Introduction to Logic

PHIL 170

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Announcements

- Quiz for Chapter 4 due Friday at 11.59pm
- Lab for Chapter 4 due Monday at 11.59pm
- In-class quiz in sections this Thursday/Friday. (Translate an argument, give the truth table, determine if the argument is valid or invalid.)

Recap: Truth-Value Assignment

A **truth-value assignment** specifies a unique truth-value (either T or F) for each atomic formula.

Recap: Truth Tables

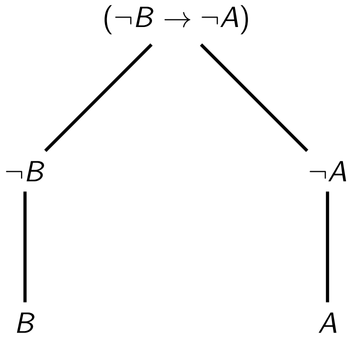
φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

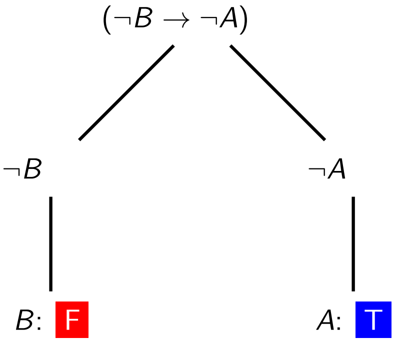
φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	$\neg\varphi$
T	F
F	T

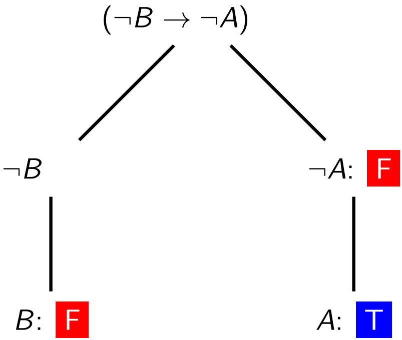
$$(\neg B \rightarrow \neg A)$$


$(\neg B \to \neg A)$
$\neg B$
$\neg A$
B
A



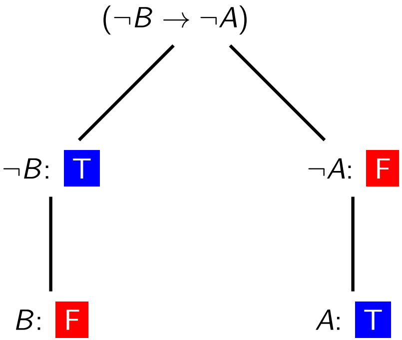
φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	$\neg\varphi$
T	F
F	T



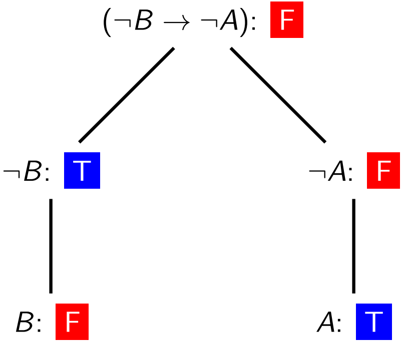
φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	$\neg\varphi$
T	F
F	T



φ	ψ	$(\varphi \rightarrow \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	$\neg\varphi$
T	F
F	T



φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	$\neg\varphi$
T	F
F	T

A	B	$\neg A$	$\neg B$	$\neg B \to \neg A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

A	B	$\neg A$	$\neg B$	$\neg B \to \neg A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

φ	$\neg\varphi$
T	F
F	T

A	B	$\neg A$	$\neg B$	$\neg B \rightarrow \neg A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

φ	$\neg\varphi$
T	F
F	T

A	B	$\neg A$	$\neg B$	$\neg B \to \neg A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

A	B	$\neg A$	$\neg B$	$\neg B \to \neg A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

A	B	$\neg A$	$\neg B$	$\neg B \to \neg A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

A	B	$\neg A$	$\neg B$	$\neg B \to \neg A$
T	T	F	F	T
T	F	F	T	F
F	T	T	F	T
F	F	T	T	T

φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

A	B	$\neg A$	$\neg B$	$\neg B \to \neg A$	$A \to B$
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

$\neg B \to \neg A$ and $A \to B$ have the same truth values, i.e., they are **logically equivalent**.

Prove that $\neg(A \vee B)$ and $(\neg A \,\&\, \neg B)$ are logically equivalent.

A	B	$\neg A$	$\neg B$	$A \vee B$	$\neg(A \vee B)$	$(\neg A \,\&\, \neg B)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

A	B	$\neg A$	$\neg B$	$A \vee B$	$\neg(A \vee B)$	$(\neg A \,\&\, \neg B)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

φ	$\neg\varphi$
T	F
F	T

A	B	$\neg A$	$\neg B$	$A \vee B$	$\neg(A \vee B)$	$(\neg A \,\&\, \neg B)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

φ	$\neg\varphi$
T	F
F	T

A	B	$\neg A$	$\neg B$	$A \vee B$	$\neg(A \vee B)$	$(\neg A \,\&\, \neg B)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

A	B	$\neg A$	$\neg B$	$A \vee B$	$\neg(A \vee B)$	$(\neg A \,\&\, \neg B)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

φ	$\neg\varphi$
T	F
F	T

A	B	$\neg A$	$\neg B$	$A \vee B$	$\neg(A \vee B)$	$(\neg A \,\&\, \neg B)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

A	B	$\neg A$	$\neg B$	$A \vee B$	$\neg(A \vee B)$	$(\neg A \mathbin{\&} \neg B)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

Tautology: A formula is called logically true or a tautology just in case it is true on every truth-value assignment.

Contradictory Formula: A formula is called a contradictory just in case it is false on every truth-value assignment.

Contingent Formula: A formula is called a contingent just in case it is true on some truth-value assignments, and false on others

Examples

$\neg\neg A \to A$ is a tautology.

A	$\neg A$	$\neg\neg A$	$(\neg\neg A \to A)$
T	F	T	T
F	T	F	T

Examples

$(A \to (B \to A))$ is a tautology.

A	B	$(B \to A)$	$(A \to (B \to A))$
T	T	T	T
T	F	T	T
F	T	F	T
F	F	T	T

Examples

$\neg(A \to B)$ & B is contradictory.

A	B	$(A \to B)$	$\neg(A \to B)$	$\neg(A \to B)$ & B
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

Examples

$\neg(A \vee B)$ is contingent.

A	B	$(A \vee B)$	$\neg(A \vee B)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

Validity: An argument is valid just in case any truth-value assignment that makes all the premises true also makes the conclusion true.

Logical Consequence: If an argument with premises $\varphi_1, \ldots, \varphi_n$ and conclusion ψ is valid, then ψ is a logical consequence of $\varphi_1, \ldots, \varphi_n$.

Invalidity: An argument is invalid just in case it is not valid, i.e., if there is some truth-value assignment that makes the premises true, but the conclusion false.

Counterexample: A truth-value assignment that makes the premises of an argument true and its conclusion false is called a counterexample to the argument.

Modus Ponens

$$\begin{array}{l} A \rightarrow B \\ A \\ \hline B \end{array}$$

A	B	$A \rightarrow B$
T	T	T
T	F	F
F	T	T
F	F	T

Modus Ponens

$$\boxed{A \to B}$$
$$\boxed{A}$$
$$\overline{B \qquad\qquad}$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Modus Ponens

$$\frac{\begin{array}{l}\boxed{A \to B}\\ \boxed{A}\end{array}}{\boxed{B}}$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Modus Ponens

$$\begin{array}{l} A \to B \\ A \\ \hline B \end{array}$$

Modus Ponens is valid because **every** truth-value function that makes all the premises true (A, $A \to B$), also makes the consequence (B) true.

Affirming the Consequent

$$\begin{array}{l} A \to B \\ B \\ \hline A \end{array}$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Affirming the Consequent

$$\frac{\boxed{A \to B} \quad \boxed{B}}{A}$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	T

Affirming the Consequent

$$\begin{array}{l} \boxed{A \to B} \\ \boxed{B} \\ \hline \boxed{A} \end{array}$$

A	B	$A \to B$
T	T	T
T	F	F
F	T	T
F	F	F

Affirming the Consequent

$$\begin{array}{l} A \to B \\ B \\ \hline A \end{array}$$

Affirming the Consequent is not valid because there is a truth-value function that makes the premises true and the conclusion false. Namely, the truth-value function that sets A to F and B to T.

Disjunctive Syllogism

$$\begin{array}{l} A \vee B \\ \neg A \\ \hline B \end{array}$$

A	B	$\neg A$	$A \vee B$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Disjunctive Syllogism

$$\frac{\begin{array}{l} \boxed{A \vee B} \\ \boxed{\neg A} \end{array}}{B}$$

A	B	$\neg A$	$A \vee B$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Disjunctive Syllogism

$$\begin{array}{l} \boxed{A \vee B} \\ \boxed{\neg A} \\ \hline \boxed{B} \end{array}$$

A	B	$\neg A$	$A \vee B$
T	T	F	T
T	F	F	T
F	T	T	T
F	F	T	F

Disjunctive Syllogism

$$\begin{array}{l} A \vee B \\ \neg A \\ \hline B \end{array}$$

Disjunctive Syllogism is valid because **every** truth-value function that makes the premises true ($\neg A$ and $A \vee B$) also makes the conclusion (B) true.

$$\begin{array}{l} A \to C \\ B \to C \\ A \vee B \\ \hline C \end{array}$$

A	B	C	$A \to C$	$B \to C$	$A \vee B$
T	T	T			
T	T	F			
T	F	T			
T	F	F			
F	T	T			
F	T	F			
F	F	T			
F	F	F			

$$\begin{array}{l} A \to C \\ B \to C \\ A \vee B \\ \hline C \end{array}$$

A	B	C	$A \to C$	$B \to C$	$A \vee B$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	F

$\boxed{A \to C}$

$\boxed{B \to C}$

$\boxed{A \vee B}$

C

A	B	C	$A \to C$	$B \to C$	$A \vee B$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	F

$$\boxed{A \to C}$$
$$\boxed{B \to C}$$
$$\boxed{A \vee B}$$
$$\overline{\boxed{C}}$$

A	B	C	$A \to C$	$B \to C$	$A \vee B$
T	T	T	T	T	T
T	T	F	F	F	T
T	F	T	T	T	T
T	F	F	F	T	T
F	T	T	T	T	T
F	T	F	T	F	T
F	F	T	T	T	F
F	F	F	T	T	F

$$\begin{array}{l} A \to C \\ B \to C \\ A \vee B \\ \hline C \end{array}$$

This argument is valid because **every** truth-value function that makes the premises true ($A \to C$, $B \to C$ and $A \vee B$) also makes the conclusion (C) true.

Truth Trees: A method to search for *counterexamples*.

Truth Trees: A method to search for *counterexamples*.

Is $((A \to \neg B) \mathbin{\&} (\neg B \vee A))$ a tautology?

Truth Trees: A method to search for *counterexamples*.

Is $((A \rightarrow \neg B) \,\&\, (\neg B \vee A))$ a tautology?

Does every truth-value assignment make $((A \rightarrow \neg B) \,\&\, (\neg B \vee A))$ true?

Is there a **counterexample** (i.e., a truth-value assignment that makes $((A \rightarrow \neg B) \,\&\, (\neg B \vee A))$ false)?

$(\varphi \,\&\, \psi)$

φ

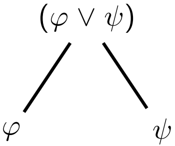
ψ

$\neg(\varphi \,\&\, \psi)$

$\neg\varphi \qquad \neg\psi$

φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \,\&\, \psi)$	$\neg(\varphi \,\&\, \psi)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T



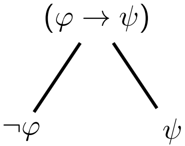
$\neg(\varphi \vee \psi)$

$\neg\varphi$

$\neg\psi$

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	ψ	$(\varphi \vee \psi)$	$\neg(\varphi \vee \psi)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T



$$\neg(\varphi \to \psi)$$
$$\varphi$$
$$\neg\psi$$

φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

φ	ψ	$(\varphi \to \psi)$	$\neg(\varphi \to \psi)$
T	T	T	F
T	F	F	T
F	T	T	F
F	F	T	F

$$\neg\neg\varphi$$
$$\varphi$$

φ	$\neg\varphi$	$\neg\neg\varphi$
T	F	T
F	T	F

$$\neg((A \to \neg B) \mathbin{\&} (\neg B \vee A))$$

$$\neg(\boxed{(A \to \neg B)} \;\&\; \boxed{(\neg B \vee A)})$$

$$\boxed{\neg(A \to \neg B)} \qquad \boxed{\neg(\neg B \vee A)}$$

$$\neg(\boxed{\varphi} \;\&\; \boxed{\psi})$$

$$\boxed{\neg\varphi} \qquad \boxed{\neg\psi}$$

$$\neg((A \to \neg B) \,\&\, (\neg B \vee A))$$

$$\neg(\boxed{A} \to \boxed{\neg B}) \qquad\qquad \neg(\neg B \vee A)$$

$$\boxed{A}$$

$$\boxed{\neg\neg B}$$

$$\neg(\boxed{\varphi} \to \boxed{\psi})$$

$$\boxed{\varphi}$$

$$\boxed{\neg\psi}$$

$$\neg((A \to \neg B) \,\&\, (\neg B \vee A))$$

$$\neg(A \to \neg B) \qquad\qquad \neg(\neg B \vee A)$$

$$A$$

$$\neg\boxed{\neg B}$$

$$\boxed{B}$$

$$\neg\boxed{\neg\varphi}$$

$$\boxed{\varphi}$$

$$\neg((A \to \neg B) \,\&\, (\neg B \vee A))$$

$$\neg(A \to \neg B)$$
$$A$$
$$\neg\neg B$$
$$B$$

$$\neg(\neg B \vee A)$$
$$\neg\neg B$$
$$\neg A$$

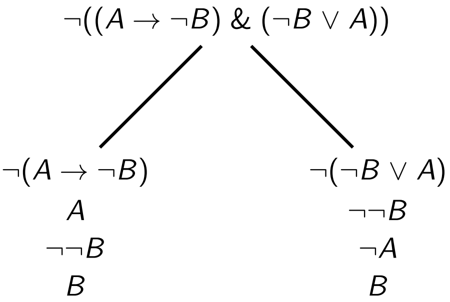
$$\neg(\varphi \vee \psi)$$
$$\neg\varphi$$
$$\neg\psi$$

$$\neg((A \to \neg B) \,\&\, (\neg B \vee A))$$

$$\neg(A \to \neg B)$$
$$A$$
$$\neg \boxed{\neg B}$$
$$\boxed{B}$$

$$\neg(\neg B \vee A)$$
$$\neg \boxed{\neg B}$$
$$\neg A$$
$$\boxed{B}$$

$$\neg \boxed{\neg \varphi}$$
$$\boxed{\varphi}$$



A	B	$(A \to \neg B)$	$(\neg B \vee A)$	$((A \to \neg B) \;\&\; (\neg B \vee A))$
T	T	F	T	F
T	F	T	T	T
F	T	T	F	F
F	F	T	T	T

$\neg((A \rightarrow \neg B) \,\&\, (\neg B \vee A))$

$\neg(A \rightarrow \neg B)$

A

$\neg\neg B$

B

$\neg(\neg B \vee A)$

$\neg\neg B$

$\neg A$

B

A	B	$(A \rightarrow \neg B)$	$(\neg B \vee A)$	$((A \rightarrow \neg B) \,\&\, (\neg B \vee A))$
T	T	F	T	F
T	F	T	T	T
F	T	T	F	F
F	F	T	T	T

$$\neg((A \to \neg B) \,\&\, (\neg B \vee A))$$

$\neg(A \to \neg B)$

A

$\neg\neg B$

B

$\neg(\neg B \vee A)$

$\neg\neg B$

$\neg A$

B

A	B	$(A \to \neg B)$	$(\neg B \vee A)$	$((A \to \neg B) \,\&\, (\neg B \vee A))$
T	T	F	T	F
T	F	T	T	T
F	T	T	F	F
F	F	T	T	T

$((A \to \neg B) \mathbin{\&} (\neg B \vee A))$

$((A \to \neg B) \;\&\; (\neg B \vee A))$

$(A \to \neg B)$

$(\neg B \vee A)$

$(\varphi \;\&\; \psi)$

φ

ψ

$$((A \to \neg B) \,\&\, (\neg B \vee A))$$

$$(\boxed{A} \to \boxed{\neg B})$$

$$(\neg B \vee A)$$

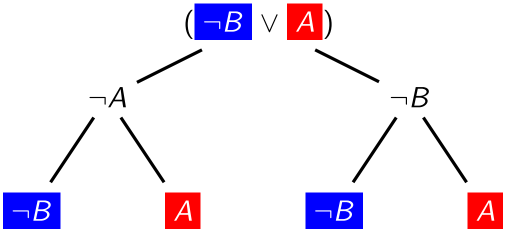
$$\boxed{\neg A} \qquad \boxed{\neg B}$$

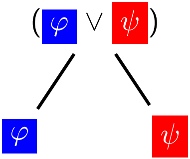
$$(\boxed{\varphi} \to \boxed{\psi})$$

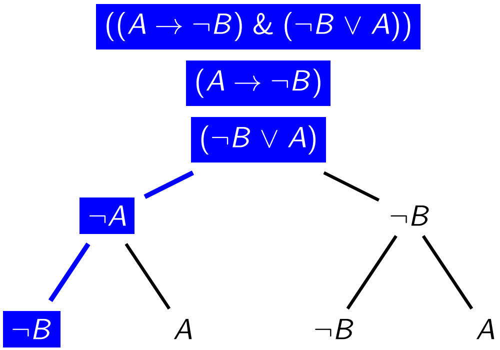
$$\boxed{\neg\varphi} \qquad \boxed{\psi}$$

$((A \to \neg B)\ \&\ (\neg B \vee A))$

$(A \to \neg B)$





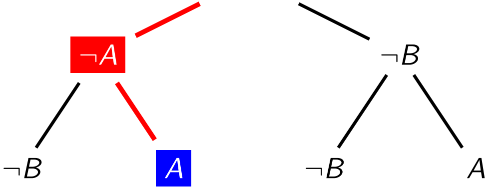


A	B	$(A \to \neg B)$	$(\neg B \vee A)$	$((A \to \neg B) \,\&\, (\neg B \vee A))$
T	T	F	T	F
T	F	T	T	T
F	T	T	F	F
F	F	T	T	T

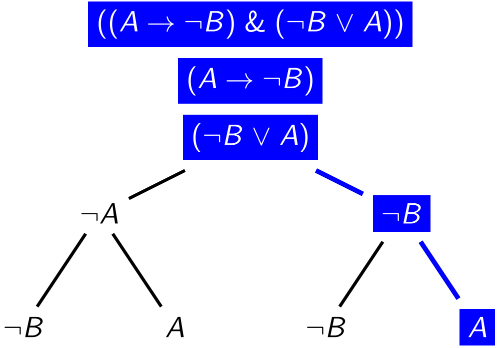
$$((A \to \neg B) \,\&\, (\neg B \vee A))$$

$$(A \to \neg B)$$

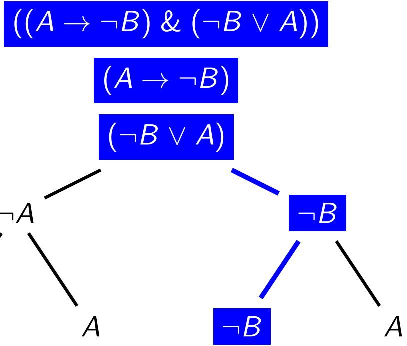
$$(\neg B \vee A)$$



A	B	$(A \to \neg B)$	$(\neg B \vee A)$	$((A \to \neg B) \,\&\, (\neg B \vee A))$
T	T	F	T	F
T	F	T	T	T
F	T	T	F	F
F	F	T	T	T



A	B	$(A \to \neg B)$	$(\neg B \vee A)$	$((A \to \neg B) \mathbin{\&} (\neg B \vee A))$
T	T	F	T	F
T	F	T	T	T
F	T	T	F	F
F	F	T	T	T



A	B	$(A \to \neg B)$	$(\neg B \vee A)$	$((A \to \neg B) \,\&\, (\neg B \vee A))$
T	T	F	T	F
T	F	T	T	T
F	T	T	F	F
F	F	T	T	T

Determine if the following formulas are tautologies, contradictories or contingent. (You must explain your answer.)

1. $((A \to B) \vee (B \to A))$
2. $((A \to (\neg A \vee \neg B))$
3. $((A \to B) \vee (A \to \neg B))$
4. $(A \to (\neg A \vee \neg B))$
5. $(\neg(A \vee B) \;\&\; (\neg A \vee \neg B))$

Fun problems to think about...

Imagine you have been transported to the mysterious "Island of Knights and Knaves" where every inhabitant is either a knight or a knave. Every knight always tells the truth and every knave always lies.

You stop for gas and the attendant says, "Either I am a knight or I am not a knight." What is he?

Suppose that you see a tall person and a short person. Suppose that you are given the following information: "The tall one is a knave and/or the short one is a knight." What can you conclude about the two people?

Fun problems to think about...

There are three suspects for a murder: Adams, Brown and Clark. Adams says "I didn't do it. The victim was an old acquaintance of Brown's. But Clark hated him." Brown states "I didn't do it. I didn't even know the guy. Besides I was out of town all that week." Clark says "I didn't do it. I saw both Adams and Brown downtown with the victim that day. One of them must have done it." Assume that the two innocent men are telling the truth, but that the guilty man might not be. Who did it?

Fun problems to think about...

An advertisement for a tennis magazine states, "If I'm not playing tennis, I'm watching tennis. And if I'm not watching tennis, I'm reading about tennis." We can assume that the speaker cannot do more than one of these activities at a time. What is the speaker doing?