# Introduction to Logic PHIL 170 

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## Announcements

- Good job on the Midterm!
- Make sure that you can solve the problems on paper!
- Summary of grades will be emailed/added to ELMS
- Quiz due on Friday, Lab due on Sunday.
- Watch for more videos about how to do the upcoming problems on LogicLab...
- Email: You should email your TA first with questions about extensions, problems with the assignments, etc.


## Fun problems to think about...

Imagine you have been transported to the mysterious "Island of Knights and Knaves" where every inhabitant is either a knight or a knave. Every knight always tells the truth and every knave always lies.

You stop for gas and the attendant says, "Either I am a knight or I am not a knight." What is he?

Suppose that you see two people $A$ and $B$. Suppose that $A$ says "Either I am a knave or $B$ is a knight." What can you conclude?

Suppose that $A$ says "Either I am a knave or $2+2=5$." What can you conclude?

## Fun problems to think about...

An advertisement for a tennis magazine states, "If I'm not playing tennis, I'm watching tennis. And if I'm not watching tennis, I'm reading about tennis." We can assume that the speaker cannot do more than one of these activities at a time. What is the speaker doing?

## General Observations

$\varphi_{1}$
$\varphi_{2}$
$\vdots \quad$ is valid if, and only if, $\left(\left(\varphi_{1} \& \varphi_{2} \& \cdots \& \varphi_{n}\right) \rightarrow \psi\right)$ is a tautology.
$\frac{\varphi_{n}}{\therefore \psi}$
$\varphi$ and $\psi$ are logically equivalent (i.e., have exactly the same truth-values) if, and only if, $((\varphi \rightarrow \psi) \&(\psi \rightarrow \varphi))$ is a tautology.

We often write $\varphi \leftrightarrow \psi$ for $((\varphi \rightarrow \psi) \&(\psi \rightarrow \varphi))$

## Logical Equivalences

$$
\begin{array}{ll}
\varphi \rightarrow \psi & \neg \varphi \vee \psi \\
\varphi \rightarrow \psi & \neg(\varphi \wedge \neg \psi) \\
\varphi \rightarrow \psi & \neg \psi \rightarrow \neg \varphi \\
\neg(\varphi \vee \psi) & (\neg \varphi \& \neg \psi) \\
\neg(\varphi \& \psi) & (\neg \varphi \vee \neg \psi) \\
\neg \neg \varphi & \varphi \\
(\varphi \&(\psi \vee \chi)) & ((\varphi \& \psi) \vee(\varphi \& \chi)) \\
(\varphi \vee(\psi \& \chi)) & ((\varphi \vee \psi) \&(\varphi \vee \chi)) \\
(\varphi \rightarrow(\psi \rightarrow \chi)) & ((\varphi \& \psi) \rightarrow \chi)
\end{array}
$$

## Questions

Why are we learning both truth-tables and truth-trees?

Why does LogicLab force you to enter truth-value assignments in the way that it does?

What's next?

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What's next? Deductions

Arguments
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Lily needs to be at the bus-stop by 9am.

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Arguments

$$
\begin{array}{ll}
X & \frac{U}{\therefore L} \\
X & \frac{\left(\left(L_{1} \& L_{2}\right) \& L_{3}\right)}{\therefore L_{4}} \\
X & \frac{B(b)}{\therefore U(b)} \\
\sqrt{ } \begin{array}{l}
A \vee S \\
\\
\therefore A
\end{array}
\end{array}
$$

## Deductions

An advertisement for a tennis magazine states, "If I'm not playing tennis, I'm watching tennis. And if I'm not watching tennis, I'm reading about tennis." We can assume that the speaker cannot do more than one of these activities at a time. What is the speaker doing?

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$$
\begin{aligned}
& \neg P \rightarrow W \\
& \neg W \rightarrow R \\
& ((P \vee W) \vee R) \\
& (P \rightarrow(\neg W \& \neg R)) \\
& (R \rightarrow(\neg P \& \neg W)) \\
& (W \rightarrow(\neg P \& \neg R)) \\
& \therefore R
\end{aligned}
$$

## Deductions

| 1. | $\neg P \rightarrow W$ | Premise |
| :--- | :--- | :--- |
| 2. | $\neg W \rightarrow R$ | Premise |
| 3. | $(P \rightarrow(\neg W \& \neg R))$ | Premise |
| 4. | $(R \rightarrow(\neg P \& \neg W))$ | Premise |
| 5. | $(W \rightarrow(\neg P \& \neg R))$ | Premise |
|  | $\vdots$ |  |
| c. | $W$ | Goal |

## Conjunction Introduction (\&I)



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| 1. | $A$ | Premise |
| :--- | :--- | :--- |
| 2. | $B$ | Premise |
| 3. | $(A \&(B \vee C))$ | Premise |

## Conjunction Introduction (\&l)

| 1. | $A$ | Premise |
| :--- | :--- | :--- |
| 2. | $B$ | Premise |
| 3. $(A \&(B \vee C))$ | Premise |  |
| 4. $(A \& B)$ | $\& I: 1,2$ |  |

## Conjunction Introduction (\&I)

| 1. | $A$ | Premise |
| :--- | :--- | :--- |
| 2. | $B$ | Premise |
| 3. $(A \&(B \vee C))$ | Premise |  |
| 4. $(A \& B)$ | $\& 1: 1,2$ |  |
| 5. $(B \& A)$ | $\& 1: 2,1$ |  |
|  |  |  |
|  |  |  |

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| 1. | $A$ | Premise |
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|  |  |  |

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| 1. | $A$ | Premise |
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| 1. | $A$ | Premise |
| :--- | :--- | :--- |
| 2. | $B$ | Premise |
| 3. | $(A \&(B \vee C))$ | Premise |
| 4. | $(A \& B)$ | $\& \mid: 1,2$ |
| 5. | $(B \& A)$ | $\& \mid: 2,1$ |
| 6. | $(A \& A)$ | $\& \mid: 1,1$ |
| 7. | $((A \&(B \vee C)) \& B)$ | $\& \mid: 3,2$ |

## Conjunction Introduction (\&I)

| 1. | A | Premise |
| :---: | :---: | :---: |
| 2. | $B$ | Premise |
| 3. | $(A \&(B \vee C))$ | Premise |
| 4. | $(A \& B)$ | \& 1: 1,2 |
| 5. | $(B \& A)$ | \& $1: 2,1$ |
| 6. | $(A \& A)$ | \& $1: 1,1$ |
| 7. | $((A \&(B \vee C)) \& B)$ | \& $1: 3,2$ |
| 8. | $((B \& A) \&(A \& A))$ | \& $1: 5,6$ |

## Example

| 1. $(\mathrm{G} \rightarrow \mathrm{H})$ | Premise |
| :---: | :--- | :--- |
| 2. $\quad \mathrm{I}$ | Premise |
| 3. $\neg \mathrm{J}$ | Premise |
| $\vdots \vdots$ |  |
| n. $\quad((\mathrm{I} \& \neg \mathrm{~J}) \&(\mathrm{G} \rightarrow \mathrm{H}))$ | Goal |

## Example



## Example




## Example



## Example



## Example



## Example

| 1. $\quad(G \rightarrow H)$ | Premise |
| :--- | :--- |
| 2. $\quad I$ | Premise |
| 3. $\neg J$ | Premise |
| 5. $\quad((I \& \neg J) \&(G \rightarrow H))$ | Goal |

## Example

| 1. $\quad(G \rightarrow H)$ | Premise |
| :--- | :--- |
| 2. $\quad I$ | Premise |
| 3. $\quad \neg J$ | Premise |
| 4. $\quad(I \& \neg J)$ | $\& I: 2,3$ |
| 5. $\quad((I \& \neg J) \&(G \rightarrow H))$ | Goal |

## Example

| 1. $\quad(G \rightarrow H)$ | Premise |
| :--- | :--- |
| 2. $\quad I$ | Premise |
| 3. $\quad \neg J$ | Premise |
| 4. $(I \& \neg J)$ | $\& I: 2,3$ |
| 5. $((I \& \neg J) \&(G \rightarrow H))$ | $\& I: 4,1$ |

## Example



## Example



Conjunction Elimination (\&EL, \&ER)

$$
\begin{array}{lll}
\text { p1. } & (\varphi \& \psi) & \\
\vdots & \\
\text { c. } & \varphi & \& E L: p 1
\end{array}
$$

p1. $\quad(\varphi \& \psi)$
c. $\psi$
\&ER: $p 1$

## Conjunction Elimination (\&EL, \&ER)

1. $((A \&(B \vee C)) \& D)$ Premise

Conjunction Elimination (\&EL, \&ER)

1. $((A \&(B \vee C)) \& D)$ Premise
2. $D$ \&ER:1

Conjunction Elimination (\&EL, \&ER)

1. $((A \&(B \vee C)) \& D)$ Premise
2. $D$ \&ER:1
3. $(A \&(B \vee C)) \& E L: 1$

Conjunction Elimination (\&EL, \&ER)

1. $((A \&(B \vee C)) \& D)$ Premise
2. $D$ \&ER:1
3. $(A \&(B \vee C)) \& E L: 1$
4. $A$ \&EL:3

Conjunction Elimination (\&EL, \&ER)

| 1. | $((A \&(B \vee C)) \& D)$ | Premise |
| :--- | :--- | :--- |
| 2. | $D$ | \&ER: 1 |
| 3. $(A \&(B \vee C))$ | $\& E L: 1$ |  |
| 4. $A$ | $\& E L: 3$ |  |
| 5. $(B \vee C)$ | $\& E R: 3$ |  |

## Example

| 1. | $(D \& E)$ | Premise |
| :--- | :--- | :--- |
| 2. | $(F \& G)$ | Premise |
| 5. | $(E \& F)$ Goal |  |

## Example

$$
\begin{array}{lll}
\text { 1. } & (D \& E) & \text { Premise } \\
\text { 2. } & (F \& G) & \text { Premise } \\
\text { 3. } & F & \& E L: 2 \\
& & \\
\text { 5. } & (E \& F) & \text { Goal }
\end{array}
$$

## Example

| 1. | $(D \& E)$ | Premise |
| :--- | :--- | :--- |
| 2. | $(F \& G)$ | Premise |
| 3. | $F$ | $\& E L: 2$ |
| 4. | $E$ | $\& E R: 1$ |
| 5. | $(E \& F)$ | Goal |

## Example

| 1. | $(D \& E)$ | Premise |
| :--- | :--- | :--- |
| 2. | $(F \& G)$ | Premise |
| 3. | $F$ | $\& E L: 2$ |
| 4. | $E$ | $\& E R: 1$ |
| 5. | $(E \& F)$ | $\& \mid: 4,3$ |

## Example



## Example



## Example



## Example



## Example



## Conditional Elimination $(\rightarrow \mathrm{E})$

$$
\begin{array}{lll}
p 1 . & \varphi & \\
\text { p2. } & (\varphi \rightarrow \psi) & \\
& \vdots & \\
\text { c. } & \psi & \rightarrow E: p 1, p 2
\end{array}
$$

## Example



## Example



## Example



## Example



## Example



