Introduction to Logic

PHIL 170

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Announcements

- Starting Part 3: Predicate Logic, read Chapter 9
- Do the practice problems!
- Online quiz due Friday, Nov. 6 at 11.59pm.
- In-class quiz: translate into predicate logic and find the truth-table.
- Lab due Sunday, Nov. 8 at 11.59pm. Start the lab early.

Predicates and Terms

Sentences about John:

- John laughed.
- John is talkative.
- John is in France.
- John likes Mary.
- John is frequently discussed in logic texts.

Predicates and Terms

Sentences about John:

- John laughed. $L(j)$
- John is talkative. $T(j)$
- John is in France. $F(j)$
- John likes Mary. $M(j)$
- John is frequently discussed in logic texts. $D(j)$

Predicates and Terms

- John likes Mary.

Predicates and Terms

- John likes Mary. $L(\quad , \quad)$

Predicates and Terms

- John likes Mary. $L(j, m)$

Predicates and Terms

Basic Expressions: Predicates and Terms

Predicates: For each non-negative integer n, we have an infinite stock of n-place predicate letters, represented by upper case letters (possibly with subscripts).

Individual Constants: We have an infinite stock of individual constants, represented by lower case letters ranging from a to t (possibly with subscripts).

Predicates and Terms

Syntax of Predicates and Terms

- Every 0-place predicate letter is a formula.
- If $n > 0$, φ is an n-place predicate letter, and $\tau_1, \ldots, \tau_n$ are n individual constants, then $\varphi(\tau_1, \ldots, \tau_n)$ is a formula.

- Maggie is a dog, and she is black.
- Snowball is a cat, and she is white.
- Zebbie is a zebra, and he is black and white.

- Maggie is a dog, and she is black.
  $(D(m) \mathbin{\&} B(m))$

- Snowball is a cat, and she is white.

- Zebbie is a zebra, and he is black and white.

- Maggie is a dog, and she is black.
  $(D(m) \,\&\, B(m))$
- Snowball is a cat, and she is white.
  $(C(s) \,\&\, W(s))$
- Zebbie is a zebra, and he is black and white.

- Maggie is a dog, and she is black.
  $(D(m) \mathbin{\&} B(m))$

- Snowball is a cat, and she is white.
  $(C(s) \mathbin{\&} W(s))$

- Zebbie is a zebra, and he is black and white.
  $(Z(z) \mathbin{\&} (B(z) \mathbin{\&} W(z)))$

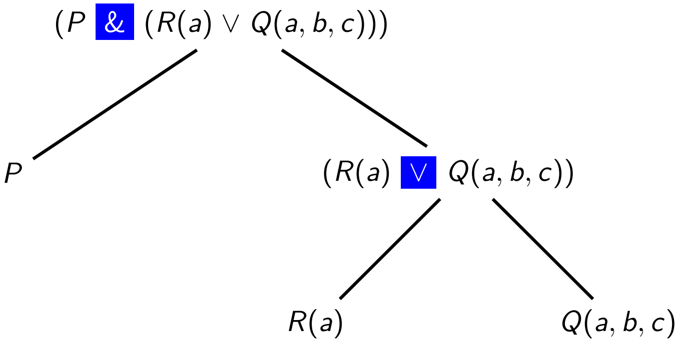
- Maggie is a dog, and she is black.
  $(D(m) \mathbin{\&} B(m))$
- Snowball is a cat, and she is white.
  $(C(s) \mathbin{\&} W(s))$
- Zebbie is a zebra, and he is black and white.
  $(Z(z) \mathbin{\&} P(z))$

Formulas of Predicate Logic

1. Every 0-place predicate letter is a formula.
2. If $n > 0$, φ is an n-place predicate letter, and $\tau_1, \ldots, \tau_n$ are n individual constants, then $\varphi(\tau_1, \ldots, \tau_n)$ is a formula.
3. If φ is a formula of predicate logic, then so is $\neg\varphi$.
4. If φ and ψ are formulae of predicate logic, then so are each of the following:
   a. $(\varphi \,\&\, \psi)$
   b. $(\varphi \vee \psi)$
   c. $(\varphi \rightarrow \psi)$
   d. $(\varphi \leftrightarrow \psi)$
5. An expression of predicate logic is a formula only if it can be constructed by one or more applications of the four three rules.

- $(P \mathbin{\&} (R(a) \vee Q(a, b, c)))$
- $(R(a, b) \to R(b, a))$
- $((P(d) \mathbin{\&} Q(c)) \leftrightarrow \neg(\neg P(d) \vee \neg Q(c)))$
- $(\neg S(a, b, c) \mathbin{\&} (Q(d) \vee Q(e)))$

$$(P \ \& \ (R(a) \vee Q(a, b, c)))$$


$(P \boxed{\&} (R(a) \vee Q(a,b,c)))$
P
$(R(a) \boxed{\vee} Q(a,b,c))$
$R(a)$
$Q(a,b,c)$

Recall: Truth-Tables

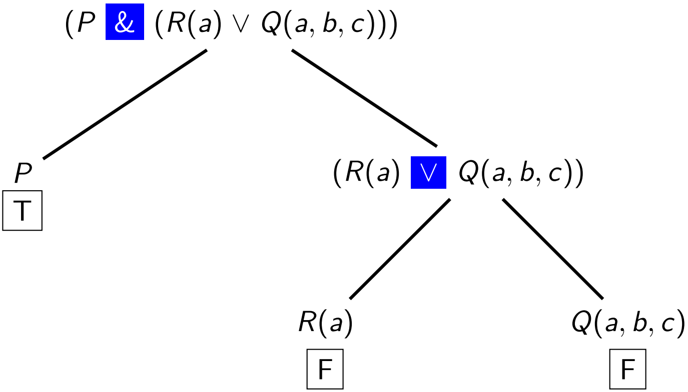
φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

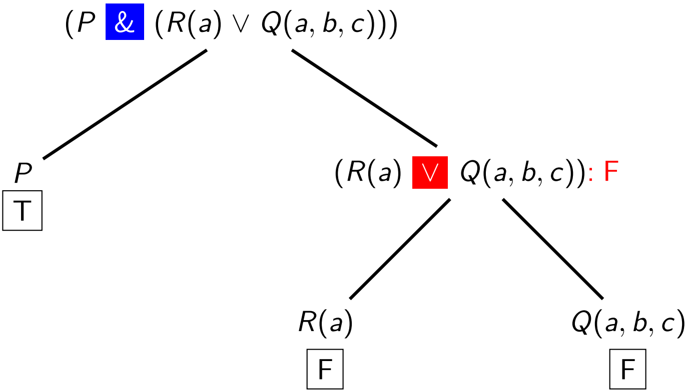
φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

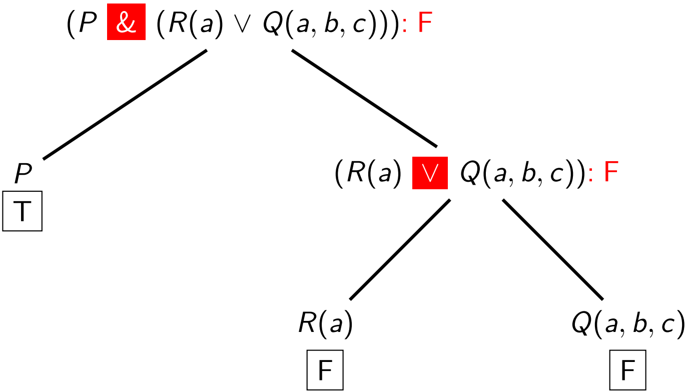
φ	ψ	$(\varphi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

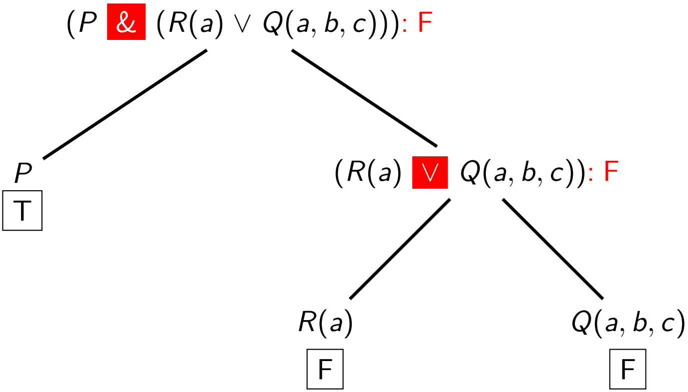
φ	$\neg\varphi$
T	F
F	T

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

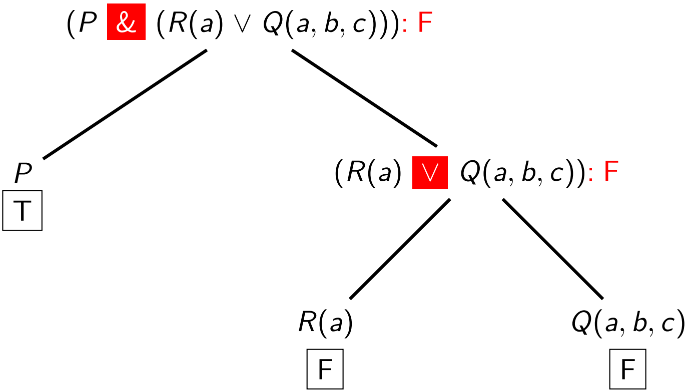

(P & (R(a) ∨ Q(a, b, c)))
P
T
(R(a) ∨ Q(a, b, c))
R(a)
F
Q(a, b, c)
F


$(P \ \& \ (R(a) \vee Q(a,b,c)))$
P
T
$(R(a) \vee Q(a,b,c))$: F
$R(a)$
F
$Q(a,b,c)$
F


(P & (R(a) ∨ Q(a, b, c))): F
P
T
(R(a) ∨ Q(a, b, c)): F
R(a)
F
Q(a, b, c)
F



Why is $R(a)$ *false*?



Why is $R(a)$ *false*? The object denoted by a does not have property R.

E.g., $R(a)$ means that 'a is red', but a denotes a black object.

$$(R(a,b) \rightarrow R(b,a))$$

$$(R(a,b) \rightarrow R(b,a))$$

$R(a,b)$	$R(b,a)$	$(R(a,b) \rightarrow R(b,a))$
T	T	T
T	F	F
F	T	T
F	F	T

Is $((P(d) \,\&\, Q(c)) \leftrightarrow \neg(\neg P(d) \vee \neg Q(c)))$ a tautology? If so, find a deduction. Otherwise, provide a counterexample.

Interpretations

An **interpretation** includes each of the following:

- A domain of discourse.
- An assignment to each individual constant of an individual from the domain of discourse.
- An assignment of truth-values to 0-place predicate letters.
- For $n > 0$, an assignment of a set of ordered n-tuples of members of the domain of discourse to each n-place predicate letter.

Domain of discourse: $\{\textbf{John}, \textbf{Mary}, \textbf{Bob}\}$

Expression	Interpretation
j	**John**
m	**Mary**
b	**Bob**
$R(x, y)$	x is sitting in front of y
$P(x)$	x is enrolled in PHIL 170

Suppose that John is sitting in row 1, Mary in row 2, and Bob is not enrolled in PHIL 170. Which of the following formulas are true:

- $\neg P(b)$ & $P(j)$
- $R(m, j)$
- $R(j, m)$
- $R(j, b)$

Definition of Truth

Suppose that I is an interpretation.

- If φ is a 0-place predicate letter, then φ is true on I just in case $I(\varphi) = \mathsf{T}$.
- If φ is of the form $\psi(\tau_1, \ldots, \tau_n)$ where ψ is an n-place predicate letter (with $n > 0$), and $\tau_1, \ldots, \tau_n$ are n terms, then φ is true on I just in case $\langle I(\tau_1), \ldots, I(\tau_n)\rangle \in I(\psi)$.

Domain of discourse: $\{A, B, C, D\}$

$I(j) = A$
$I(k) = B$
$I(l) = D$
$I(P) = \{\langle B\rangle, \langle D\rangle\}$
$I(Q) = \{\langle A, D\rangle, \langle D, B\rangle, \langle B, A\rangle\}$
$I(R) = \text{F (false)}$

Which of the following formulas are true:

- $\neg P(k)\ \&\ P(j)$
- $Q(j, l)\ \&\ Q(l, j)$
- $Q(l, l)$
- $Q(k, j) \rightarrow Q(j, l)$
- $R \rightarrow Q(j, k)$

Find the deduction:

1.	$P \vee Q$	Premise
2.	$P \to R$	Premise
3.	$Q \to S$	Premise
	$\vdots$	
n.	$R \vee S$	Goal

1.	$P \vee Q$	Premise
2.	$P \to R$	Premise
3.	$Q \to S$	Premise
4.	P	Assumption
5.	R	$\to$E : 2, 4
6.	$R \vee S$	$\vee$IR : 5
7.	Q	Assumption
8.	S	$\to$E : 3, 7
9.	$R \vee S$	$\vee$IL: 8
10.	$R \vee S$	$\vee$E : 1, 6, 9

Find the deduction:

1.	$V(h,e,m) \vee V(h,p,m)$	Premise
2.	$V(h,e,m) \to V(h,l,t)$	Premise
3.	$V(h,p,m) \to V(h,l,d)$	Premise
	$\vdots$	
n.	$V(h,l,t) \vee V(h,l,d)$	Goal

Find the deduction:

1.	$\boxed{V(h,e,m)} \vee V(h,p,m)$	Premise
2.	$\boxed{V(h,e,m)} \rightarrow V(h,l,t)$	Premise
3.	$V(h,p,m) \rightarrow V(h,l,d)$	Premise
	$\vdots$	
$n.$	$V(h,l,t) \vee V(h,l,d)$	Goal

1.	$\boxed{P} \vee Q$	Premise
2.	$\boxed{P} \rightarrow R$	Premise
3.	$Q \rightarrow S$	Premise
	$\vdots$	
$n.$	$R \vee S$	Goal

Find the deduction:

1.	$V(h, e, m) \lor \boxed{V(h, p, m)}$	Premise
2.	$V(h, e, m) \to V(h, l, t)$	Premise
3.	$\boxed{V(h, p, m)} \to V(h, l, d)$	Premise
	$\vdots$	
n.	$V(h, l, t) \lor V(h, l, d)$	Goal

1.	$P \lor \boxed{Q}$	Premise
2.	$P \to R$	Premise
3.	$\boxed{Q} \to S$	Premise
	$\vdots$	
n.	$R \lor S$	Goal

Find the deduction:

1.	$V(h,e,m) \vee V(h,p,m)$	Premise
2.	$V(h,e,m) \to \boxed{V(h,l,t)}$	Premise
3.	$V(h,p,m) \to V(h,l,d)$	Premise
	$\vdots$	
$n.$	$\boxed{V(h,l,t)} \vee V(h,l,d)$	Goal

1.	$P \vee Q$	Premise
2.	$P \to \boxed{R}$	Premise
3.	$Q \to S$	Premise
	$\vdots$	
$n.$	$\boxed{R} \vee S$	Goal

Find the deduction:

1.	$V(h,e,m) \vee V(h,p,m)$	Premise
2.	$V(h,e,m) \to V(h,l,t)$	Premise
3.	$V(h,p,m) \to \boxed{V(h,l,d)}$	Premise
	$\vdots$	
$n.$	$V(h,l,t) \vee \boxed{V(h,l,d)}$	Goal

1.	$P \vee Q$	Premise
2.	$P \to R$	Premise
3.	$Q \to \boxed{S}$	Premise
	$\vdots$	
$n.$	$R \vee \boxed{S}$	Goal

Recall: Truth-Tables

φ	ψ	$(\varphi \mathbin{\&} \psi)$
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φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	$\neg\varphi$
T	F
F	T

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Basic Rules

Conjunction Introduction (&I)

$p1.$	φ	
$p2.$	ψ	
	$\vdots$	
c.	$(\varphi \,\&\, \psi)$	&I: $p1, p2$

Conjunction Elimination (&EL, &ER)

$p1.$	$(\varphi \,\&\, \psi)$	
	$\vdots$	
c.	φ	&EL : $p1$

$p1.$	$(\varphi \,\&\, \psi)$	
	$\vdots$	
c.	ψ	&ER : $p1$

Conditional Introduction ($\rightarrow$ I)

*a*1.	φ	Assumption
	$\vdots$	
*p*1.	ψ	Goal
c.	$(\varphi \rightarrow \psi)$	$\rightarrow$I : *p*1

*p*1.	ψ	
	$\vdots$	
c.	$(\varphi \rightarrow \psi)$	$\rightarrow$I : *p*1

Conditional Elimination ($\rightarrow$ E)

*p*1.	φ	
*p*2.	$(\varphi \rightarrow \psi)$	
	$\vdots$	
c.	ψ	$\rightarrow$E : *p*1, *p*2

Disjunction Introduction ($\vee$IL, $\vee$IR)

$p1.$	φ	
	$\vdots$	
c.	$(\varphi \vee \psi)$	$\vee$IR : $p1$

$p1.$	φ	
	$\vdots$	
c.	$(\psi \vee \varphi)$	$\vee$IL: $p1$

Disjunction Elimination ($\vee$E)

$p1.$	$(\varphi \vee \psi)$	Premise
$a1.$	φ	Assumption
	$\vdots$	
$p2.$	ρ	Goal
$a2.$	ψ	Assumption
	$\vdots$	
$p3.$	ρ	Goal
$c.$	ρ	$\vee$E : $p1, p2, p3$

Negation Introduction/Elimination ($\neg$I, $\neg$E)

$a1.$	$\boxed{\varphi}$	Assumption
	$\vdots$	
$p1.$	$\bot$	Goal
c.	$\neg\varphi$	$\neg$I : $p1$

$a1.$	$\neg\varphi$	Assumption
	$\vdots$	
$p1.$	$\bot$	Goal
c.	φ	$\neg$E : $p1$

Falsum Introduction/Elimination ($\bot$I, $\bot$E)

$p1.$	φ	
$p2.$	$\neg\varphi$	
	$\vdots$	
c.	$\bot$	$\bot$I : $p1, p2$

$p1.$	$\bot$	
	$\vdots$	
c.	φ	$\bot$E : $p1$