

# Introduction to Logic

## PHIL 170

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# Announcements

- ▶ No problem sets this week.
- ▶ Read chapter 3 (Answer the “Did I get it?” questions before Wednesday).
- ▶ Quiz due Sunday, 11.59pm. (Lots of questions, so start the quiz early!).
- ▶ Problems with LogicLab...
- ▶ Extra credit for Chapter 2: Up to 4 extra points towards the problem sets based on the percentage of activity for Chapter 2 (the “Did I get it?” and “Learning by doing” questions).

# Recap

- ▶ An argument is a set of statements, one of which is singled out as the conclusion, the other statements are called premises.
- ▶ An argument is **valid** if it is impossible that the premises are all true and the conclusions is false.
- ▶ An argument is **sound** if it is valid and all the premises are true.
- ▶ Introduction to logical form: Structure vs. unstructured statements
- ▶ Identifying inference patterns: Modus Ponens, Modus Tollens, Adjunction, Disjunctive Syllogism, Affirming the Consequent, ...

# Inference Patterns, I

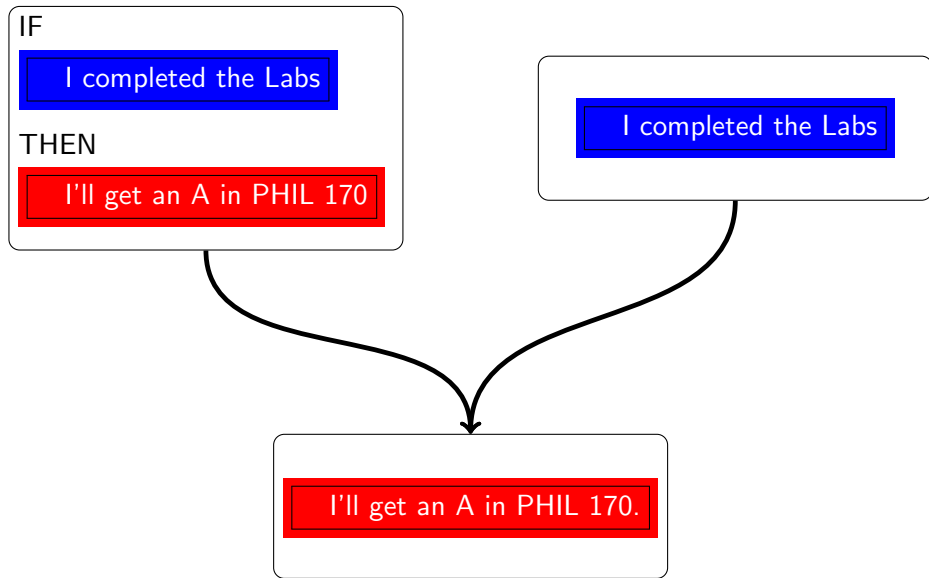
- ▶ Modus Ponens
- ▶ Modus Tollens
- ▶ Disjunctive Syllogism (left), Disjunctive Syllogism (right)
- ▶ Simplification (left), Simplification (right)
- ▶ Addition (left), Addition (right)

# Inference Patterns, II

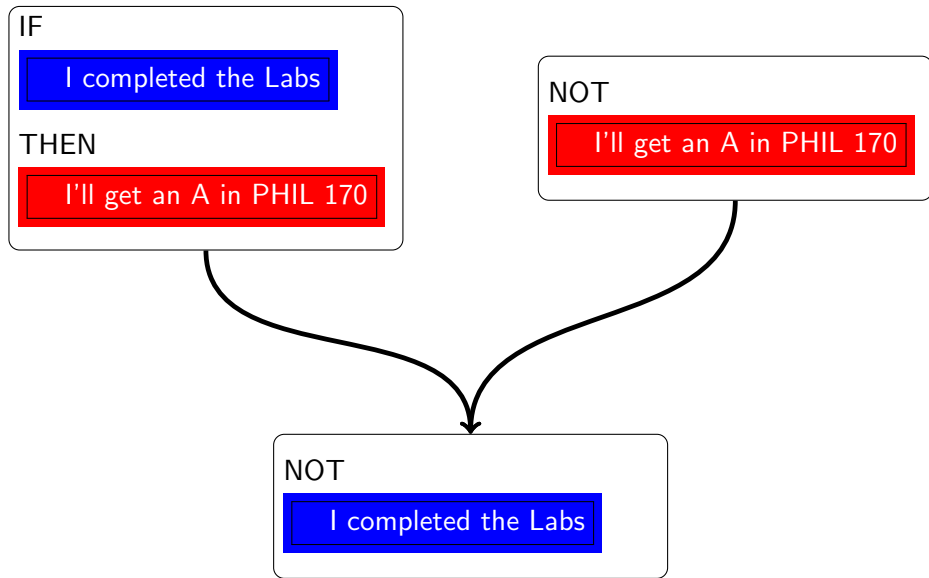
- ▶ Denying the Antecedent
- ▶ Affirming the Consequent
- ▶ Affirming a (Left) Disjunct, Affirming a (Right) Disjunct
- ▶ Denying a (Left) Conjunct, Denying a (Right) Conjunct

# Valid inference patterns

# Modus Ponens

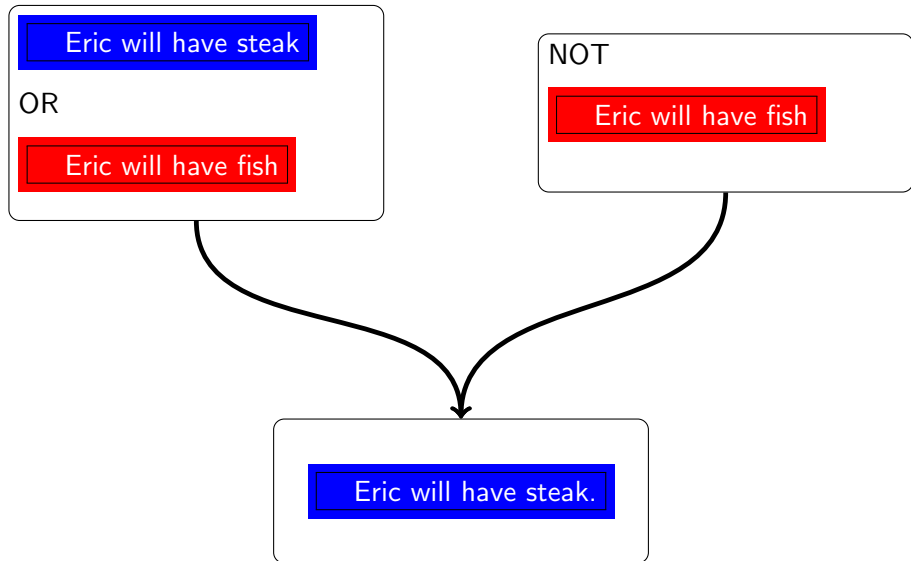


# Modus Tollens

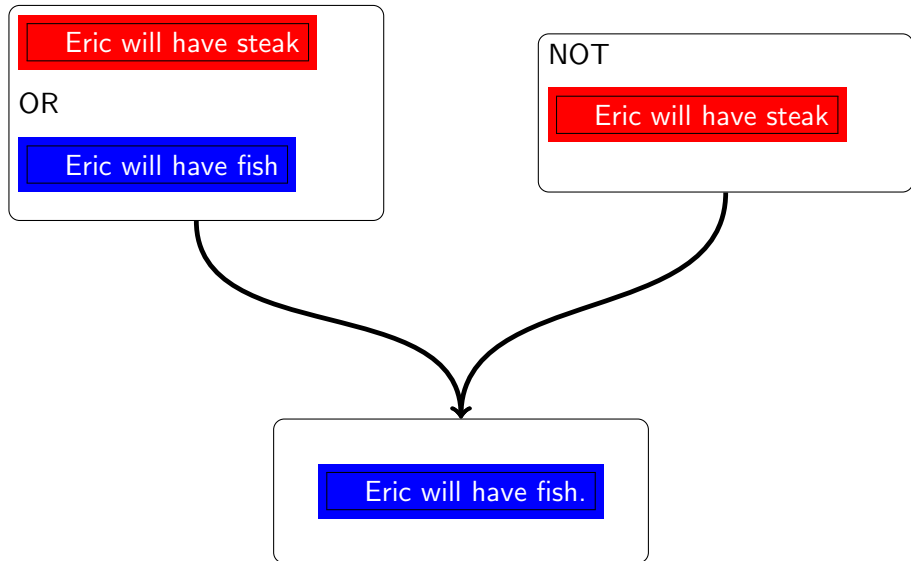




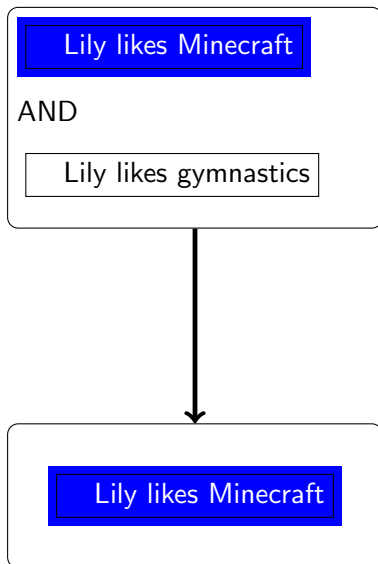
## Disjunctive Syllogism (left)



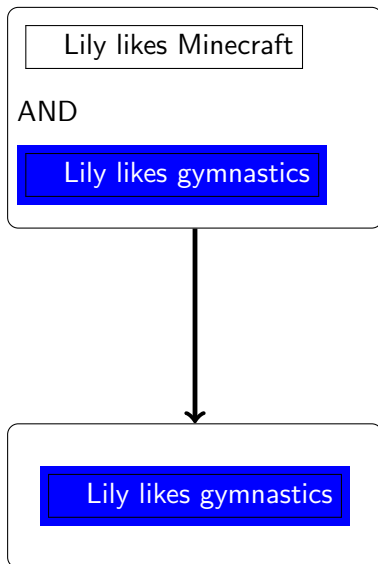
## Disjunctive Syllogism (right)



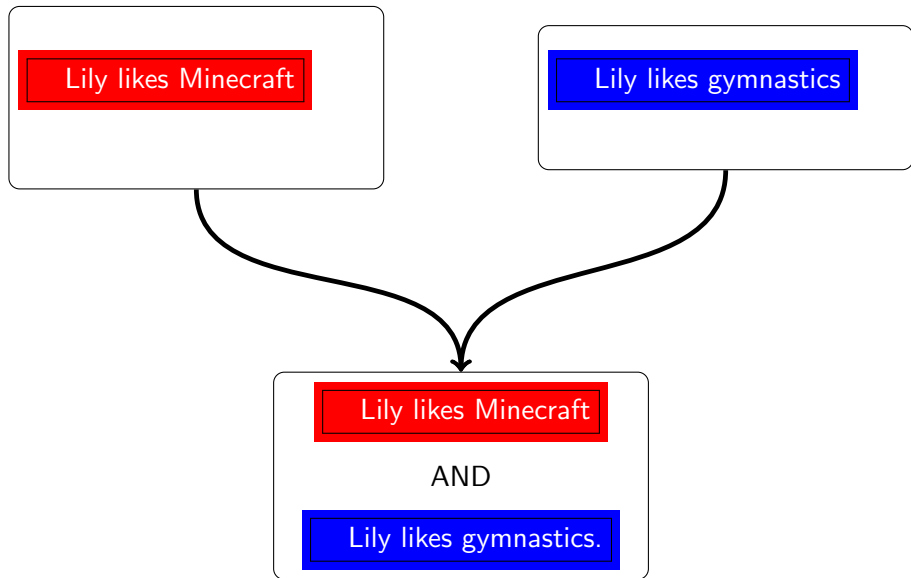
## Simplification (left)



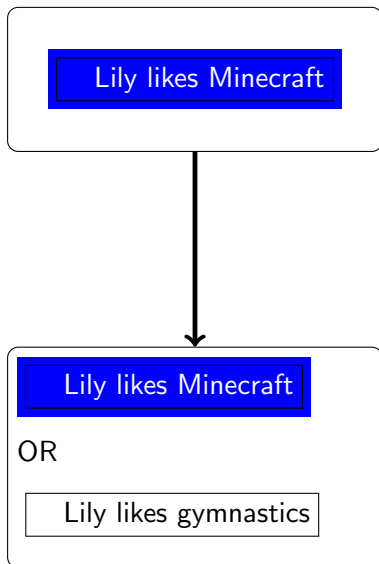
## Simplification (right)



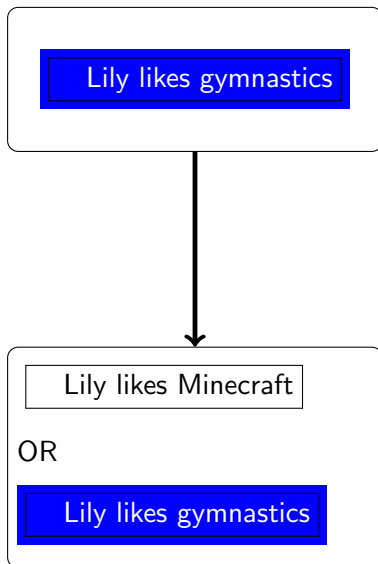
## Adjunction



## Addition (right)



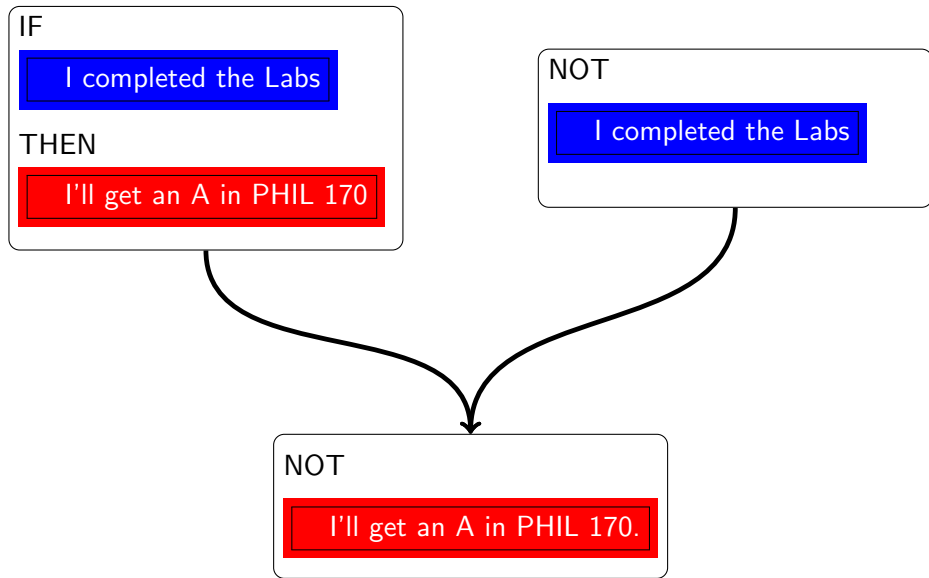
## Addition (left)



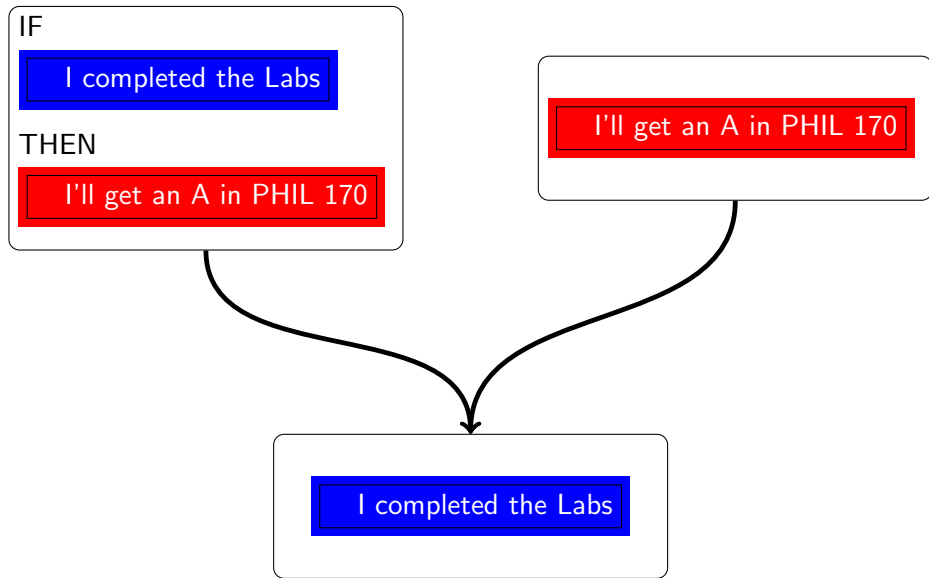
# Invalid inference patterns



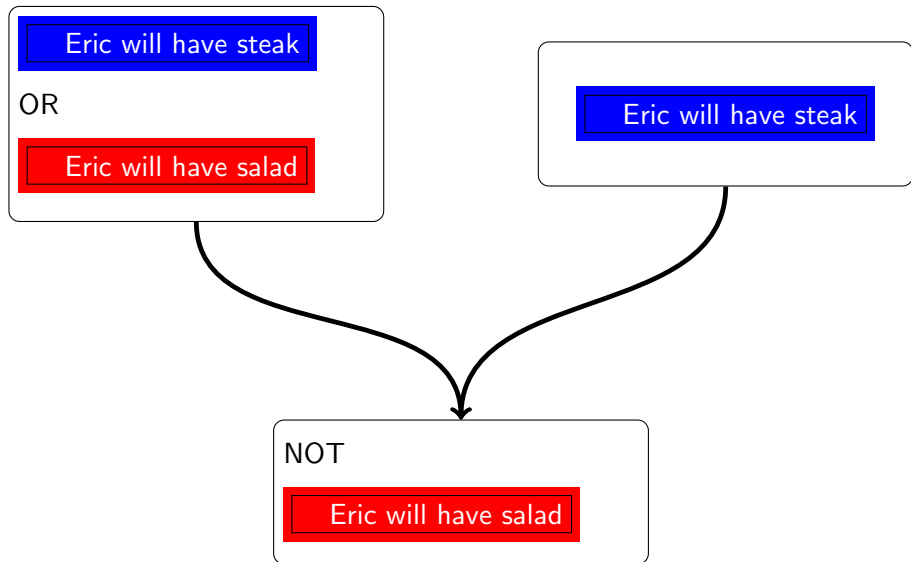
## Denying the Antecedent



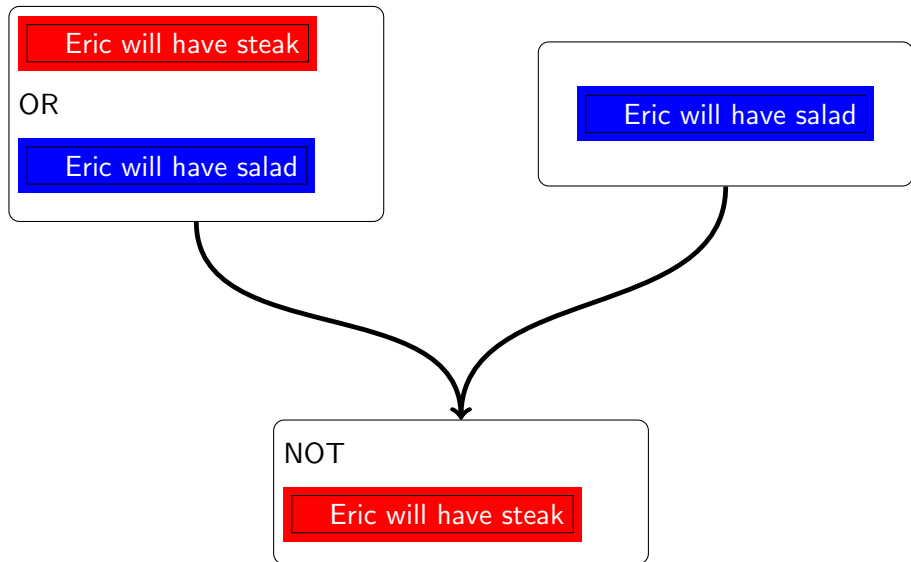
## Affirming the Consequent



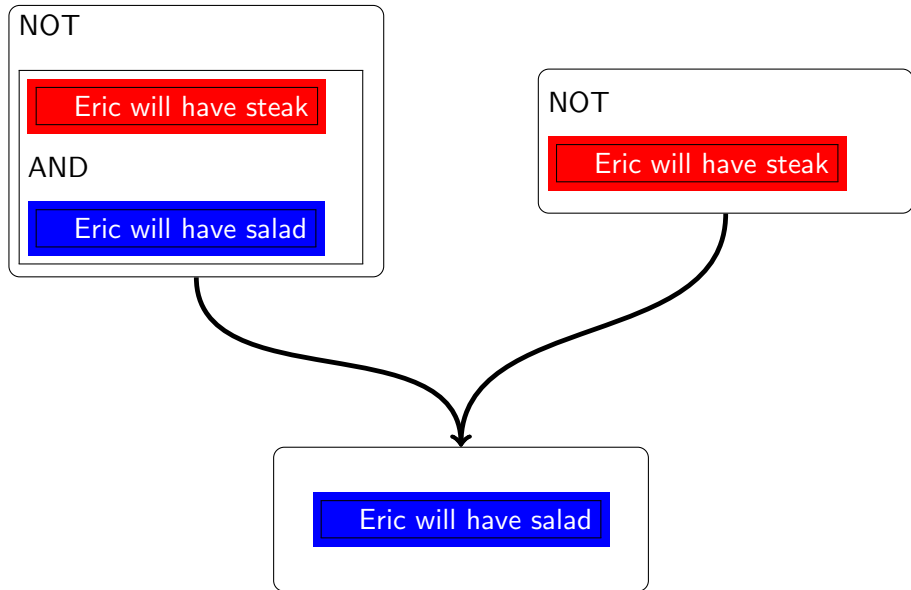
## Affirming a Disjunct (left)



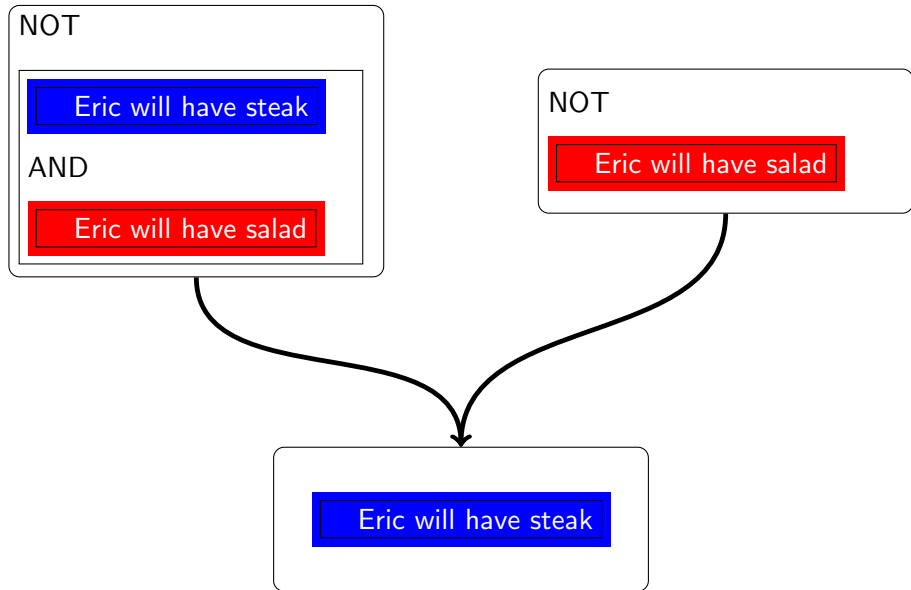
## Affirming a Disjunct (right)



## Denying a Conjunct (left)



## Denying a Conjunct (left)



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Formulas of sentential logic correspond to statements (and so to English sentences that express those statements)

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# Sentential Logic: Atomic Formulae and Logical Connectives

Formulas of sentential logic correspond to statements (and so to English sentences that express those statements)

Two categories of words or basic expressions: **atomic formulae** and **logical connectives**.

**Atomic formulae** correspond to sentences that, *from the point of view of sentential logic*, have no logically relevant internal structure.

**Logical connectives** connect formulae in order to create new and more complex formulae. We start with four logical connectives: conjunction, disjunction, the conditional, and negation.

# Atomic Sentences

1. John ran.
2. Mary laughed.
3. Harry said that Mary laughed.
4. John thinks that Mary laughed at his running.
5. John ran and Mary laughed.
6. Either John ran, or Mary laughed.
7. If Mary laughed, then John ran.
8. John didn't run.
9. It is not the case that Mary laughed.

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# Identifying Atomic Sentences

Eric believes that the LogicLab is a wonderful program.

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What is the relevant difference between these two sentences?

# Identifying Atomic Sentences

Eric believes that the LogicLab is a wonderful program.

It is not the case that the LogicLab is a wonderful program.

What is the relevant difference between these two sentences? Logical operators are **truth-functional**.



# Conjunction, I

Ann had coffee and Bob had tea.

# Conjunction, I

Ann had coffee and Bob had tea.

$[ \text{Ann had coffee} ]_1$  and  $[ \text{Bob had tea} ]_2$ .

# Conjunction, I

Ann had coffee and Bob had tea.

$[ \text{Ann had coffee} ]_1$  and  $[ \text{Bob had tea} ]_2$ .

$C$  and  $T$ .

$C$	Ann had coffee.
$T$	Bob had tea.

# Conjunction, I

Ann had coffee and Bob had tea.

$[ \text{Ann had coffee} ]_1$  and  $[ \text{Bob had tea} ]_2$ .

$C$	Ann had coffee.
$T$	Bob had tea.

$C$  and  $T$

$C \ \& \ T$

## Conjunction, II

Ann and Bob had cake.

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## Conjunction, II

Ann and Bob had cake.

Ann had cake and Bob had cake.

$[Ann\ had\ cake]_1$  and  $[Bob\ had\ cake]_2$ .

$A$	Ann had cake.
$B$	Bob had cake.

$A$  and  $B$ .

$A \& B$

# Conjunction, III

## Non-Truth-Functional Conjunction

- ▶ Ann and Bob fell in love.



# Conjunction, III

## Non-Truth-Functional Conjunction

- ▶ Ann and Bob fell in love. ( $L$ )

## Not-Just-Truth-Functional Conjunction

- ▶ Ann had a drink and went to the police station.
- ▶ Ann went to the police station and had a drink.

# Conjunction, III

## Non-Truth-Functional Conjunction

- ▶ Ann and Bob fell in love. ( $L$ )

## Not-Just-Truth-Functional Conjunction

- ▶ Ann had a drink and went to the police station. ( $D \ \& \ P$ )
- ▶ Ann went to the police station and had a drink. ( $P \ \& \ D$ )

# Conjunction, IV

## Conjunction Words other Than “And”

- ▶ The cat is napping, but the dog is chasing his tail.
- ▶ Although the cat is sharpening her claws on the dog, the dog is sleeping soundly.
- ▶ The cat is purring, though the dog is howling.
- ▶ Mary has just taken the dog to the vet; however, the dog's appointment is tomorrow.
- ▶ Mary is fond of cats, whereas John likes dogs.

# Conjunction, IV

## Conjunction Words other Than “And”

- ▶ The cat is napping, **but** the dog is chasing his tail. (*N & C*)
- ▶ **Although** the cat is sharpening her claws on the dog, the dog is sleeping soundly. (*S & D*)
- ▶ The cat is purring, **though** the dog is howling. (*P & H*)
- ▶ Mary has just taken the dog to the vet; **however**, the dog's appointment is tomorrow. (*V & A*)
- ▶ Mary is fond of cats, **whereas** John likes dogs. (*M & J*)

# Adjunction

$$\frac{P \quad Q}{P \& Q}$$

# Simplification

$$\frac{P \& Q}{P}$$

$$\frac{P \& Q}{Q}$$

# Dijunction, I

Ann had coffee or Bob had tea.

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# Dijunction, I

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$C$  or  $T$ .

$C$	Ann had coffee.
$T$	Bob had tea.

# Dijunction, I

Ann had coffee or Bob had tea.

$[\text{Ann had coffee}]_1$  or  $[\text{Bob had tea}]_2$ .

$C$	Ann had coffee.
$T$	Bob had tea.

$C$  or  $T$

$C \vee T$

# Disjunction, II

## Non-Sentential Disjunction

- ▶ Either John or Mary laughed.
- ▶ Mary either laughed or sneezed.

# Disjunction, II

## Non-Sentential Disjunction

- ▶ Either John or Mary laughed. ( $J \vee M$ )
- ▶ Mary either laughed or sneezed. ( $L \vee S$ )

# Disjunction, II

## Non-Sentential Disjunction

- ▶ Either John or Mary laughed. ( $J \vee M$ )
- ▶ Mary either laughed or sneezed. ( $L \vee S$ )

## Inclusive vs. Exclusive “Or”

- ▶ Ann will have steak or Bob will have fish.
- ▶ Either Mary will buy ice cream, or John will buy ice cream.
- ▶ Ann will have steak or fish.
- ▶ Either Ann will get an A or a B in PHIL 170

# Addition

$$\frac{P}{P \vee Q}$$

$$\frac{Q}{P \vee Q}$$

## Conditional, I

If Ann had coffee, then Bob had tea.

## Conditional, I

If Ann had coffee, then Bob had tea.

If [Ann had coffee]<sub>1</sub>, then [Bob had tea]<sub>2</sub>.



# Conditional, I

If Ann had coffee, then Bob had tea.

If [Ann had coffee]<sub>1</sub>, then [Bob had tea]<sub>2</sub>.

$C$	Ann had coffee.
$T$	Bob had tea.

If  $C$ , then  $T$ .

# Conditional, I

If Ann had coffee, then Bob had tea.

If [Ann had coffee]<sub>1</sub>, then [Bob had tea]<sub>2</sub>.

$C$	Ann had coffee.
$T$	Bob had tea.

If  $C$ , then  $T$ .

$$C \rightarrow T$$

## Conditional, II

- ▶ If John ran, then Mary laughed.
- ▶ If John ran, Mary laughed.
- ▶ Mary laughed, provided that John ran.
- ▶ Given that John ran, Mary laughed.
- ▶ Mary laughed if John ran.
- ▶ John ran only if Mary laughed.

## Conditional, II

- ▶ If John ran, then Mary laughed. ( $J \rightarrow M$ )
- ▶ If John ran, Mary laughed. ( $J \rightarrow M$ )
- ▶ Mary laughed, provided that John ran. ( $J \rightarrow M$ )
- ▶ Given that John ran, Mary laughed. ( $J \rightarrow M$ )
- ▶ Mary laughed if John ran. ( $J \rightarrow M$ )
- ▶ John ran only if Mary laughed. ( $J \rightarrow M$ )

# Modus Ponens

$$\frac{P \quad P \rightarrow Q}{Q}$$

# Negation

Ann didn't have coffee.

# Negation

Ann didn't have coffee.

It's not the case that Ann had coffee.

# Negation

Ann didn't have coffee.

It's not the case that Ann had coffee.

$C$	Ann had coffee.
-----	-----------------

It's not the case that  $[Ann\ had\ coffee]_1$ .



# Negation

Ann didn't have coffee.

It's not the case that Ann had coffee.

$C$	Ann had coffee.
-----	-----------------

It's not the case that [Ann had coffee]<sub>1</sub>.

$\neg C$