Introduction to Logic

PHIL 170

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Announcements

- You should have completed chapters 1, 2 and 3.
- Start chapter 4 (read up to pg. 53 by Wednesday)
- Questions about the quiz?

Recap: Logical Connectives

English expression	Logical connective
not, it is not the case that, it is false that	$\neg$
and, yet, but, however, both, also, although, nevertheless, still, also, although, moreover, additionally, furthermore	&
or, unless, either … or …	$\vee$
if … then …, only if, given that, in case, provided that, on condition that, sufficient condition, necessary condition, unless (Note: don't confuse antecedents/consequents!)	$\rightarrow$

Recap: Expressions

Basic Expressions

- The capital letters we use to symbolize atomic sentences, called atomic formulae or occasionally sentential letters: A, B, C, and so on (possibly with numeric subscripts).
- The symbols for the logical connectives: $\&$, $\vee$, $\rightarrow$, and $\neg$
- The parentheses used to disambiguate the scope of the connectives: (and).

Recap: Expressions

Basic Expressions

- The capital letters we use to symbolize atomic sentences, called atomic formulae or occasionally sentential letters: A, B, C, and so on (possibly with numeric subscripts).
- The symbols for the logical connectives: $\&$, $\vee$, $\rightarrow$, and $\neg$
- The parentheses used to disambiguate the scope of the connectives: (and).

Expressions of Sentential Logic

Any finite sequence or string of basic expressions is an expression of sentential logic.

Recap: Formulae

1. Every atomic formula φ is a formula of sentential logic.
2. If φ is a formula of sentential logic, then so is $\neg\varphi$.
3. If φ and ψ are formulae of sentential logic, then so are each of the following:
   a. $(\varphi \,\&\, \psi)$
   b. $(\varphi \vee \psi)$
   c. $(\varphi \rightarrow \psi)$
4. An expression of sentential logic is a formula only if it can be constructed by one or more applications of the first three rules.

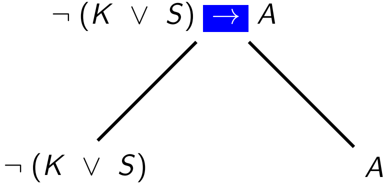
Recap: Parse Tree

$$\neg (K \vee S) \rightarrow A$$

Recap: Parse Tree

$$\neg (K \lor S) \boxed{\rightarrow} A$$

Recap: Parse Tree



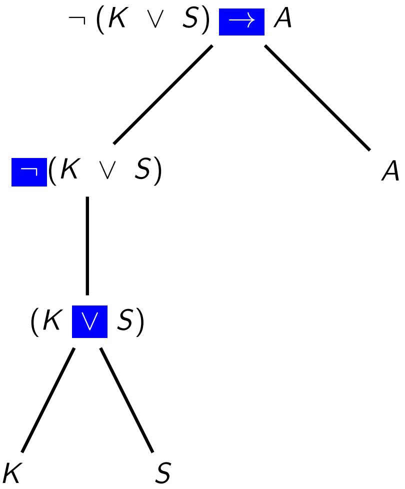
Recap: Parse Tree

$$\neg(K \vee S) \to A$$

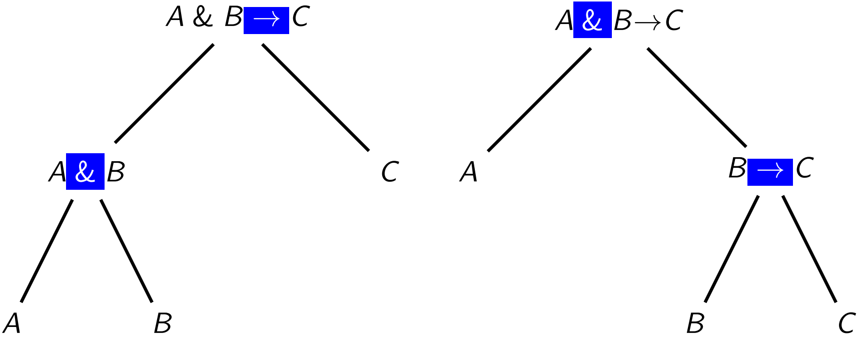
$$\neg(K \vee S) \qquad\qquad A$$

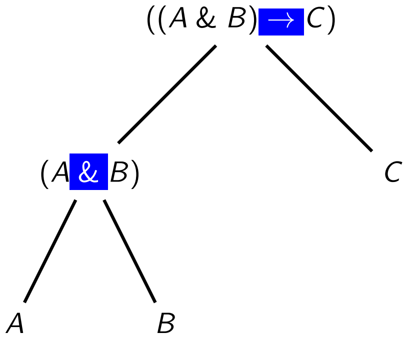
$$(K \vee S)$$

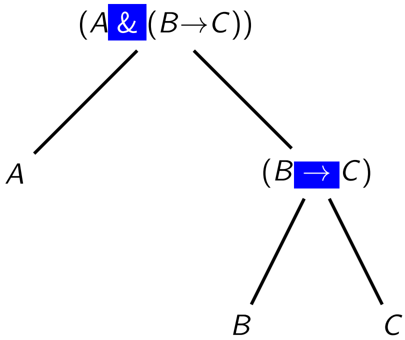
Recap: Parse Tree



$$A \,\&\, B \rightarrow C$$


A & B → C
A & B
C
A
B
A & B → C
A
B → C
B
C


((A & B) → C)
(A & B)
C
A
B


$(A \,\&\, (B \to C))$
A
$(B \to C)$
B
C

Eric had steak and wine.

$(S \,\&\, W)$ $(W \,\&\, S)$

Eric had steak and wine.

✓ $(S \& W)$ ✗ $(W \& S)$

Eric had steak and wine.

✓ $(S \,\&\, W)$ ✗ $(W \,\&\, S)$

It's not the case that it is not raining.

$\neg\neg R$ R

Eric had steak and wine.

✓ $(S \mathbin{\&} W)$ ✗ $(W \mathbin{\&} S)$

It's not the case that it is not raining.

✓ $\neg\neg R$ ✗ R

Eric had steak and wine.

✓ $(S \,\&\, W)$ ✗ $(W \,\&\, S)$

It's not the case that it is not raining.

✓ $\neg\neg R$ ✗ R

Eric had neither red wine nor white wine for dinner.

$\neg(R \vee W)$ $(\neg R \,\&\, \neg W)$

Eric had steak and wine.

✓ $(S \,\&\, W)$ ✗ $(W \,\&\, S)$

It's not the case that it is not raining.

✓ $\neg\neg R$ ✗ R

Eric had neither red wine nor white wine for dinner.

✓ $\neg(R \vee W)$ ✗ $(\neg R \,\&\, \neg W)$

Eric had steak and wine.

✓ $(S \,\&\, W)$ ✗ $(W \,\&\, S)$

It's not the case that it is not raining.

✓ $\neg\neg R$ ✗ R

Eric had neither red wine nor white wine for dinner.

✓ $\neg(R \vee W)$ ✗ $(\neg R \,\&\, \neg W)$

Neither Eric nor Lauren had wine.

$\neg(E \vee L)$ $(\neg E \,\&\, \neg L)$

Eric had steak and wine.

✓ $(S \,\&\, W)$ ✗ $(W \,\&\, S)$

It's not the case that it is not raining.

✓ $\neg\neg R$ ✗ R

Eric had neither red wine nor white wine for dinner.

✓ $\neg(R \vee W)$ ✗ $(\neg R \,\&\, \neg W)$

Neither Eric nor Lauren had wine.

✓? $\neg(E \vee L)$ ✗? $(\neg E \,\&\, \neg L)$

did I get this

Hint

Translate each sentence using the given translation key:

B Bob is gathering eggs.

C Bob has fed the chickens.

D Bob has fed the ducks.

P The ducks are playing in the pond.

Neither Bob has fed the chickens, nor are the ducks playing in the pond.

Click here for info on typing symbols and formulae in activities

(~C & ~P)

Either Bob has fed the chickens and the ducks, or the ducks are playing in the pond.

Click here for info on typing symbols and formulae in activities

Bob has fed the ducks, and either he has fed the chickens or the ducks are playing in the pond.

Click here for info on typing symbols and formulae in activities

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The phrase 'neither...nor...' corresponds to a negated disjunction, rather than a conjunction of negations. The conjunction of negations will always have the same truth-value as the negated disjunction, i.e., it means the same thing, but it doesn't accurately reflect the structure of the sentence.

Neither Bob has fed the chickens, nor are the ducks playing in the pond.

Click here for info on typing symbols and formulae in activities

(~C & ~P)

The phrase 'neither...nor...' corresponds to a negated disjunction, rather than a conjunction of negations. The conjunction of negations will always have the same truth-value as the negated disjunction, i.e., it means the same thing, but it doesn't accurately reflect the structure of the sentence.

Truth-Value Assignment

A **truth-value assignment** specifies a unique truth-value (either T or F) for each atomic formula.

How many truth value assignments are there for a single atomic propositions A?

How many truth value assignments are there for a single atomic propositions A? **2**

How many truth value assignments are there for a single atomic propositions A? **2**

How many truth value assignments are there for two atomic propositions A and B?

How many truth value assignments are there for a single atomic propositions A? **2**

How many truth value assignments are there for two atomic propositions A and B? **4**

How many truth value assignments are there for a single atomic propositions A? **2**

How many truth value assignments are there for two atomic propositions A and B? **4**

How many truth value assignments are there for three atomic propositions A, B, and C?

How many truth value assignments are there for a single atomic propositions A? **2**

How many truth value assignments are there for two atomic propositions A and B? **4**

How many truth value assignments are there for three atomic propositions A, B, and C? **8**

How many truth value assignments are there for a single atomic propositions A? **2**

How many truth value assignments are there for two atomic propositions A and B? **4**

How many truth value assignments are there for three atomic propositions A, B, and C? **8**

How many truth value assignments are there for four atomic propositions A, B, C and D?

How many truth value assignments are there for a single atomic propositions A? **2**

How many truth value assignments are there for two atomic propositions A and B? **4**

How many truth value assignments are there for three atomic propositions A, B, and C? **8**

How many truth value assignments are there for four atomic propositions A, B, C and D? **16**

How many truth value assignments are there for a single atomic propositions A? **2**

How many truth value assignments are there for two atomic propositions A and B? **4**

How many truth value assignments are there for three atomic propositions A, B, and C? **8**

How many truth value assignments are there for four atomic propositions A, B, C and D? **16**

How many truth value assignments are there for n atomic propositions $A_1, A_2, \ldots, A_n$?

How many truth value assignments are there for a single atomic propositions A? **2**

How many truth value assignments are there for two atomic propositions A and B? **4**

How many truth value assignments are there for three atomic propositions A, B, and C? **8**

How many truth value assignments are there for four atomic propositions A, B, C and D? **16**

How many truth value assignments are there for n atomic propositions $A_1, A_2, \ldots, A_n$? $\mathbf{2^n}$

Conjunction

Eric had steak and wine. $(S \,\&\, W)$

Conjunction

Eric had steak and wine. $(S \;\&\; W)$

S	W
T	T
T	F
F	T
F	F

Conjunction

Eric had steak and wine. $(S \,\&\, W)$

S	W	$(S \,\&\, W)$
T	T	
T	F	
F	T	
F	F	

Conjunction

Eric had steak and wine. $(S \,\&\, W)$

S	W	$(S \,\&\, W)$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction

Eric had steak and wine. $(S \,\&\, W)$

S	W	$(S \,\&\, W)$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction

Eric had steak and wine. $(S \,\&\, W)$

S	W	$(S \,\&\, W)$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction

Eric had steak and wine. $(S \,\&\, W)$

S	W	$(S \,\&\, W)$
T	T	T
T	F	F
F	T	F
F	F	F

Conjunction

Eric had steak and wine. $(S \,\&\, W)$

S	W	$(S \,\&\, W)$
T	T	T
T	F	F
F	T	F
F	F	F

Truth-Table for Conjunction

φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

Truth-Table for Conjunction

(	φ	&	ψ	)
	T	T	T	
	T	F	F	
	F	F	T	
	F	F	F	

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Disjunction

Eric had steak or wine. $(S \vee W)$

S	W	$(S \vee W)$
T	T	T
T	F	T
F	T	T
F	F	F

Truth-Table for Disjunction

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

Truth-Table for Disjunction

(	φ	$\vee$	ψ	)
	T	T	T	
	T	T	F	
	F	T	T	
	F	F	F	

Negation

Eric didn't have steak. $\neg S$

S	$\neg S$
T	F
F	T

Negation

Eric didn't have steak. $\neg S$

S	$\neg S$
T	F
F	T

Negation

Eric didn't have steak. $\neg S$

S	$\neg S$
T	F
F	T

Truth-Table for Negation

φ	$\neg\varphi$
T	F
F	T

Truth-Table for Negation

$\neg$	φ
F	T
T	F

Conditional

If Eric had steak, then he had wine. $(S \to W)$

S	W	$(S \to W)$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional

If Eric had steak, then he had wine. $(S \to W)$

S	W	$(S \to W)$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional

If Eric had steak, then he had wine. $(S \to W)$

S	W	$(S \to W)$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional

If Eric had steak, then he had wine. $(S \to W)$

S	W	$(S \to W)$
T	T	T
T	F	F
F	T	T
F	F	T

Conditional

If Eric had steak, then he had wine. $(S \to W)$

S	W	$(S \to W)$
T	T	T
T	F	F
F	T	T
F	F	T

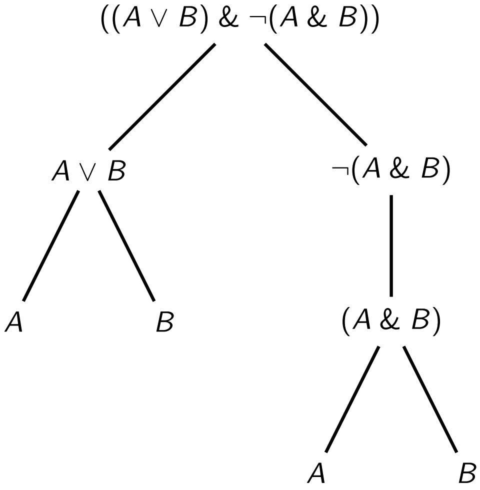
Truth-Table for the Conditional

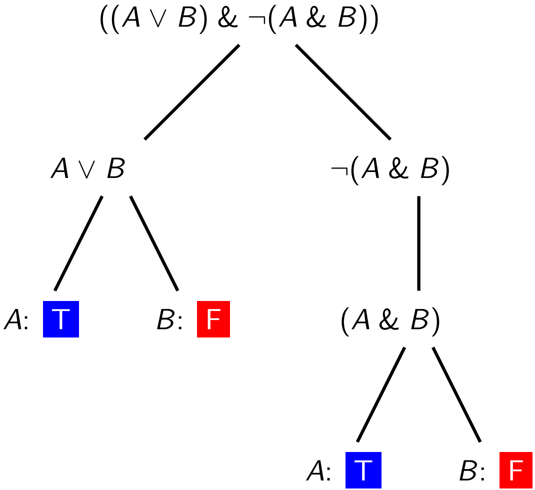
φ	ψ	$(\varphi \to \psi)$
T	T	T
T	F	F
F	T	T
F	F	T

Truth-Table for the Conditional

(	φ	$\rightarrow$	ψ	)
	T	T	T	
	T	F	F	
	F	T	T	
	F	T	F	

$$((A \vee B) \,\&\, \neg(A \,\&\, B))$$

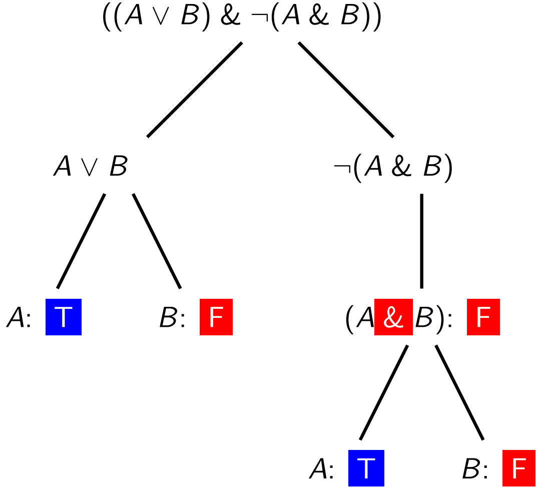

((A ∨ B) & ¬(A & B))
A ∨ B
¬(A & B)
A
B
(A & B)
A
B



φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

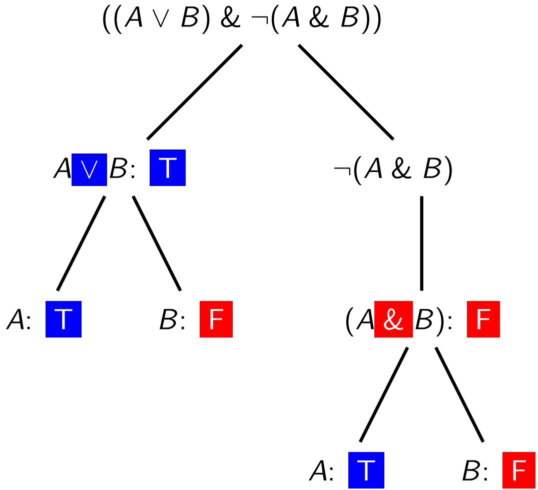
φ	$\neg\varphi$
T	F
F	T



φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

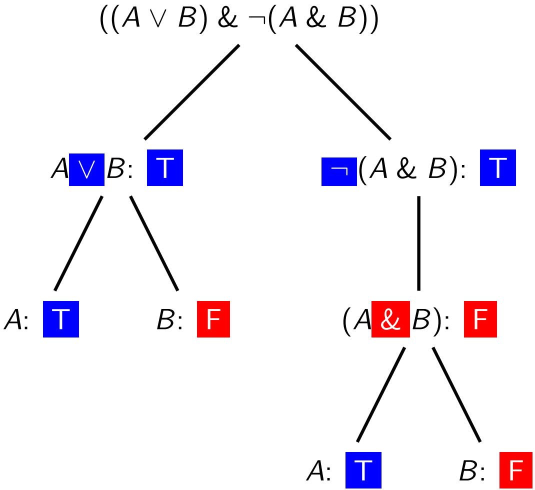
φ	$\neg\varphi$
T	F
F	T



φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

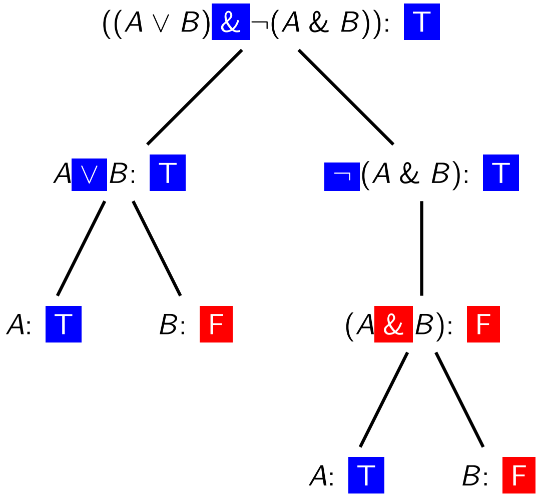
φ	$\neg\varphi$
T	F
F	T



φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

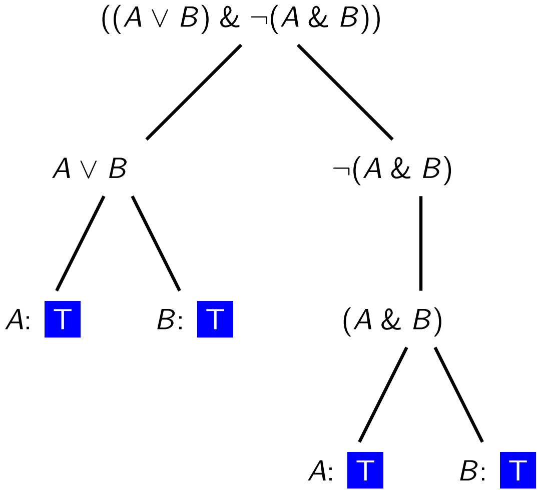
φ	$\neg\varphi$
T	F
F	T



φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

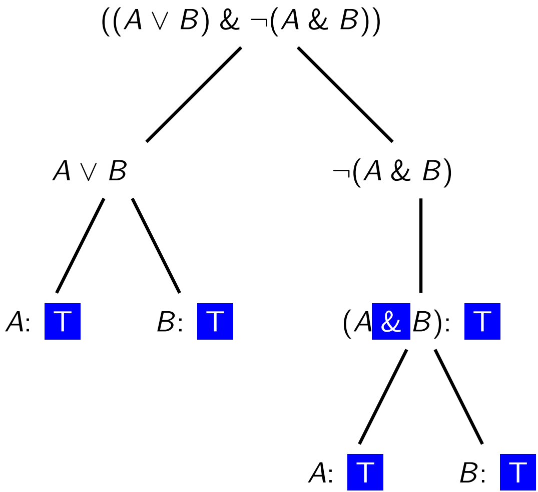
φ	$\neg\varphi$
T	F
F	T



φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

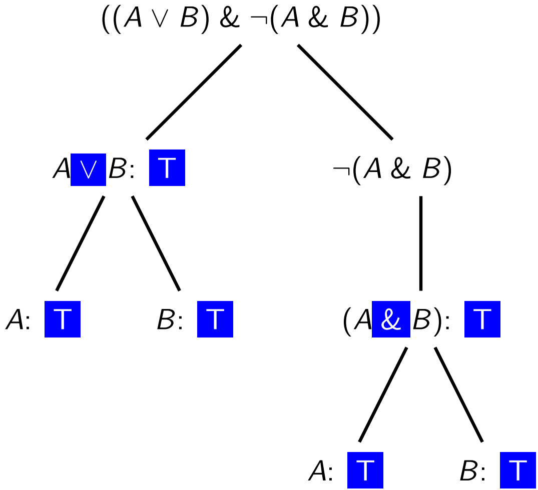
φ	$\neg\varphi$
T	F
F	T



φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

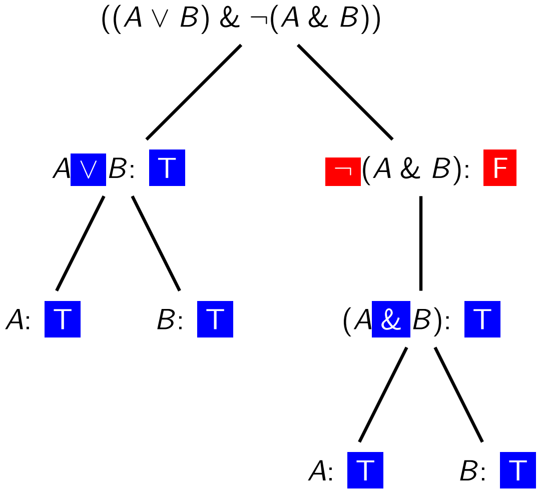
φ	$\neg\varphi$
T	F
F	T



φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

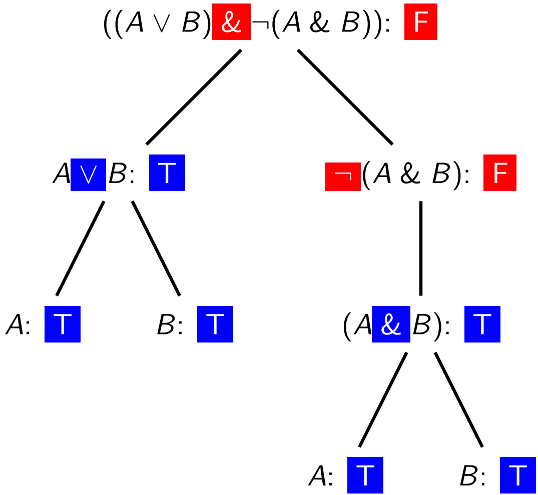
φ	$\neg\varphi$
T	F
F	T



φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	$\neg\varphi$
T	F
F	T



φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

φ	$\neg\varphi$
T	F
F	T

A	B	$(A \vee B)$	$(A \,\&\, B)$	$\neg(A \,\&\, B)$	$((A \vee B) \,\&\, \neg(A \,\&\, B))$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

A	B	$(A \vee B)$	$(A \,\&\, B)$	$\neg(A \,\&\, B)$	$((A \vee B) \,\&\, \neg(A \,\&\, B))$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

φ	ψ	$(\varphi \vee \psi)$
T	T	T
T	F	T
F	T	T
F	F	F

A	B	$(A \vee B)$	$(A \,\&\, B)$	$\neg(A \,\&\, B)$	$((A \vee B) \,\&\, \neg(A \,\&\, B))$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

φ	ψ	$(\varphi \,\&\, \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

A	B	$(A \vee B)$	$(A \,\&\, B)$	$\neg(A \,\&\, B)$	$((A \vee B) \,\&\, \neg(A \,\&\, B))$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

φ	$\neg\varphi$
T	F
F	T

A	B	$(A \vee B)$	$(A \mathbin{\&} B)$	$\neg(A \mathbin{\&} B)$	$((A \vee B) \mathbin{\&} \neg(A \mathbin{\&} B))$
T	T	T	T	F	F
T	F	T	F	T	T
F	T	T	F	T	T
F	F	F	F	T	F

φ	ψ	$(\varphi \mathbin{\&} \psi)$
T	T	T
T	F	F
F	T	F
F	F	F

(	(	A	$\vee$	B	)	&	$\neg$	(	A	&	B	)	)
		T	T	T		F	F		T	T	T		
		T	T	F		T	T		T	F	F		
		F	T	T		T	T		F	F	T		
		F	F	F		F	T		F	F	F		