((\forall z)(\exists y)P(z,y) \to (\forall u)(P(x,u) \to Q(x)))$$

$$(\forall x)((\forall z)(\exists y)P(z,y) \to (\forall u)(P(x,u) \to Q(x)))$$

$$((\forall z)(\exists y)P(z,y) \to (\forall u)(P(x,u) \to Q(x)))$$

$$(\forall z)(\exists y)P(z,y) \qquad\qquad (\forall u)(P(x,u) \to Q(x)))$$

$$(\forall x)((\forall z)(\exists y)P(z, y) \to (\forall u)(P(x, u) \to Q(x)))$$

$$((\forall z)(\exists y)P(z, y) \to (\forall u)(P(x, u) \to Q(x)))$$

$$(\forall z)(\exists y)P(z, y) \qquad\qquad (\forall u)(P(x, u) \to Q(x)))$$

$$(\exists y)P(z, y)$$

$$\boxed{(\forall x)}((\forall z)(\exists y)P(z,y) \rightarrow (\forall u)(P(x,u) \rightarrow Q(x)))$$

$$((\forall z)(\exists y)P(z,y) \boxed{\rightarrow} (\forall u)(P(x,u) \rightarrow Q(x)))$$

$$\boxed{(\forall z)}(\exists y)P(z,y) \qquad\qquad (\forall u)(P(x,u) \rightarrow Q(x)))$$

$$\boxed{(\exists y)}P(z,y)$$

$$P(z,y)$$

$$(\forall x)((\forall z)(\exists y)P(z,y) \to (\forall u)(P(x,u) \to Q(x)))$$

$$((\forall z)(\exists y)P(z,y) \to (\forall u)(P(x,u) \to Q(x)))$$

$$(\forall z)(\exists y)P(z,y) \qquad\qquad (\forall u)(P(x,u) \to Q(x)))$$

$$(\exists y)P(z,y) \qquad\qquad (P(x,u) \to Q(x)))$$

$$P(z,y)$$

$$\boxed{(\forall x)}((\forall z)(\exists y)P(z,y) \rightarrow (\forall u)(P(x,u) \rightarrow Q(x)))$$

$$((\forall z)(\exists y)P(z,y) \boxed{\rightarrow} (\forall u)(P(x,u) \rightarrow Q(x)))$$

$$\boxed{(\forall z)}(\exists y)P(z,y) \qquad \boxed{(\forall u)}(P(x,u) \rightarrow Q(x)))$$

$$\boxed{(\exists y)}P(z,y) \qquad (P(x,u) \boxed{\rightarrow} Q(x)))$$

$$P(z,y) \qquad P(x,u) \qquad Q(x)$$

Translations

$H(x)$	x is happy.
$S(x)$	x is a student.

All students are happy.

Some students are happy.

Translations

$H(x)$	x is happy.
$S(x)$	x is a student.

All students are happy.

? $(\forall x)(S(x) \rightarrow H(x))$

? $(\forall x)(S(x) \,\&\, H(x))$

Some students are happy.

Translations

$H(x)$	x is happy.
$S(x)$	x is a student.

All students are happy.

- ✓ $(\forall x)(S(x) \to H(x))$
- ✗ $(\forall x)(S(x) \,\&\, H(x))$ (Everyone is a student and happy.)

Some students are happy.

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$H(x)$	x is happy.
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- ✓ $(\forall x)(S(x) \rightarrow H(x))$
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Some students are happy.

- ? $(\exists x)(S(x) \rightarrow H(x))$
- ? $(\exists x)(S(x) \,\&\, H(x))$

Translations

$H(x)$	x is happy.
$S(x)$	x is a student.

All students are happy.

- ✓ $(\forall x)(S(x) \rightarrow H(x))$
- ✗ $(\forall x)(S(x) \,\&\, H(x))$ (Everyone is a student and happy.)

Some students are happy.

- ✗ $(\exists x)(S(x) \rightarrow H(x))$ (There is someone that is either not a student or happy.)
- ✓ $(\exists x)(S(x) \,\&\, H(x))$

every **A** is **B**	$(\forall x)(A(x) \rightarrow B(x))$
some **A** is **B**	$(\exists x)(A(x) \mathbin{\&} B(x))$
no **A** is **B**	$\neg(\exists x)(A(x) \mathbin{\&} B(x))$
some **A** is not **B**	$(\exists x)(A(x) \mathbin{\&} \neg B(x))$
every **A** is a non-**B**	$(\forall x)(A(x) \rightarrow \neg B(x))$
not every **A** is **B**	$\neg(\forall x)(A(x) \rightarrow B(x))$

All logicians are mathematicians.

Some logicians are mathematicians.

$L(x)$	x is a logician
$M(x)$	x is a mathematician

No logician is a mathematician.

Some logicians are not mathematicians.

Every logician is not a mathematician.

Not every logician is a mathematician.

All logicians are mathematicians.

$(\forall x)(L(x) \to M(x))$

Some logicians are mathematicians.

$(\exists x)(L(x) \; \& \; M(x))$

$L(x)$	x is a logician
$M(x)$	x is a mathematician

No logician is a mathematician.

Some logicians are not mathematicians.

Every logician is not a mathematician.

Not every logician is a mathematician.

All logicians are mathematicians.

$(\forall x)(L(x) \to M(x))$

Some logicians are mathematicians.

$(\exists x)(L(x) \,\&\, M(x))$

$L(x)$	x is a logician
$M(x)$	x is a mathematician

No logician is a mathematician.

$\neg(\exists x)(L(x) \,\&\, M(x))$

Some logicians are not mathematicians.

Every logician is not a mathematician.

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