Introduction to Logic

PHIL 170

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September 16, 2015

Announcements

- No problem sets this week.
- Read chapter 3.
- Quiz due Sunday, 11.59pm. (**Lots of questions, so start the quiz early!**).
- **Answer the "Did I get it?" and "Learn by doing" questions** (currently, only 51 out of 217 students have been working on this part of the course...)

Recap

- An argument is **valid** if it is impossible that the premises are all true and the conclusions is false.
- **Atomic formulae** correspond to sentences that, *from the point of view of sentential logic*, have no logically relevant internal structure.
- **Logical connectives** connect formulae in order to create new and more complex formulae. We start with four logical connectives: conjunction, disjunction, the conditional, and negation.
- Conjunction (&), disjunction ($\vee$) and negation ($\neg$)

Conditional, I

If Ann had coffee, then Bob had tea.

Conditional, I

If Ann had coffee, then Bob had tea.

If [Ann had coffee]$_1$, then [Bob had tea]$_2$.

Conditional, I

If Ann had coffee, then Bob had tea.

If [Ann had coffee]$_1$, then [Bob had tea]$_2$.

C	Ann had coffee.
T	Bob had tea.

If C, then T.

Conditional, I

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If [Ann had coffee]$_1$, then [Bob had tea]$_2$.

C	Ann had coffee.
T	Bob had tea.

If C, then T.

$C \to T$

Conditional, II

- If John ran, then Mary laughed.
- If John ran, Mary laughed.
- Mary laughed, provided that John ran.
- Given that John ran, Mary laughed.
- Mary laughed if John ran.
- John ran only if Mary laughed.

Conditional, II

- If John ran, then Mary laughed. $(J \rightarrow M)$
- If John ran, Mary laughed. $(J \rightarrow M)$
- Mary laughed, provided that John ran. $(J \rightarrow M)$
- Given that John ran, Mary laughed. $(J \rightarrow M)$
- Mary laughed if John ran. $(J \rightarrow M)$
- John ran only if Mary laughed. $(J \rightarrow M)$

Modus Ponens

$$\frac{\begin{array}{c} P \\ P \to Q \end{array}}{Q}$$

Exclusive Disjunction

Eric will have steak or fish.

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Either Eric will have steak, or Eric will have fish (but not both).

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Either Eric will have steak, or Eric will have fish (but not both).

Either Eric will have steak, or Eric will have fish (but it is not the case that Eric will have steak and Eric will have fish).

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$(S \vee F) \,\&\, \neg(S \,\&\, F)$

Unless

John will pick up Henry at the airport, unless Mary does it.

Unless

John will pick up Henry at the airport, unless Mary does it.

If Mary doesn't pick up Henry at the airport, then John will pick up Henry at the airport.

Unless

John will pick up Henry at the airport, unless Mary does it.

If Mary doesn't pick up Henry at the airport, then John will pick up Henry at the airport.

$\neg M \to J$

Practice

1. I will graduate this semester only if I pass intro logic.
2. Although it is raining, I plan to go jogging this afternoon.
3. Ann and Bob are Republicans, but they both hate Nixon.
4. If Ann will go only if Bob does not go, then either we will cancel the trip or we will not invite Jay
5. Both Ann and Bob will go to the beach this weekend, provided that neither of them is sick.
6. If I study too hard I will not enjoy college, but at the same time I will not enjoy college if I flunk out.
7. Neither Ann nor Bob is able to attend the meeting.
8. I will not graduatate if I don't pass both Logic and Philosophy.
9. Unless logic is easy, I will pass only if I study.

Practice, I

I will graduate this semester only if I pass intro logic.

If I will graduate this semester (G), then I will pass intro logic (P).

$G \to P$

Practice, I

I will graduate this semester only if I pass intro logic.

If I will graduate this semester (G), then I will pass intro logic (P).

$G \to P$

Although it is raining, I plan to go jogging this afternoon.

It is raining this afternoon (R) and I plan to go jogging this afternoon (J).

$R \,\&\, J$

Practice, I

I will graduate this semester only if I pass intro logic.

If I will graduate this semester (G), then I will pass intro logic (P).

$G \to P$

Although it is raining, I plan to go jogging this afternoon.

It is raining this afternoon (R) and I plan to go jogging this afternoon (J).

$R \,\&\, J$

Ann and Bob are Republicans, but they both hate Nixon.

Ann is a Republican (A) and Bob is a Republican (B); and
Ann hates Nixon (H) and Bob hates Nixon (N).

$(A \,\&\, B) \,\&\, (H \,\&\, N)$

Practice, II

If Ann will go only if Bob does not go, then either we will cancel the trip or we will not invite Jay

If it is the case that, if Ann will go on the trip (A), then Bob will not go on the trip (B), then either we will cancel the trip (C) or we will not invite Bob (I).

$(A \to \neg B) \to (C \vee \neg I)$

Practice, II

If Ann will go only if Bob does not go, then either we will cancel the trip or we will not invite Jay

If it is the case that, if Ann will go on the trip (A), then Bob will not go on the trip (B), then either we will cancel the trip (C) or we will not invite Bob (I).

$(A \rightarrow \neg B) \rightarrow (C \vee \neg I)$

Both Ann and Bob will go to the beach this weekend, provided that neither of them is sick.

If Ann is not sick (S) and Bob is not sick (T), then Ann will go to the beach (A) and Bob will go to the beach (B).

$(\neg S \,\&\, \neg T) \rightarrow (J \,\&\, K)$

Practice, III

If I study too hard I will not enjoy college, but at the same time I will not enjoy college if I flunk out.

If I study too hard (S), then I will not enjoy college (E),

and if I flunk out of college (F), then I will not enjoy college (E).

$(S \to \neg E)\ \&\ (F \to \neg E)$

Practice, III

If I study too hard I will not enjoy college, but at the same time I will not enjoy college if I flunk out.

If I study too hard (S), then I will not enjoy college (E),

and if I flunk out of college (F), then I will not enjoy college (E).

$(S \to \neg E) \;\&\; (F \to \neg E)$

Neither Ann nor Bob is able to attend the meeting.

Ann is not able to attend the meeting (A) and Bob is not able to attend the meeting (B).

$\neg A \;\&\; \neg B$ (or: $\neg(A \vee B)$)

Practice, IV

I will not graduatate if I don't pass both Logic and Philosophy.

If it is not the case that I pass logic (L) and I pass Philosophy (P), then I will not graduate (G).

$\neg(L \,\&\, P) \rightarrow \neg G$

Practice, IV

I will not graduatate if I don't pass both Logic and Philosophy.

If it is not the case that I pass logic (L) and I pass Philosophy (P),

then I will not graduate (G).

$\neg(L \,\&\, P) \rightarrow \neg G$

Unless logic is easy, I will pass only if I study.

If logic is not easy (E), then it is the case that

if I pass logic (P), then I study for the exams (S).

$\neg E \rightarrow (P \rightarrow S)$

Expressions

Basic Expressions

- The capital letters we use to symbolize atomic sentences, called atomic formulae or occasionally sentential letters: A, B, C, and so on (possibly with numeric subscripts).
- The symbols for the logical connectives: & , $\vee$, $\rightarrow$, and $\neg$
- The parentheses used to disambiguate the scope of the connectives: (and).

Expressions

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- The capital letters we use to symbolize atomic sentences, called atomic formulae or occasionally sentential letters: A, B, C, and so on (possibly with numeric subscripts).
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Expressions of Sentential Logic

Any finite sequence or string of basic expressions is an expression of sentential logic.

Formulae

1. Every atomic formula φ is a formula of sentential logic.
2. If φ is a formula of sentential logic, then so is $\neg\varphi$.
3. If φ and ψ are formulae of sentential logic, then so are each of the following:
   a. $(\varphi \,\&\, \psi)$
   b. $(\varphi \vee \psi)$
   c. $(\varphi \to \psi)$
4. An expression of sentential logic is a formula only if it can be constructed by one or more applications of the first three rules.

Expressions: $A \& B$, $\neg A \neg\neg \& B$, $(A \vee \neg, \rightarrow A \neg B$, ...

Formulae: $\neg A$, $\neg\neg A$, $A \& B$, $\neg A \& B$, $(A \vee \neg A)$, $A \rightarrow \neg B$, $\neg(A \,\&\, \neg B) \rightarrow (C \vee \neg D)$, ...

Why are $A \,\&\, B$, $\neg A \neg\neg \& B$, $(A \vee \neg$, and $\to A \neg B$ expressions?

Why are $A \,\&\, B$, $\neg A \neg\neg \& B$, $(A \vee \neg$, and $\rightarrow A \neg B$ expressions?

Answer: Because they are strings of atomic formulae and logical connectives.

Why is $\neg(A \,\&\, \neg B) \rightarrow (C \vee \neg D)$ a formula?

- A, B, C and D are all formulae (since they are atomic formulae)

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- A, B, C and D are all formulae (since they are atomic formulae)
- Since B and D are formula, so is $\neg B$ and $\neg D$
- Since A and $\neg B$ are formulae then so is $A \,\&\, \neg B$

Why is $\neg(A \,\&\, \neg B) \to (C \vee \neg D)$ a formula?

- A, B, C and D are all formulae (since they are atomic formulae)
- Since B and D are formula, so is $\neg B$ and $\neg D$
- Since A and $\neg B$ are formulae then so is $A \,\&\, \neg B$
- Since C and $\neg D$ are formulae, then so is $C \vee \neg D$

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- Since A and $\neg B$ are formulae then so is $A \,\&\, \neg B$
- Since C and $\neg D$ are formulae, then so is $C \vee \neg D$
- Since $A \,\&\, \neg B$ is a formula, then so is $\neg(A \,\&\, \neg B)$

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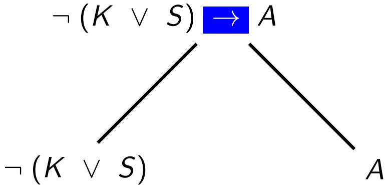
- Since B and D are formula, so is $\neg B$ and $\neg D$
- Since A and $\neg B$ are formulae then so is $A \,\&\, \neg B$
- Since C and $\neg D$ are formulae, then so is $C \vee \neg D$
- Since $A \,\&\, \neg B$ is a formula, then so is $\neg(A \,\&\, \neg B)$
- Since $\neg(A \,\&\, \neg B)$ and $C \vee \neg D$ are formulae, then so is $\neg(A \,\&\, \neg B) \rightarrow (C \vee \neg D)$

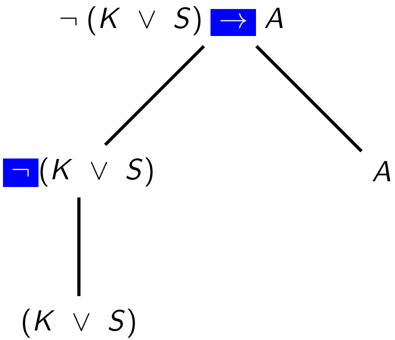
He has an Ace if he does not have a Knight or a Spade.

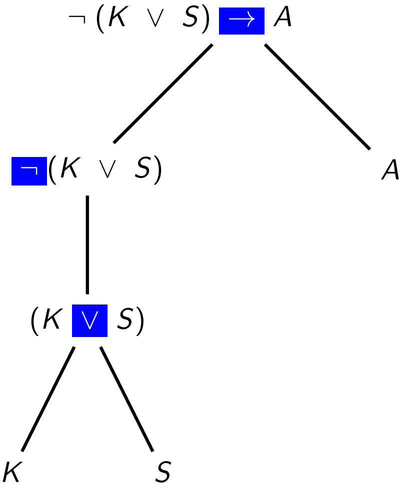
$$\neg(K \vee S) \to A$$

$$\neg(K \vee S) \to A$$

$$\neg(K \lor S) \boxed{\rightarrow} A$$

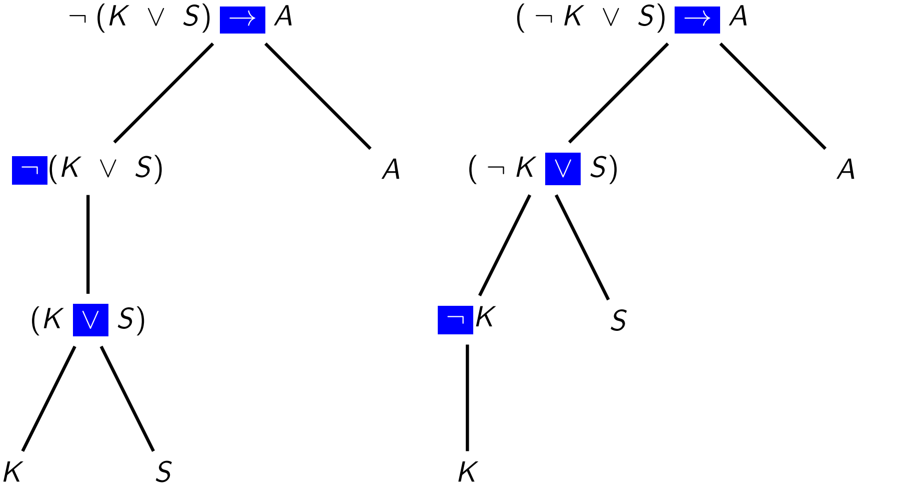

$\neg(K \vee S) \boxed{\rightarrow} A$
$\neg(K \vee S)$
A


$\neg(K \vee S) \rightarrow A$
$\neg(K \vee S)$
A
$(K \vee S)$

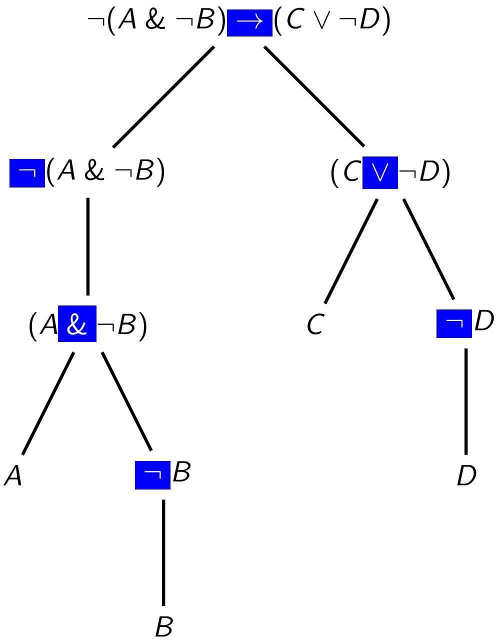

$\neg(K \vee S) \boxed{\rightarrow} A$
$\boxed{\neg}(K \vee S)$
A
$(K \boxed{\vee} S)$
K
S

$\neg(K \vee S) \rightarrow A$

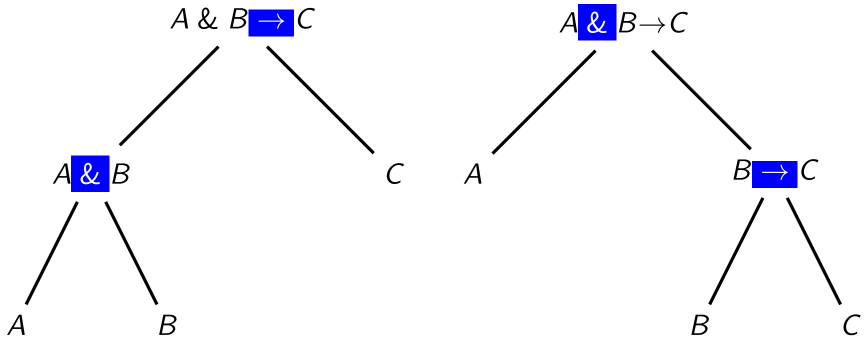
$(\neg K \vee S) \rightarrow A$

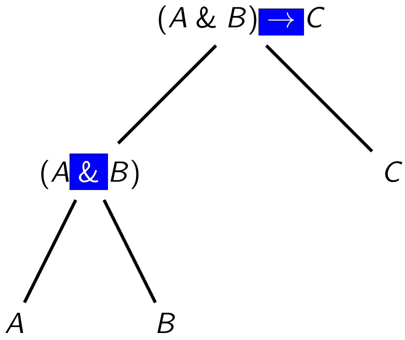

¬(K ∨ S) → A
¬(K ∨ S)
A
(K ∨ S)
K
S
(¬K ∨ S) → A
(¬K ∨ S)
A
¬K
S
K

Draw the parse tree for $\neg(A \,\&\, \neg B) \to (C \vee \neg D)$.


$\neg(A \,\&\, \neg B) \rightarrow (C \vee \neg D)$
$\neg(A \,\&\, \neg B)$
$(C \vee \neg D)$
$(A \,\&\, \neg B)$
C
$\neg D$
A
$\neg B$
D
B

$$A \mathbin{\&} B \to C$$


A & B → C
A & B
C
A
B
A & B → C
A
B → C
B
C


$(A \,\&\, B) \rightarrow C$
$(A \,\&\, B)$
C
A
B

Find the parse trees

$$A \mathbin{\&} B \mathbin{\&} C$$

$$A \mathbin{\&} B \vee C$$

$$\neg A \to C$$

$$\neg A \mathbin{\&} B \to C$$

$$\neg A \mathbin{\&} B \vee C$$

Fun problems to think about...

Imagine you have been transported to the mysterious "Island of Knights and Knaves" where every inhabitant is either a knight or a knave. Every knight always tells the truth and every knave always lies.

You stop for gas and the attendant says, "Either I am a knight or I am not a knight." What is he?

Suppose that you see a tall person and a short person. Suppose that you are given the following information: "The tall one is a knave and/or the short one is a knight." What can you conclude about the two people?

Fun problems to think about...

There are three suspects for a murder: Adams, Brown and Clark. Adams says "I didn't do it. The victim was an old acquaintance of Brown's. But Clark hated him." Brown states "I didn't do it. I didn't even know the guy. Besides I was out of town all that week." Clark says "I didn't do it. I saw both Adams and Brown downtown with the victim that day. One of them must have done it." Assume that the two innocent men are telling the truth, but that the guilty man might not be. Who did it?

Fun problems to think about...

An advertisement for a tennis magazine states, “If I’m not playing tennis, I’m watching tennis. And if I’m not watching tennis, I’m reading about tennis.” We can assume that the speaker cannot do more than one of these activities at a time. What is the speaker doing?