Introduction to Logic

PHIL 170

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Announcements

- I am away next week (I am speaking at a conference in Tawain). *Class is not canceled*. There will be a problem session led by the TAs.
- I will have a few videos to cover any material needed for the midterm.
- Labs are due Wednesday at 11.59pm. *Start working on the lab problems early!*
- Do the practice quiz problems!

Chapter 7 Quiz

Chapter 7 quiz is due next **Wednesday, Oct 28, 11.59pm**.

Advice:

- Read Chapter 7.
- For questions 6 - 16 (pg. 2), refer to the derived rules from Chapter 7 (see pg. 108 for a summary of all the derived rules).
- Do the practice quizzes.

Extra Credit: Chapter 7 Lab

- You will receive 1 extra credit point for each problem you solve from any of the questions 4 - 12 (no extra credit for questions 1, 2 or 3).
- **This is not required**.
- The strict deadline is **Thursday Oct. 29 at 11.59pm** (No extensions will be granted. The deadline will remain on the Course syllabus, but this is only extra credit.)

Midterm 2

- Midterm 2 quiz is due **Friday, Oct. 30 at 11.59pm**.
- Midterm 2 labs are due **Monday, Nov. 2 at 11.59pm**. There are three labs (truth-tables, truth-trees and deductions).
- The midterm quiz and labs **must be completed on your own**.
- Covers material from Chapters 4, 5, 6 and 7 plus pgs. 115 & 116 (Chapter 8).

Midterm 2 Quiz: Answer

Question 8

Points: *1 out of 1*

For which of the following formulae is $((M \; \& \; N) \; \& \; \neg L)$ the canonically constructed disjunctive normal form equivalent?

A. $(\neg(M \& N) \rightarrow L)$
B. $\neg((M \& N) \rightarrow L)$
C. $((M \& N) \rightarrow L)$
D. $((M \& N) \rightarrow \neg L)$

Feedback

That's right!

Midterm 2 Quiz: Answer

Question 15

(A v B) is not a literal.

Points: *1 out of 1*

Feedback

That's right!

Question 16

A is a literal.

Points: *1 out of 1*

Feedback

That's right!

Midterm 2 Quiz: Answer

Question 17

Points: *2 out of 2*

$(P \vee \neg Q)$

A. Conjunctive normal form

B. Disjunctive normal form

C. Both conjunctive and disjunctive normal form

D. Neither

Feedback

That's right!

Midterm 2 Quiz: Challenging Question

Question 25

✖ Points: *0 out of 2*

Consider the Sheffer Stroke connective (also called **nand**):

φ	ψ	$(\varphi \uparrow \psi)$
T	**T**	**F**
T	**F**	**T**
F	**T**	**T**
F	**F**	**T**

View as SVG

What would the Sheffer stroke elimination inference rule(s) look like?

```
p1. (φ ↑ ψ)
a1. | ¬φ        Assumption
    |  :
p2. | ρ
a2. | ¬ψ        Assumption
    |  :
p3. | ρ
 c. ρ           ↑E: p1,p2,p3
```

```
p1. (φ ↑ ψ)
      :
 c. ¬φ          ↑EL: p1

p1. (φ ↑ ψ)
      :
 c. ¬ψ          ↑ER: p1
```

```
p1. (φ ↑ ψ)
a1. | (φ & ¬ψ)  Assumption
    |  :
p2. | ρ
a2. | (¬φ & ψ)  Assumption
    |  :
p3. | ρ
 c. ρ           ↑E: p1,p2,p3
```

none of these

Tactics (pg. 83)

Tactic 1: Extraction
Apply elimination rules forward in order to extract goals that occur as positive subformulae of the formulae on available lines.

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Invert non-atomic goals by applying introduction rules backward to them.

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Use disjunction elimination on any goals for which the previous three tactics have either not applied, or not been successful.

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Use disjunction elimination on any goals for which the previous three tactics have either not applied, or not been successful.

Tactic 5: Refutation
Apply negation elimination backward to goals that cannot be obtained by any other means.

Finding Candidate Contradictions (pg. 85)

Make a list of every negation that occurs as a positive subformula of any premise or available assumption.

If you are using $\neg$E, don't forget to include the assumption you just made.

Pair each negation $\neg\varphi$ you have collected in this way with its immediate subformula φ.

1.	$\neg P \vee \neg Q$	Premise
2.	$\neg P \to R$	Premise
3.	$\neg Q \vee \neg S$	Premise
4.	$\neg(R \;\&\; \neg S)$	Premise
5.	$(R \to \neg(S \vee T))$	Premise
6.	$\neg(P \vee T)$	Assumption
	$\vdots$	
$n-1$.	$\bot$	Goal
n.	$P \vee T$	Goal

<u>Cand. Contradictions</u>

$\neg P$, P

(line 1)

$\neg Q$, Q

(line 1)

$\neg S$, S

(line 3)

$\neg(R \;\&\; \neg S)$, $(R \;\&\; \neg S)$

(line 4)

$\neg(S \vee T)$, $(S \vee T)$

(line 5)

$\neg(P \vee T)$, $(P \vee T)$

(line 6)

From Tactics to Strategies

Read pgs. 93 & 94

From Tactics to Strategies

Read pgs. 93 & 94

If you follow this procedure to the letter you are guaranteed to successfully complete your derivation—eventually...At first, you might find it a bit tedious to work through the procedure step by step, but if you keep with it, eventually it will become second nature and you will find that you are able to complete derivations with a minimum of effort.

Basic Rules

Conjunction Introduction (&I)

$p1.$	φ	
$p2.$	ψ	
	$\vdots$	
c.	$(\varphi \mathbin{\&} \psi)$	&I: $p1, p2$

Conjunction Elimination (&EL, &ER)

$p1.$	$(\varphi \,\&\, \psi)$	
	$\vdots$	
c.	φ	&EL : $p1$

$p1.$	$(\varphi \,\&\, \psi)$	
	$\vdots$	
c.	ψ	&ER : $p1$

Conditional Introduction ($\rightarrow$ I)

*a*1.	φ	Assumption
	$\vdots$	
*p*1.	ψ	Goal
c.	$(\varphi \rightarrow \psi)$	$\rightarrow$I : *p*1

*p*1.	ψ	
	$\vdots$	
c.	$(\varphi \rightarrow \psi)$	$\rightarrow$I : *p*1

Conditional Elimination ($\rightarrow$ E)

*p*1.	φ	
*p*2.	$(\varphi \rightarrow \psi)$	
	$\vdots$	
c.	ψ	$\rightarrow$E : *p*1, *p*2

Disjunction Introduction ($\vee$IL, $\vee$IR)

*p*1.	φ	
	$\vdots$	
c.	$(\varphi \vee \psi)$	$\vee$IR : *p*1

*p*1.	φ	
	$\vdots$	
c.	$(\psi \vee \varphi)$	$\vee$IL: *p*1

Disjunction Elimination ($\vee$E)

$p1.$	$(\varphi \vee \psi)$	Premise
$a1.$	φ	Assumption
	$\vdots$	
$p2.$	ρ	Goal
$a2.$	ψ	Assumption
	$\vdots$	
$p3.$	ρ	Goal
$c.$	ρ	$\vee$E: $p1, p2, p3$

Negation Introduction/Elimination ($\neg$I, $\neg$E)

$a1.$	φ	Assumption
	$\vdots$	
$p1.$	$\bot$	Goal
c.	$\neg\varphi$	$\neg$I : $p1$

$a1.$	$\neg\varphi$	Assumption
	$\vdots$	
$p1.$	$\bot$	Goal
c.	φ	$\neg$E : $p1$

Falsum Introduction/Elimination ($\perp$I, $\perp$E)

$p1.$	φ	
$p2.$	$\neg\varphi$	
	$\vdots$	
c.	$\perp$	$\perp$I : $p1, p2$

$p1.$	$\perp$	
	$\vdots$	
c.	φ	$\perp$E : $p1$

Derived Rules

(Chapter 7, see pg. 108 for a summary)

Double Negation Elimination/Introduction (DNE, DNI)

$p1.$	$\neg\neg\varphi$	
	$\vdots$	
c.	φ	DNE : $p1$

$p1.$	φ	
	$\vdots$	
c.	$\neg\neg\varphi$	DNI : $p1$

Disjunction/Conjunction are Commutative

*p*1.	$(\varphi \vee \psi)$	
	$\vdots$	
c.	$(\psi \vee \varphi)$	Comm $\vee$: *p*1

*p*1.	$(\varphi \,\&\, \psi)$	
	$\vdots$	
c.	$(\psi \,\&\, \varphi)$	Comm & : *p*1

Hypothetical Syllogism

The conditional is transitive.

$p1.$	$(\varphi \to \psi)$	
$p2.$	$(\psi \to \rho)$	
	$\vdots$	
c.	$(\varphi \to \rho)$	HS : $p1, p2$

Distributivity, I

Conjunction distributes over disjunction.

$p1.$	$(\varphi \,\&\, (\psi \vee \rho))$	
	$\vdots$	
c.	$((\varphi \,\&\, \psi) \vee (\varphi \,\&\, \rho))$	Distr & : $p1$

$p1.$	$((\varphi \,\&\, \psi) \vee (\varphi \,\&\, \rho))$	
	$\vdots$	
c.	$(\varphi \,\&\, (\psi \vee \rho))$	Distr & C : $p1$

Distributivity, II

Disjunction distributes over conjunction.

$p1.$	$(\varphi \vee (\psi \,\&\, \rho))$	
	$\vdots$	
c.	$((\varphi \vee \psi) \,\&\, (\varphi \vee \rho))$	Distr$\vee$: $p1$

$p1.$	$((\varphi \vee \psi) \,\&\, (\varphi \vee \rho))$	
	$\vdots$	
c.	$(\varphi \vee (\psi \,\&\, \rho))$	Distr$\vee$C : $p1$

DeMorgan's Laws

$p1.$	$\neg(\varphi \vee \psi)$	
	$\vdots$	
c.	$(\neg\varphi \,\&\, \neg\psi)$	DeM : $p1$

$p1.$	$\neg(\varphi \,\&\, \psi)$	
	$\vdots$	
c.	$(\neg\varphi \vee \neg\psi)$	DeM : $p1$

$p1.$	$(\neg\varphi \,\&\, \neg\psi)$	
	$\vdots$	
c.	$\neg(\varphi \vee \psi)$	DeM : $p1$

$p1.$	$(\neg\varphi \vee \neg\psi)$	
	$\vdots$	
c.	$\neg(\varphi \,\&\, \psi)$	DeM : $p1$

See pg. 108 for a summary of the other rules.

General Observations

$$\begin{array}{l} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_n \\ \hline \therefore \psi \end{array}$$

is valid if, and only if, $((\varphi_1 \mathbin{\&} \varphi_2 \mathbin{\&} \cdots \mathbin{\&} \varphi_n) \rightarrow \psi)$ is a tautology.

(We say that ψ is a **logical consequence** of $\varphi_1, \ldots, \varphi_n$).

φ and ψ are **logically equivalent** (i.e., have exactly the same truth-values) if, and only if, $((\varphi \rightarrow \psi) \mathbin{\&} (\psi \rightarrow \varphi))$ is a tautology.

Logical Equivalences

$\varphi \to \psi$	$\neg\varphi \vee \psi$
$\varphi \to \psi$	$\neg(\varphi \wedge \neg\psi)$
$\varphi \to \psi$	$\neg\psi \to \neg\varphi$
$\neg(\varphi \vee \psi)$	$(\neg\varphi \mathbin{\&} \neg\psi)$
$\neg(\varphi \mathbin{\&} \psi)$	$(\neg\varphi \vee \neg\psi)$
$\neg\neg\varphi$	φ
$(\varphi \mathbin{\&} (\psi \vee \chi))$	$((\varphi \mathbin{\&} \psi) \vee (\varphi \mathbin{\&} \chi))$
$(\varphi \vee (\psi \mathbin{\&} \chi))$	$((\varphi \vee \psi) \mathbin{\&} (\varphi \vee \chi))$
$(\varphi \to (\psi \to \chi))$	$((\varphi \mathbin{\&} \psi) \to \chi)$

We often write $\varphi \leftrightarrow \psi$ for "φ if, and only if, ψ".

$\varphi \leftrightarrow \psi$ is "shorthand" for $((\varphi \rightarrow \psi)$ & $(\psi \rightarrow \varphi))$

What if we add $\leftrightarrow$ as a new logical connective to our language? (pgs. 115 & 116 from Chapter 8)

Biconditional: $\leftrightarrow$

1. What new formulas can be constructed using this new connective?
2. What is the characteristic truth table for the connective?
3. What are the truth-tree decomposition rules for the connective?
4. What are the deduction rules for the connective?

Formulae

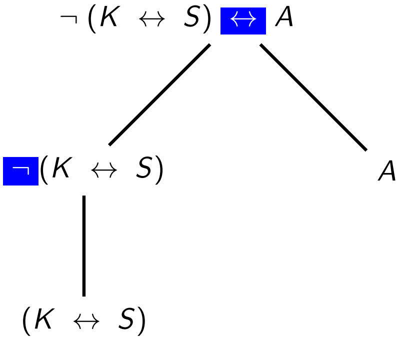
1. Every atomic formula φ is a formula of sentential logic.
2. If φ is a formula of sentential logic, then so is $\neg\varphi$.
3. If φ and ψ are formulae of sentential logic, then so are each of the following:
   a. $(\varphi \,\&\, \psi)$
   b. $(\varphi \vee \psi)$
   c. $(\varphi \rightarrow \psi)$
   d. $(\varphi \leftrightarrow \psi)$
4. An expression of sentential logic is a formula only if it can be constructed by one or more applications of the first three rules.

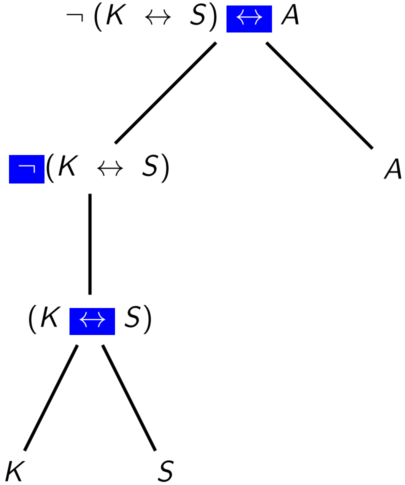
$$\neg(K \leftrightarrow S) \leftrightarrow A$$

$$\neg (K \leftrightarrow S) \boxed{\leftrightarrow} A$$

$$\neg(K \leftrightarrow S) \boxed{\leftrightarrow} A$$

$$\neg(K \leftrightarrow S) \qquad\qquad\qquad A$$


¬(K ↔ S) ↔ A
¬(K ↔ S)
A
(K ↔ S)


$\neg(K \leftrightarrow S) \leftrightarrow A$
$\neg(K \leftrightarrow S)$
A
$(K \leftrightarrow S)$
K
S

Truth Table for $\leftrightarrow$

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Truth Tables

φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

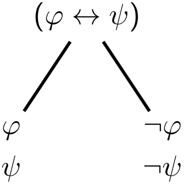
φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$(\varphi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

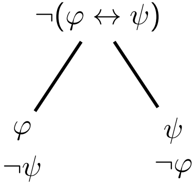
φ	$\neg\varphi$
T	F
F	T

φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Truth-Tree Decomposition Rules for $\leftrightarrow$



φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T



φ	ψ	$(\varphi \leftrightarrow \psi)$
T	T	T
T	F	F
F	T	F
F	F	T

Biconditional Introduction ($\leftrightarrow$ I)

*a*1.	φ	Assumption
	$\vdots$	
*p*1.	ψ	Goal
*a*2.	ψ	Assumption
	$\vdots$	
*p*2.	φ	Goal
c.	$(\varphi \leftrightarrow \psi)$	$\leftrightarrow$I: *p*1, *p*2

*p*1.	φ	
*p*2.	ψ	
	$\vdots$	
c.	$(\varphi \leftrightarrow \psi)$	$\leftrightarrow$I: *p*1, *p*2

Biconditional Elimination ($\leftrightarrow$ E)

$p1.$	$(\varphi \leftrightarrow \psi)$	
$p2.$	φ	
	$\vdots$	
c.	ψ	$\leftrightarrow$E : $p1, p2$

$p1.$	$(\varphi \leftrightarrow \psi)$	
$p2.$	ψ	
	$\vdots$	
c.	φ	$\leftrightarrow$E : $p1, p2$