Introduction to Logic

PHIL 170

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Announcements

- Read Chapter 6.
- Practice, Practice, Practice!
- Quiz is due Sunday at 11.59pm. Labs are due next Wednesday at 11.59pm. *Start working on the lab problems early!*
- In-class quiz on Friday.
- Do the practice quiz problems!
- Watch the videos about how to do deductions on LogicLab...
- Watch the video about conditional elimination (but note the lecture today...).

Questions

Why are we learning deductions?

Why are we using LogicLab to learn deductions?

Conjunction Introduction (&I)

$p1.$	φ	
$p2.$	ψ	
	$\vdots$	
c.	$(\varphi \,\&\, \psi)$	$\&\mathsf{I} : p1, p2$

Conjunction Elimination (&EL, &ER)

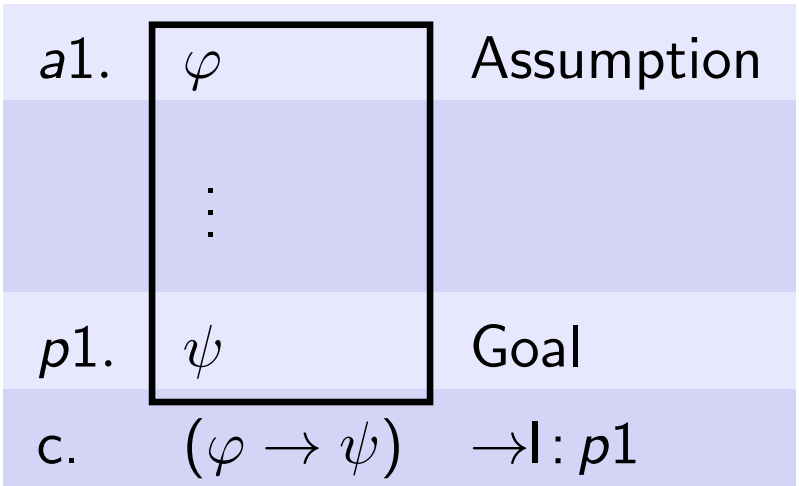
$p1.$	$(\varphi \,\&\, \psi)$	
	$\vdots$	
c.	φ	&EL : $p1$

$p1.$	$(\varphi \,\&\, \psi)$	
	$\vdots$	
c.	ψ	&ER : $p1$

Conditional Elimination ($\rightarrow$ E)

$p1.$	φ	
$p2.$	$(\varphi \rightarrow \psi)$	
	$\vdots$	
c.	ψ	$\rightarrow$E : $p1, p2$

Conditional Introduction ($\rightarrow$ I)



Conditional Introduction ($\rightarrow$ I)

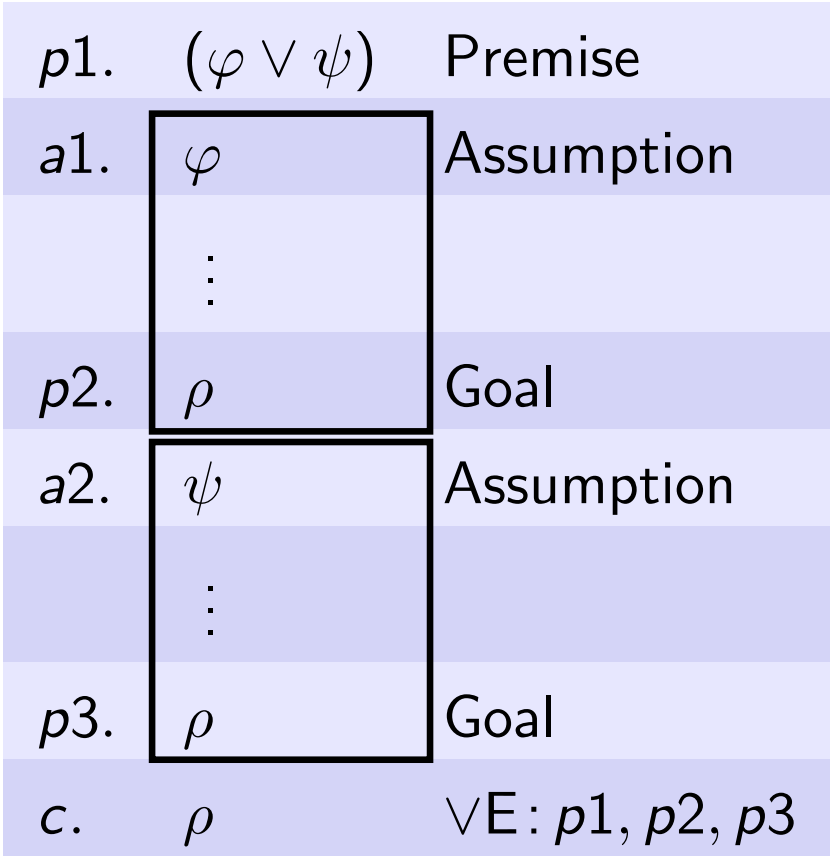
$p1.$	ψ	
	$\vdots$	
c.	$(\varphi \rightarrow \psi)$	$\rightarrow \mathsf{I} : p1$

Disjunction Introduction ($\vee$IL, $\vee$IR)

$p1.$	φ	
	$\vdots$	
c.	$(\varphi \vee \psi)$	$\vee$IR : $p1$

$p1.$	φ	
	$\vdots$	
c.	$(\psi \vee \varphi)$	$\vee$IL: $p1$

Disjunction Elimination ($\vee$E)

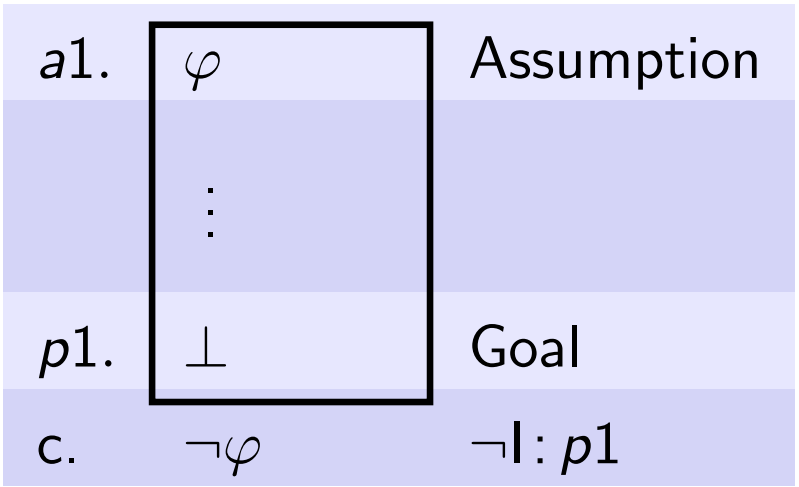


Dealing with negation

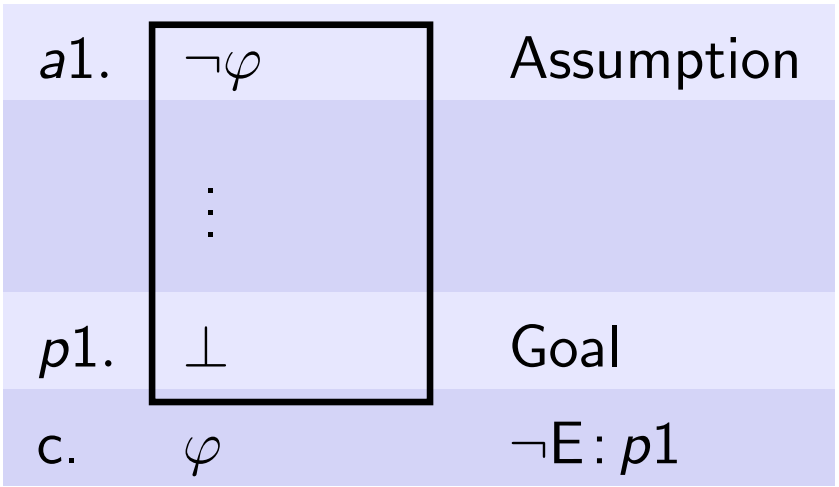
$\bot$: A new atomic proposition that represents a **contradiction**.

This symbol is called the **falsum**, and we treat it almost like any other atomic formula of sentential logic, with one exception: $\bot$ is never used as a constituent of a compound formula—it appears only in derivations as an indication that a contradiction has been derived.

Negation Introduction ($\neg$I)



Negation Eliminiation ($\neg$E)



1.	$(P \rightarrow Q)$	Premise
2.	$(P \rightarrow \neg Q)$	Premise
	$\vdots$	
n.	$\neg P$	Goal

1.	$(P \to Q)$	Premise
2.	$(P \to \neg Q)$	Premise
3.	P	Assumption
	$\vdots$	
n.	$\bot$	Goal
4.	$\neg P$	$\neg \mathsf{I} : n$

1.	$(P \to Q)$	Premise
2.	$(P \to \neg Q)$	Premise
3.	P	Assumption
4.	Q	$\to$E : 1, 3
5.	$\neg Q$	$\to$E : 2, 3
6.	$\bot$	???
7.	$\neg P$	$\neg$I : 6

Falsum Introduction ($\bot$I)

$p1.$	φ	
$p2.$	$\neg\varphi$	
	$\vdots$	
c.	$\bot$	$\bot\mathsf{I} : p1, p2$

1.	$(P \to Q)$	Premise
2.	$(P \to \neg Q)$	Premise
3.	P	Assumption
4.	Q	$\to$E : 1, 3
5.	$\neg Q$	$\to$E : 2, 3
6.	$\bot$	$\bot$I : 4, 5
7.	$\neg P$	$\neg$I : 6

Find the deductions

1.	$(\neg P \,\&\, \neg Q)$	Premise
	$\vdots$	
n.	$\neg(P \vee Q)$	Goal

1.	$\neg(P \vee Q)$	Premise
	$\vdots$	
n.	$(\neg P \,\&\, \neg Q)$	Goal

1.	$(P \vee Q)$	Premise
2.	$\neg Q$	Premise
	$\vdots$	
n.	P	Goal

1.	$(P \vee Q)$	Premise
2.	$\neg Q$	Premise
3.	P	Assumption
	$\vdots$	
n.	P	Goal
6.	Q	Assumption
	$\vdots$	
n.	P	Goal
7.	P	$\vee$E: ???

1.	$(P \vee Q)$	Premise
2.	$\neg Q$	Premise
3.	P	Assumption
4.	Q	Assumption
	$\vdots$	
n.	P	Goal
5.	P	$\vee$E : ???

1.	$(P \vee Q)$	Premise
2.	$\neg Q$	Premise
3.	P	Assumption
4.	Q	Assumption
5.	$\neg P$	Assumption
6.	$\bot$	$\bot$I : 2, 4
7.	P	$\neg$E : 6
8.	P	$\vee$E : 3, 7

1.	$(B \rightarrow \neg C)$	Premise
2.	$(\neg C \rightarrow A)$	Premise
3.	$\neg\neg B$	Premise
	$\vdots$	
n.	$\neg\neg A$	Goal

Double Negation Elimination/Introduction (DNE, DNI)

$p1.$	$\neg\neg\varphi$	
	$\vdots$	
c.	φ	DNE : $p1$

$p1.$	φ	
	$\vdots$	
c.	$\neg\neg\varphi$	DNI : $p1$