Introduction to Logic

PHIL 170

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Announcements

- Midterm 2 labs are due **Monday, Nov. 2 at 11.59pm**. There are three labs (truth-tables, truth-trees and deductions).
- The midterm quiz and labs **must be completed on your own**.
- Covers material from Chapters 4, 5, 6 and 7 plus pgs. 115 & 116 (Chapter 8).
- Starting Part 3: Predicate Logic (read Chapter 9, in-class and online quiz on Friday)

General Observations

$$\begin{array}{l} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_n \\ \hline \therefore \psi \end{array}$$

is valid if, and only if, $((\varphi_1 \ \& \ \varphi_2 \ \& \cdots \& \ \varphi_n) \to \psi)$ is a tautology.

(We say that ψ is a **logical consequence** of $\varphi_1, \ldots, \varphi_n$).

φ and ψ are **logically equivalent** (i.e., have exactly the same truth-values) if, and only if, $((\varphi \to \psi) \ \& \ (\psi \to \varphi))$ is a tautology.

Logical Equivalences

$\varphi \rightarrow \psi$	$\neg\varphi \vee \psi$
$\varphi \rightarrow \psi$	$\neg(\varphi \wedge \neg\psi)$
$\varphi \rightarrow \psi$	$\neg\psi \rightarrow \neg\varphi$
$\neg(\varphi \vee \psi)$	$(\neg\varphi \,\&\, \neg\psi)$
$\neg(\varphi \,\&\, \psi)$	$(\neg\varphi \vee \neg\psi)$
$\neg\neg\varphi$	φ
$(\varphi \,\&\, (\psi \vee \chi))$	$((\varphi \,\&\, \psi) \vee (\varphi \,\&\, \chi))$
$(\varphi \vee (\psi \,\&\, \chi))$	$((\varphi \vee \psi) \,\&\, (\varphi \vee \chi))$
$(\varphi \rightarrow (\psi \rightarrow \chi))$	$((\varphi \,\&\, \psi) \rightarrow \chi)$

We often write $\varphi \leftrightarrow \psi$ for "φ if, and only if, ψ".

$\varphi \leftrightarrow \psi$ is "shorthand" for $((\varphi \rightarrow \psi)$ & $(\psi \rightarrow \varphi))$

What if we add $\leftrightarrow$ as a new logical connective to our language? (pgs. 115 & 116 from Chapter 8)

Biconditional: $\leftrightarrow$

1. What new formulas can be constructed using this new connective?
2. What is the characteristic truth table for the connective?
3. What are the truth-tree decomposition rules for the connective?
4. What are the deduction rules for the connective?

Formulae

1. Every atomic formula φ is a formula of sentential logic.
2. If φ is a formula of sentential logic, then so is $\neg\varphi$.
3. If φ and ψ are formulae of sentential logic, then so are each of the following:
   a. $(\varphi \,\&\, \psi)$
   b. $(\varphi \vee \psi)$
   c. $(\varphi \rightarrow \psi)$
   d. $(\varphi \leftrightarrow \psi)$
4. An expression of sentential logic is a formula only if it can be constructed by one or more applications of the first three rules.

Truth Tables

φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$(\varphi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	$\neg\varphi$
T	F
F	T

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Truth-Tree Decomposition Rules for $\leftrightarrow$

$(\varphi \leftrightarrow \psi)$

φ	$\neg\varphi$
ψ	$\neg\psi$

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

$\neg(\varphi \leftrightarrow \psi)$

φ	ψ
$\neg\psi$	$\neg\varphi$

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditional Introduction ($\leftrightarrow$ I)

*a*1.	φ	Assumption
	$\vdots$	
*p*1.	ψ	Goal
*a*2.	ψ	Assumption
	$\vdots$	
*p*2.	φ	Goal
c.	$(\varphi \leftrightarrow \psi)$	$\leftrightarrow$I: *p*1, *p*2

*p*1.	φ	
*p*2.	ψ	
	$\vdots$	
c.	$(\varphi \leftrightarrow \psi)$	$\leftrightarrow$I: *p*1, *p*2

Biconditional Elimination ($\leftrightarrow$ E)

$p1.$	$(\varphi \leftrightarrow \psi)$	
$p2.$	φ	
	$\vdots$	
c.	ψ	$\leftrightarrow$E : $p1, p2$

$p1.$	$(\varphi \leftrightarrow \psi)$	
$p2.$	ψ	
	$\vdots$	
c.	φ	$\leftrightarrow$E : $p1, p2$

Propositional logic (also called Sentential Logic, Boolean logic)

- Syntax (Formulas, e.g., $(A \,\&\, B) \to \neg C$)
- Semantics (Truth-Tables)
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Is propositional logic enough?

Wason Selection Task

P. C. Wason. *Reasoning about a rule*. Quarterly Journal of Experimental Psychology, 20:273 - 281, 1968.

Wason Selection Task

You are shown a set of four cards placed on a table, each of which has a number on one side and a letter on the other side. Also below is a rule which applies only to the four cards. Your task is to decide which if any of these four cards you *must* turn in order to decide if the rule is true. Don't turn unnecessary cards.

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Rule: If there is a vowel on one side, then there is an even number on the other side.

A	K	4	7

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Which card(s) should we turn over?

1. A
2. A and 4
3. K and 4
4. A and 7
5. All of them
6. Other

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A K 4   7

Which card(s) should we turn over?

1. A
2. A and 4 (half the subjects)
3. K and 4
4. A and 7 (Very few)
5. All of them
6. Other

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P	Q	$P \to Q$
1	1	1
1	0	0
0	1	1
0	0	1

P: vowel
Q: even number

Rule: If there is a vowel on one side, then there is an even number on the other side.

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Responses

Rule: If there is a vowel on one side (P), then there is an even number on the other side (Q).

$$P \rightarrow Q$$

Responses

Rule: If there is a vowel on one side (P), then there is an even number on the other side (Q).

$$P \to Q$$

P	P, Q	$P, \neg Q$	$P, Q, \neg Q$	misc
35%	45%	5%	7%	8%

Responses

Wason (and, until fairly recently, the great majority of researchers) assumed, without considering alternatives, that the correct performance is to turn the A and 7 cards only.

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Positions

- Human reasoning is normatively correct. What appears to be incorrect reasoning can be explained by various maneuvers, such as different interpretation of logical terms, etc.

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- Actual human performance follows prescriptive rules, but they are not the normative rules because of the heavy demands of normatively correct reasoning
- Actual human reasoning falls short of prescriptive standards, so there is room for improvement by suitable education
- Reasoning rarely happens in real life: we have developed "fast and frugal algorithms" which allow us to take quick decisions which are optimal given constraints of time and energy.

Non-Monotonicity

Valid Argument: $A \to B \vdash (A \wedge C) \to B$

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'If you put sugar in the coffee, then it will taste good' can be true without 'If you put sugar and gasoline in the coffee, then it will taste good' being true.

Reasoning May Lead to Revising

Modus Ponens: $P, P \to Q \vdash Q$

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Modus Ponens: $P, P \rightarrow Q \vdash Q$

Suppose that Ann believes that if she will attend Yale, then she will become an atheist. She also believes that she will attend Yale.

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She concludes that she will become an atheist.

Reasoning May Lead to Revising

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Suppose that Ann believes that if she will attend Yale, then she will become an atheist. She also believes that she will attend Yale.

She concludes that she will become an atheist.

But although MP gives Ann a reason to believe the conclusion, it does not decide that she will believe it. Instead of believing the conclusion, she may decide to drop her belief in the conditional.

If Oswald didn't kill Kennedy, then someone else did.

If Oswald didn't kill Kennedy, then someone else would have.

According to Propositional Logic, both sentences have the same truth-value.

First-steps in Predicate Logic

All humans are mortal.

Socrates is human.

$\therefore$ Socrates is mortal.

All humans are mortal.	H
Socrates is human.	M
$\therefore$ Socrates is mortal.	$\therefore S$

All humans are mortal.

Socrates is human.

If all humans are mortal and Socrates is a man, then Socrates is mortal

---

$\therefore$ Socrates is mortal.

$$\begin{array}{l} H \\ M \\ (H \;\&\; M) \rightarrow S \\ \hline \therefore S \end{array}$$

Predicates and Terms

Sentences about John:

- John laughed.
- John is talkative.
- John is in France.
- John likes Mary.
- John is frequently discussed in logic texts.

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- John laughed. $L(j)$
- John is talkative. $T(j)$
- John is in France. $F(j)$
- John likes Mary. $M(j)$
- John is frequently discussed in logic texts. $D(j)$

Predicates and Terms

- John likes Mary.

Predicates and Terms

- John likes Mary. $L(\quad , \quad)$

Predicates and Terms

- John likes Mary. $L(j, m)$