Introduction to Logic
PHIL 170

Eric Pacuit

University of Maryland, College Park
pacuit.org
epacuit@umd.edu

September 28, 2015

Announcements

- Lab due on Monday. Good job!
- Midterm 1: Quiz due Friday, Lab (Truth-Table and Truth-Tree) due Sunday
- Thanks for answering the survey!
- Quizzes in Sections this week: Write down the truth table for the conditional and the two decomposition rules for truth-trees.
- **Email**: You should email your TA first with questions about extensions, problems with the assignments, etc.
- Watch for more videos about how to do the upcoming problems on LogicLab...

Truth Tables

φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	$\neg\varphi$
T	F
F	T

A formula is a

Tautology

A formula is a

Tautology when it is always true.

A formula is a

Tautology when it is always true.

(In the truth-table, every truth-value underneath the formula is T. Every branch in the truth-tree starting with the negation of the formula closes)

A formula is a

Tautology when it is always true.

(In the truth-table, every truth-value underneath the formula is T. Every branch in the truth-tree starting with the negation of the formula closes)

Contradictory

A formula is a

Tautology when it is always true.

(In the truth-table, every truth-value underneath the formula is T. Every branch in the truth-tree starting with the negation of the formula closes)

Contradictory when it is always false.

A formula is a

Tautology when it is always true.

(In the truth-table, every truth-value underneath the formula is T. Every branch in the truth-tree starting with the negation of the formula closes)

Contradictory when it is always false.

(In the truth-table, every truth-value underneath the formula is F. Every branch in the truth-tree starting with the formula closes)

A formula is a

Tautology when it is always true.

(In the truth-table, every truth-value underneath the formula is T. Every branch in the truth-tree starting with the negation of the formula closes)

Contradictory when it is always false.

(In the truth-table, every truth-value underneath the formula is F. Every branch in the truth-tree starting with the formula closes)

Contingent

A formula is a

Tautology when it is always true.

(In the truth-table, every truth-value underneath the formula is T. Every branch in the truth-tree starting with the negation of the formula closes)

Contradictory when it is always false.

(In the truth-table, every truth-value underneath the formula is F. Every branch in the truth-tree starting with the formula closes)

Contingent when it is sometimes true and sometimes false.

A formula is a

Tautology when it is always true.

(In the truth-table, every truth-value underneath the formula is T. Every branch in the truth-tree starting with the negation of the formula closes)

Contradictory when it is always false.

(In the truth-table, every truth-value underneath the formula is F. Every branch in the truth-tree starting with the formula closes)

Contingent when it is sometimes true and sometimes false.

(In the truth-table, there are some T and some F underneath the formula. There are open branches in the truth-tree for the negation of the formula.)

An argument is

Valid

An argument is

Valid when it is impossible for the premises to all be true and the conclusion to be false, or when truth of all the premises force the conclusion to be true.

An argument is

Valid when it is impossible for the premises to all be true and the conclusion to be false, or when truth of all the premises force the conclusion to be true.

(In the truth-table, every row in which all the premises are true, also has the conclusion true. In every row in which the conclusion is false, at least one of the premises is false. Every branch in the truth-tree starting with each premise and the negation of the conclusion closes.)

An argument is

Valid when it is impossible for the premises to all be true and the conclusion to be false, or when truth of all the premises force the conclusion to be true.

(In the truth-table, every row in which all the premises are true, also has the conclusion true. In every row in which the conclusion is false, at least one of the premises is false. Every branch in the truth-tree starting with each premise and the negation of the conclusion closes.)

Invalid

An argument is

Valid when it is impossible for the premises to all be true and the conclusion to be false, or when truth of all the premises force the conclusion to be true.

(In the truth-table, every row in which all the premises are true, also has the conclusion true. In every row in which the conclusion is false, at least one of the premises is false. Every branch in the truth-tree starting with each premise and the negation of the conclusion closes.)

Invalid when there is a **counterexample** (a truth-value assignment making all the premises true and the conclusion false).

An argument is

Valid when it is impossible for the premises to all be true and the conclusion to be false, or when truth of all the premises force the conclusion to be true.

(In the truth-table, every row in which all the premises are true, also has the conclusion true. In every row in which the conclusion is false, at least one of the premises is false. Every branch in the truth-tree starting with each premise and the negation of the conclusion closes.)

Invalid when there is a **counterexample** (a truth-value assignment making all the premises true and the conclusion false).

(In the truth-table, there is a row in which all the premises are true and the conclusion if false. There is a branch in the truth-tree starting with each premise and the negation of the conclusion that is open.)

$$
\begin{array}{c}
(\varphi \,\&\, \psi) \\
\varphi \\
\psi
\end{array}
\qquad\qquad
\begin{array}{ccc}
 & \neg(\varphi \,\&\, \psi) & \\
\swarrow & & \searrow \\
\neg\varphi & & \neg\psi
\end{array}
$$

φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \,\&\, \psi)$	$\neg(\varphi \,\&\, \psi)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

$(\varphi \vee \psi)$

$\varphi \qquad \psi$

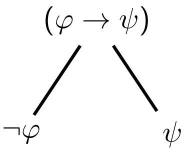
$\neg(\varphi \vee \psi)$

$\neg\varphi$

$\neg\psi$

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$(\varphi \vee \psi)$	$\neg(\varphi \vee \psi)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T



$\neg(\varphi \to \psi)$

φ

$\neg\psi$

φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	ψ	$(\varphi \to \psi)$	$\neg(\varphi \to \psi)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

$$\neg\neg\varphi$$
$$\varphi$$

φ	$\neg\varphi$	$\neg\neg\varphi$
T	F	T
F	T	F

Determine if the following formulas are tautologies, contradictories or contingent. (You must explain your answers.)

1. $((A \to B) \vee (B \to A))$
2. $((A \to (\neg A \vee \neg B))$
3. $((A \to B) \vee (A \to \neg B))$
4. $(A \to (\neg A \vee \neg B))$
5. $(\neg(A \vee B) \,\&\, (\neg A \vee \neg B))$

Add the parentheses (according to the rules in the textbook).

1. $A \to B \to C$
2. $\neg A \,\&\, B \vee C \to \neg\neg A \,\&\, B$
3. $A \,\&\, B \to C \to D \vee C$

Determine if the following arguments are valid or invalid. (You must explain your answers.)

1. $$\begin{array}{l} (A \vee B) \\ (B \rightarrow C) \\ \hline \therefore (B \,\&\, C) \end{array}$$

2. $$\begin{array}{l} (A \rightarrow (B \rightarrow C)) \\ (B \,\&\, C) \\ \hline \therefore \neg\neg A \end{array}$$

3. $$\begin{array}{l} (A \rightarrow B) \\ (A \rightarrow C) \\ \hline \therefore A \rightarrow (B \,\&\, C) \end{array}$$

4. $$\begin{array}{l} ((A \,\&\, B) \vee (A \rightarrow \neg B)) \\ (B \rightarrow C) \\ \hline \therefore (\neg C \rightarrow A) \end{array}$$

5. $$\begin{array}{l} ((A \,\&\, B) \rightarrow (B \,\&\, C)) \\ (B \,\&\, D) \\ \hline \therefore (A \rightarrow (C \rightarrow \neg D)) \end{array}$$

Fun problems to think about...

Imagine you have been transported to the mysterious "Island of Knights and Knaves" where every inhabitant is either a knight or a knave. Every knight always tells the truth and every knave always lies.

You stop for gas and the attendant says, "Either I am a knight or I am not a knight." What is he?

Suppose that you see two people A and B. Suppose that A says "Either I am a knave or B is a knight." What can you conclude?

Suppose that A says "Either I am a knave or $2 + 2 = 5$." What can you conclude?

Fun problems to think about...

An advertisement for a tennis magazine states, "If I'm not playing tennis, I'm watching tennis. And if I'm not watching tennis, I'm reading about tennis." We can assume that the speaker cannot do more than one of these activities at a time. What is the speaker doing?

Fun problems to think about...

There are three suspects for a murder: Adams, Brown and Clark. Adams says “I didn’t do it. The victim was an old acquaintance of Brown’s. But Clark hated him.” Brown states “I didn’t do it. I didn’t even know the guy. Besides I was out of town all that week.” Clark says “I didn’t do it. I saw both Adams and Brown downtown with the victim that day. One of them must have done it.” Assume that the two innocent men are telling the truth, but that the guilty man might not be. Who did it?

General Observations

$$\begin{array}{c} \varphi_1 \\ \varphi_2 \\ \vdots \\ \varphi_n \\ \hline \therefore \psi \end{array}$$

is valid if, and only if, $((\varphi_1 \mathbin{\&} \varphi_2 \mathbin{\&} \cdots \mathbin{\&} \varphi_n) \to \psi)$ is a tautology.

φ and ψ are **logically equivalent** (i.e., have exactly the same truth-values) if, and only if, $((\varphi \to \psi) \mathbin{\&} (\psi \to \varphi))$ is a tautology.

We often write $\varphi \leftrightarrow \psi$ for $((\varphi \to \psi) \mathbin{\&} (\psi \to \varphi))$

Logical Equivalences

$\varphi \to \psi$	$\neg\varphi \vee \psi$
$\varphi \to \psi$	$\neg(\varphi \wedge \neg\psi)$
$\varphi \to \psi$	$\neg\psi \to \neg\varphi$
$\neg(\varphi \vee \psi)$	$(\neg\varphi \,\&\, \neg\psi)$
$\neg(\varphi \,\&\, \psi)$	$(\neg\varphi \vee \neg\psi)$
$\neg\neg\varphi$	φ
$(\varphi \,\&\, (\psi \vee \chi))$	$((\varphi \,\&\, \psi) \vee (\varphi \,\&\, \chi))$
$(\varphi \vee (\psi \,\&\, \chi))$	$((\varphi \vee \psi) \,\&\, (\varphi \vee \chi))$
$(\varphi \to (\psi \to \chi))$	$((\varphi \,\&\, \psi) \to \chi)$

Questions

Why are we learning both truth-tables *and* truth-trees?

Why does LogicLab force you to enter truth-value assignments in the way that it does?

What's next?