Reasoning about Knowledge and Beliefs

Lecture 20

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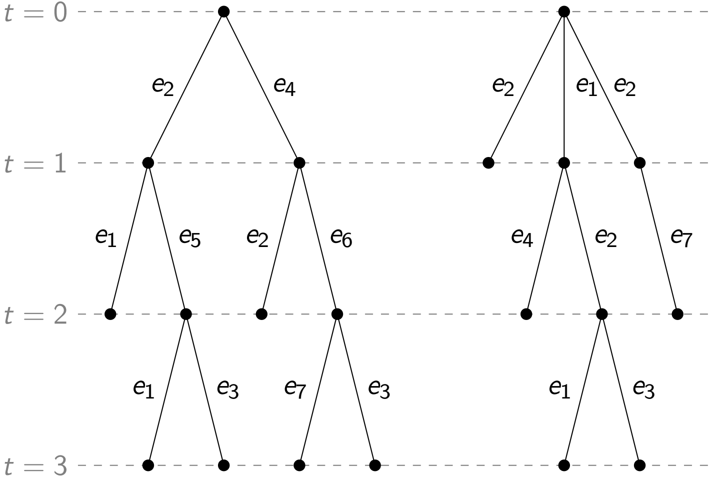
V. Goranko and EP. *Temporal Aspects of the Dynamics of Knowledge*. 2013.

Epistemic Temporal Logic

R. Parikh and R. Ramanujam. *A Knowledge Based Semantics of Messages*. *Journal of Logic, Language and Information*, 12: 453 – 467, 1985, 2003.

FHMV. Reasoning about Knowledge. MIT Press, 1995.

The 'Playground'



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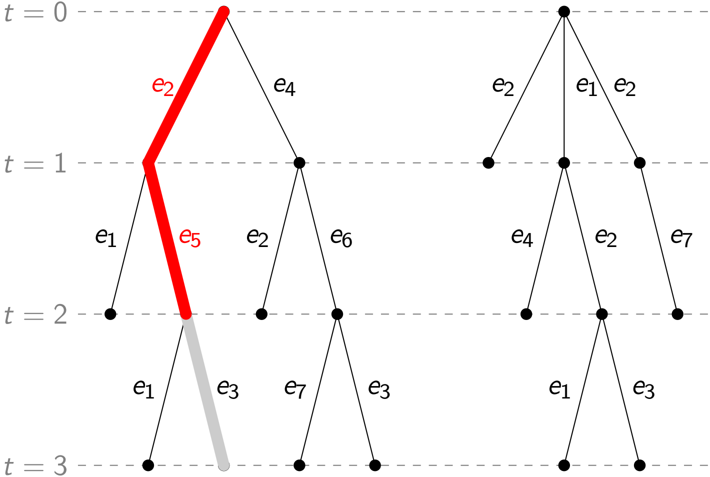

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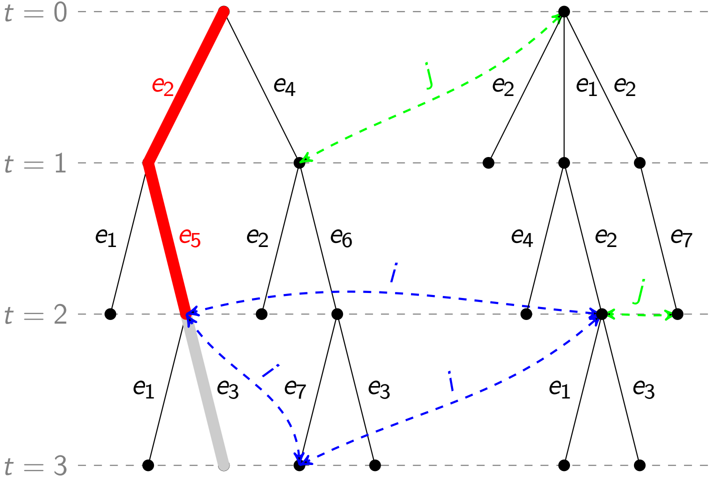

Formal Languages

- $P\varphi$ (φ is true *sometime* in the past),
- $F\varphi$ (φ is true *sometime* in the future),
- $Y\varphi$ (φ is true at *the* previous moment),
- $N\varphi$ (φ is true at *the* next moment),
- $N_e\varphi$ (φ is true after event e)
- $K_i\varphi$ (agent i knows φ) and
- $C_B\varphi$ (the group $B \subseteq \mathcal{A}$ commonly knows φ).

History-based Models

An ETL **model** is a structure $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}, V\rangle$ where $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}\rangle$ is an ETL frame and

$V : \mathsf{At} \to 2^{\mathsf{finite}(\mathcal{H})}$ is a valuation function.

Formulas are interpreted at pairs H, t:

$$H, t \models \varphi$$

Truth in a Model

- $H, t \models P\varphi$ iff there exists $t' \leq t$ such that $H, t' \models \varphi$
- $H, t \models F\varphi$ iff there exists $t' \geq t$ such that $H, t' \models \varphi$
- $H, t \models N\varphi$ iff $H, t+1 \models \varphi$
- $H, t \models Y\varphi$ iff $t > 1$ and $H, t-1 \models \varphi$
- $H, t \models K_i\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_i H'_m$ then $H', m \models \varphi$
- $H, t \models C\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_* H'_m$ then $H', m \models \varphi$.

where $\sim_*$ is the reflexive transitive closure of the union of the $\sim_i$.

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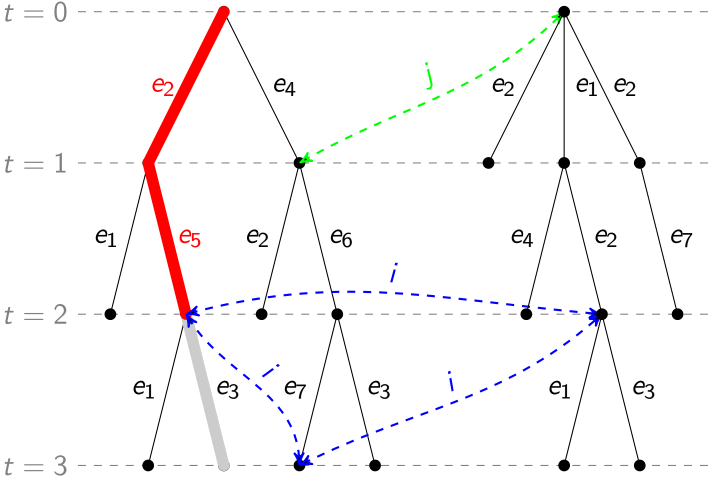
$t=0$
$t=1$
$t=2$
$t=3$
e_2
e_4
j
e_2
e_1
e_2
e_1
e_5
e_2
e_6
e_4
e_2
e_7
i
e_1
e_3
i
e_7
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An Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

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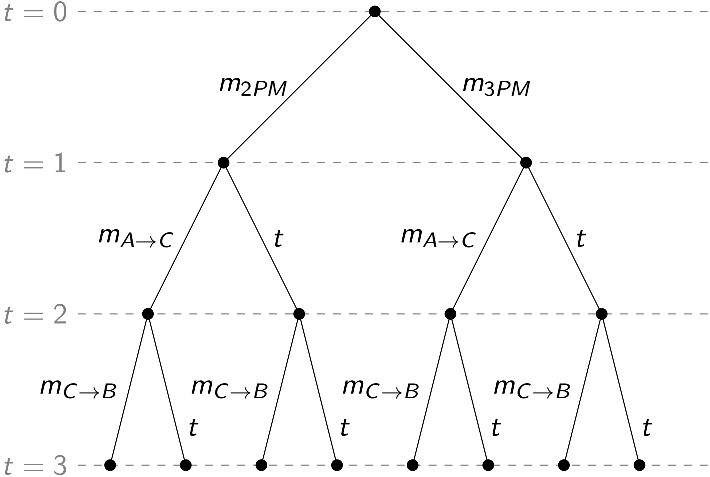
Is this procedure correct?



$t = 0$
$t = 1$
$t = 2$
$t = 3$
m_{2PM}
m_{3PM}
$m_{A \to C}$
t
$m_{A \to C}$
t
$m_{C \to B}$
t
$m_{C \to B}$
t
$m_{C \to B}$
t
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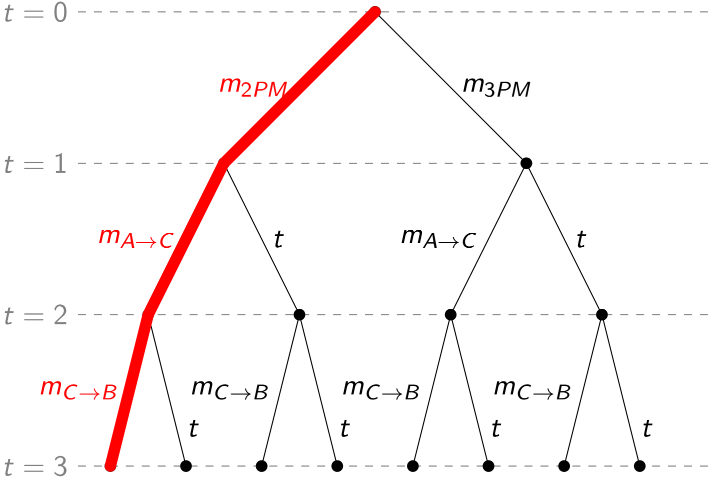
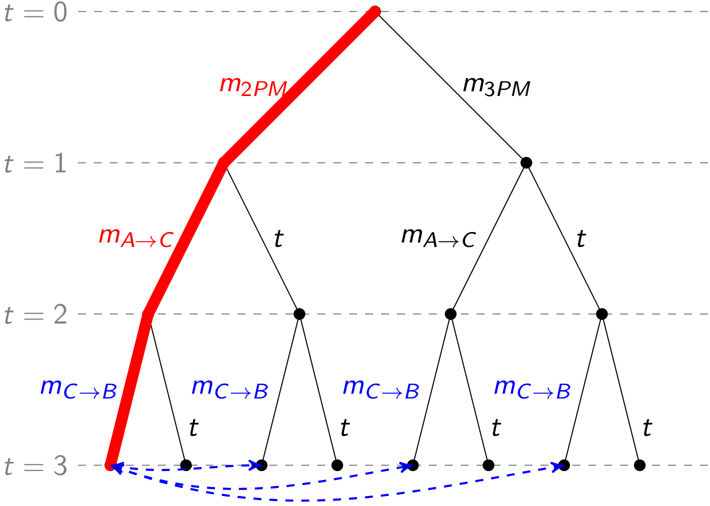
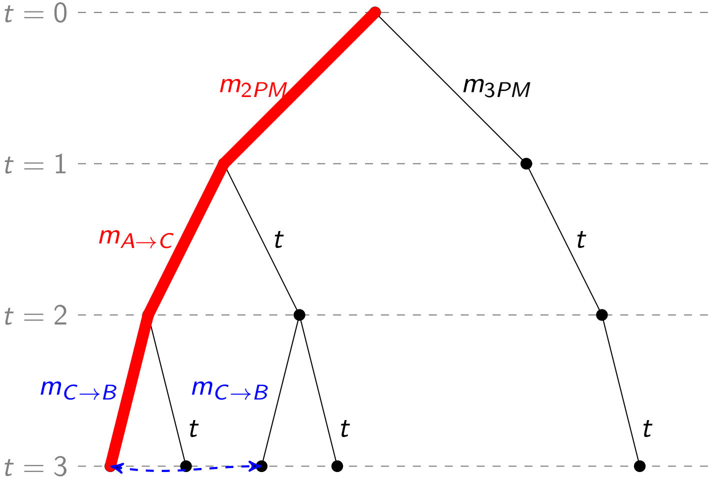
t



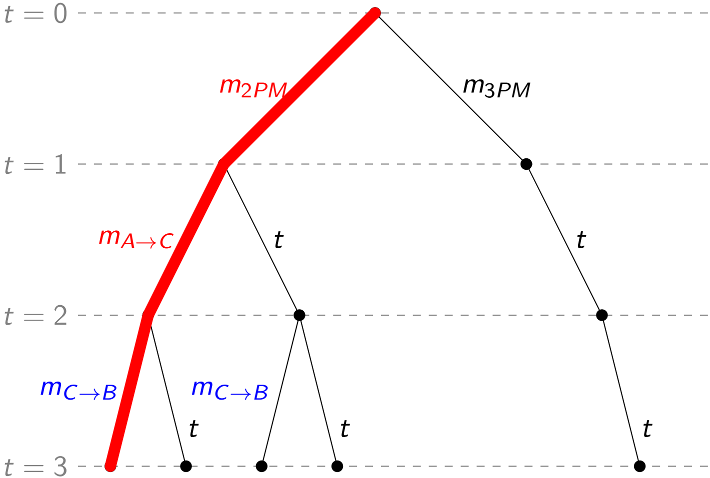
$$H, 3 \models \varphi$$



Bob's uncertainty: $H, 3 \models \neg K_B P_{2PM}$



Bob's uncertainty + 'Protocol information': $H, 3 \models K_B P_{2PM}$



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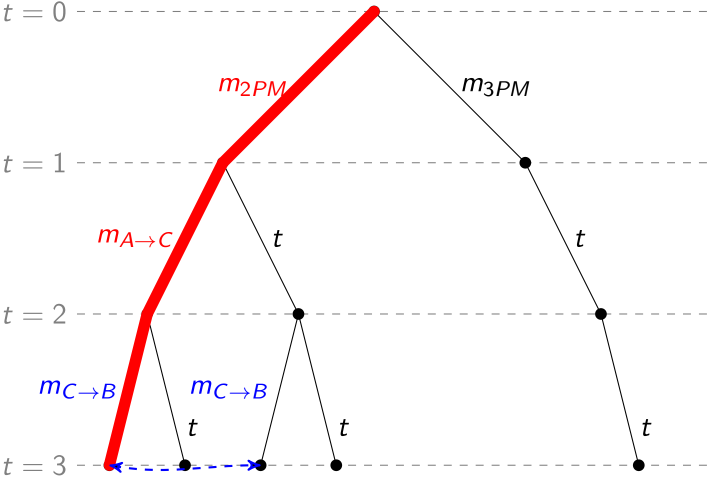

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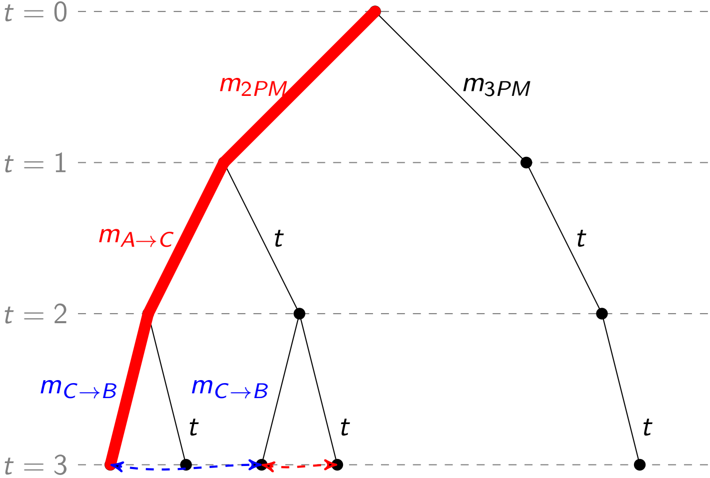

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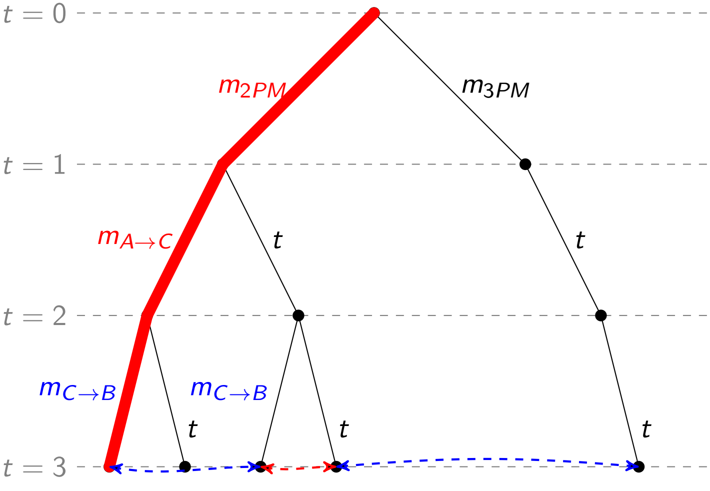

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Parameters of the Logical Framework

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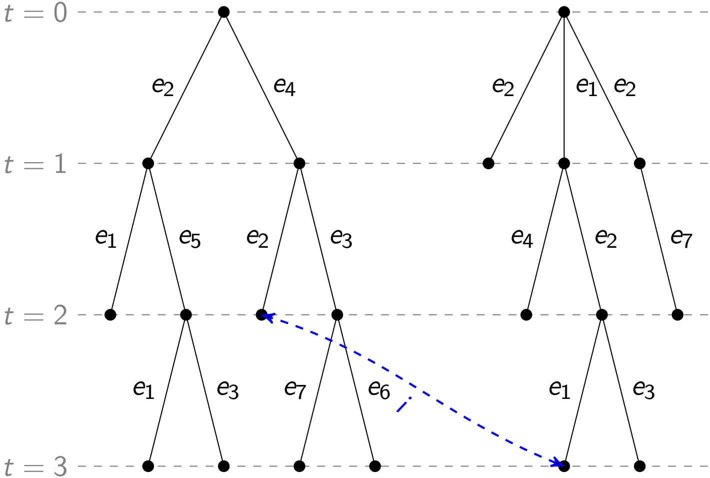
3. Conditions on the reasoning abilities of the agents. Do the agents satisfy perfect recall? No miracles? Do they agents' know what time it is?

Agent Oriented Properties:

- **No Miracles**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $H \sim_i H'$ then $He \sim_i H'e$.
- **Perfect Recall**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $He \sim_i H'e$ then $H \sim_i H'$.
- **Synchronous**: For all finite histories $H, H' \in \mathcal{H}$, if $H \sim_i H'$ then $\mathrm{len}(H) = \mathrm{len}(H')$.

Perfect Recall



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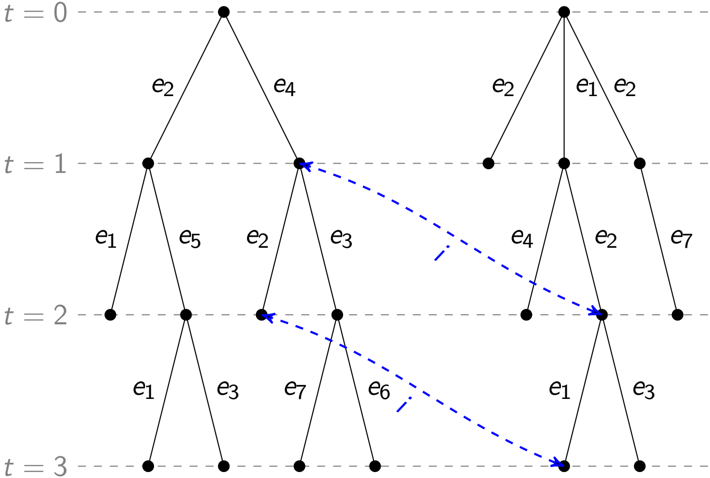

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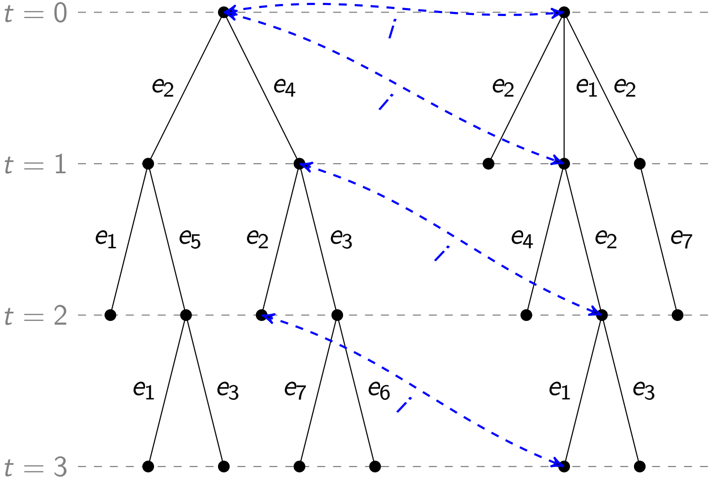

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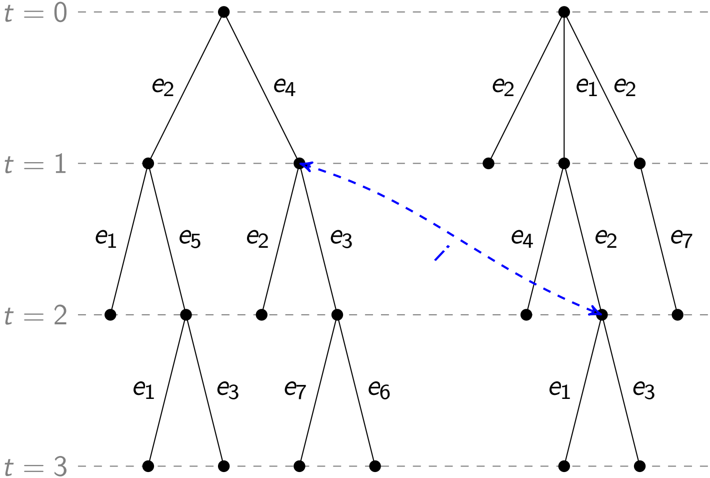

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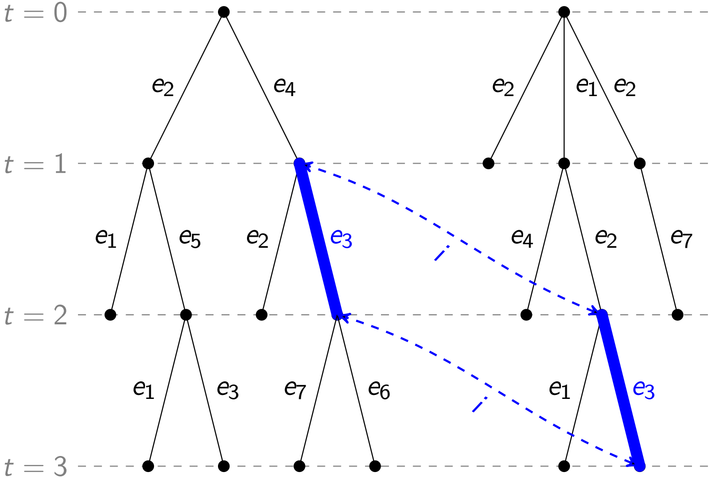

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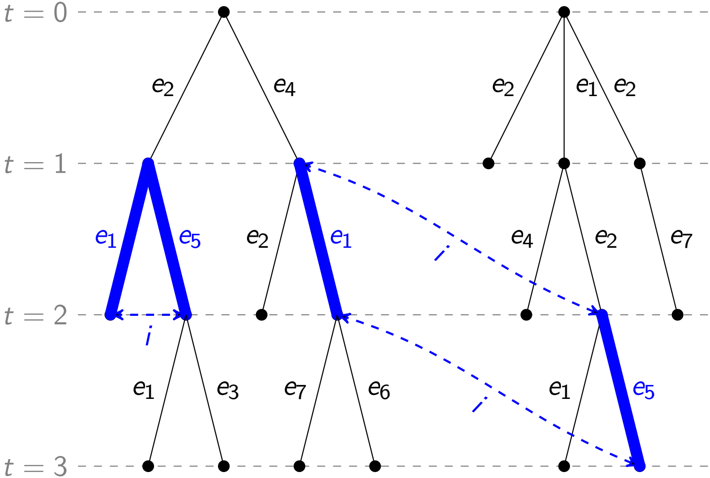

Ideal Agents

Assume there are two agents

Theorem

The logic of ideal *agents with respect to a language with common knowledge and future is highly undecidable (for example, by assuming perfect recall).*

J. Halpern and M. Vardi.. *The Complexity of Reasoning abut Knowledge and Time*. *J. Computer and Systems Sciences*, 38, 1989.

J. van Benthem and EP. *The Tree of Knowledge in Action*. Proceedings of AiML, 2006.

Constrained Public Announcement

1. $A \to \langle A \rangle \top$ vs. $\langle A \rangle \top \to A$

Constrained Public Announcement

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3. $\langle A \rangle K_i P \leftrightarrow \langle A \rangle \top \wedge K_i (A \to \langle A \rangle P)$

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