Reasoning about Knowledge and Beliefs

Lecture 19

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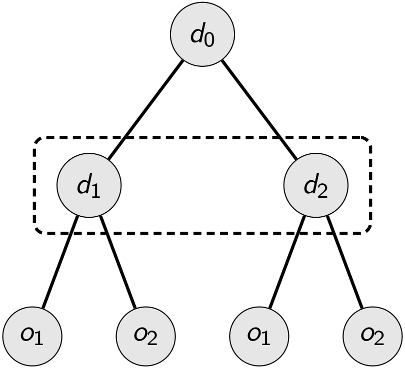
The Absent-Minded Driver

Games of Imperfect Information



The Absent-Minded Driver

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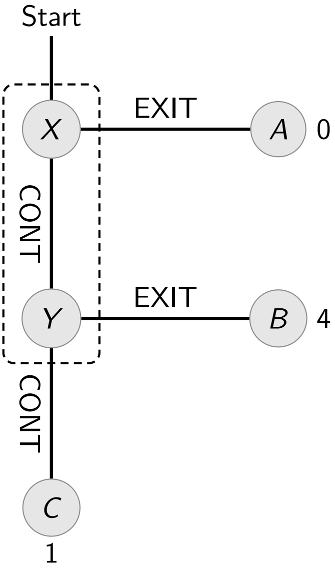
The driver is absentminded and is aware of this fact. At an intersection, he cannot tell whether it is the first or the second intersection and he cannot remember how many he has passed (one can make the situation more realistic by referring to the 17th intersection). While sitting at the bar, all he can do is to decide whether or not to exit at an intersection. (pg. 7)

M. Piccione and A. Rubinstein. *On the Interpretation of Decision Problems with Imperfect Recall*. Games and Econ Behavior, 20, pgs. 3- 24, 1997.


Start
EXIT
EXIT
CONT
CONT
X
Y
A
B
C
0
4
1

Planning stage: While planning his trip home at the bar, the decision maker is faced with a choice between "Continue; Continue" and "Exit". Since he cannot distinguish between the two intersections, he cannot plan to "Exit" at the second intersection (he must plan the same behavior at both X and Y). Since "Exit" will lead to the worst outcome (with a payoff of 0), the optimal strategy is "Continue; Continue" with a guaranteed payoff of 1.

Action stage: When arriving at an intersection, the decision maker is faced with a local choice of either “Exit” or “Continue” (possibly followed by another decision). Now the decision maker knows that since he committed to the plan of choosing “Continue” at each intersection, it is possible that he is at the second intersection. Indeed, the decision maker concludes that he is at the first intersection with probability $1/2$. But then, his expected payoff for “Exit” is 2, which is greater than the payoff guaranteed by following the strategy he previously committed to. Thus, he chooses to “Exit”.

"...the driver's example has a paradoxical flavor due to the conflict between two ways of reasoning at an intersection. The first instructs the decision maker to follow his initial decision not to exit, following an intuitive principle of rationality that unless new information is received or there isa change in tastes, previous decisions should not be changed. The second way of reasoning, maximizing the expected payoff given the belief, suggests he should deviate from his initial decision. "

Planning-Optimal Decision

Let p be the probability of CONT.

THe **planning-optimal decision** is to find a p such that

$$(1-p)\cdot 0 + p(1-p)\cdot 4 + p^2 \cdot 1$$

is maximal.

The above equation is maximal when $p = \frac{2}{3}$. So, the planning-optimal strategy is to CONT with probability $\frac{2}{3}$ and EXIT with probability $\frac{1}{3}$

Action Stage

1. The driver makes a decision at *each* intersection through which he passes. Moreover, when at one intersection, he can determine the action *only there*, and *not* at the other intersection—where he isn't.
2. Since he is in completely indistinguishable situations at the two intersections, whatever reasoning obtains at one must obtain also at the other, and he is aware of this.

Let p be the probabilities of CONT at the current and at the "other" intersections, respectively. Let α be the probability that the decision maker is at the first intersection.

The expected payoff at the acton stage is:

$$H(p,q,\alpha) = \alpha[(1-p)\cdot 0 + p(1-q)\cdot 4 + pq\cdot 1] + (1-\alpha)[(1-p)\cdot 4 + p\cdot 1]$$

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Piccione and Rubinstein maximize $H(p, p, \alpha)$ over p, holding α fixed.

"This makes sense only if the driver controls the probabilities at both intersections—a violation of the first observation. But even if, by some magical process, the driver *could* control the probability q at the other intersection, surely α depends on q, and cannot be held fixed in the maximization!" (pg. 104)

1. The optimal decision is the same at both intersections; call it p^*
2. Therefore, at each intersection, the driver believes that p^* is chosen at the other intersection.
3. At each intersection, the driver optimizes his decision given his beliefs. Therefore, choosing p at the current intersection to be p^* must be optimal given the belief that p^* is chosen at the other intersection.

Let q be the behavior at the other intersection. the probability that the current intersection is the first one is $\alpha = \frac{1}{1+q}$. Let $h(p, q) = H(p, q, \frac{1}{1+q})$, item 3. implies that p^* is action-optimal if the maximum of $h(p, p^*)$ over p is attained at $p = p^*$.

That is, p^* is a fixed point of the mapping $q \mapsto \arg\max_p h(p, q)$

Formally, (p^*, p^*) is a symmetric Nash equilibrium in the symmetric game between "the driver at the current intersection and the "the driver at the other intersection".

The planning-optimal decision—CONT with probability $\frac{2}{3}$—is also action-optimal.

If $q = \frac{2}{3}$, then $\alpha = \frac{3}{5}$.

Thus,

$$h(p, \frac{2}{3}) = \frac{3}{5}[(1-p) \cdot 0 + p(\frac{1}{3}) \cdot 4 + p \cdot \frac{2}{3} \cdot 1] + \frac{2}{5}[(1-p) \cdot 4 + p \cdot 1]$$

which equals $\frac{8}{5}$ for all p. So, $p = \frac{2}{3}$ maximizes it; thus $p^* = \frac{2}{3}$.

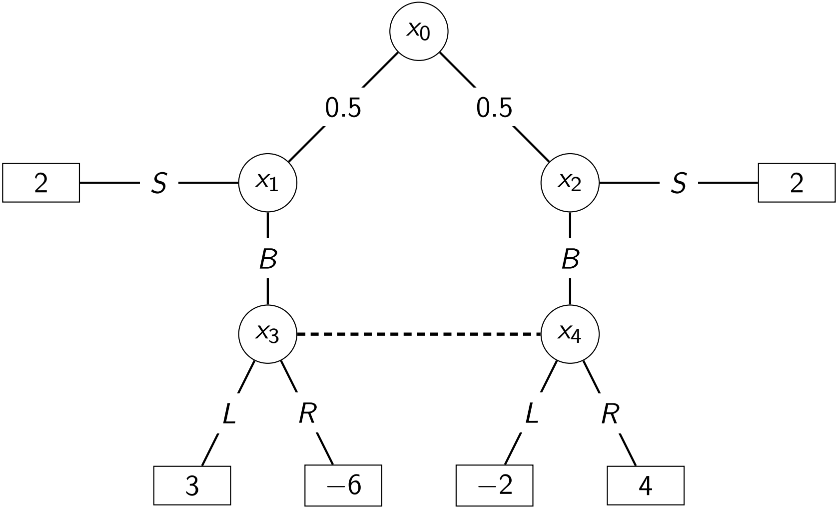
$p^* = \frac{2}{3}$ is the unique action-optional decision.

$$h(p,q) = \frac{1}{1+q}[(1-p)\cdot 0 + p(1-q)\cdot 4 + pq\cdot 1] + \frac{q}{1+q}[(1-p)\cdot 4 + p\cdot 1]$$

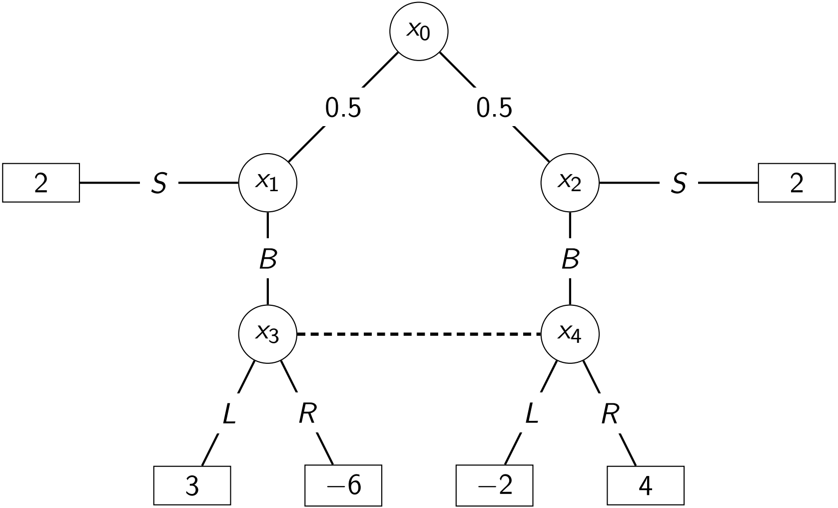
$$= \frac{(4-6q)p + 4q}{1+q}$$

Given q, the maximizing p therefore is: 0 for $q > \frac{2}{3}$, 1 for $q < \frac{2}{3}$ and anything for $q = \frac{2}{3}$. So, the only fixed-point is $p^* = \frac{2}{3}$.

1. What decisions can be made? In particular, can a decision maker decide about when to make a decision?
2. What tis the timing of decisions? Is there a planning stage or are decisions made only at the time actions are executed?
3. Can a decision maker change his strategy along its execution? And, if he does change his strategy, can he change it again?
4. Can a decision maker use random devices?



The optimal strategy is $f(x_1) = S$, $f(x_2) = B$ and $f(\{x_3, x_4\}) = R$



At x_1, $f'(x_1) = B$, $f'(x_2) = S$, $f'(\{x_3, x_4\}) = L$ is best

...if node x_1 is reached, the agent should reconsider, and decide to switch from f to f'. *If the agent is able to remember that he switched strategies*, then this is correct; the agent is indeed better off (under any reasonable notion of "better off") if he switches. "

J. Halpern. *On Ambiguities in the Interpretation of Game Tree*. Games and Economic Behavior, 1996.

Temporal Aspects of the Dynamics of Knowledge

Ingredients of a Logical Analysis of Rational Agency

- $\Rightarrow$ informational attitudes (eg., knowledge, belief, certainty)
- $\Rightarrow$ time, actions and ability
- $\Rightarrow$ motivational attitudes (eg., preferences)
- $\Rightarrow$ group notions (eg., common knowledge and coalitional ability)
- $\Rightarrow$ normative attitudes (eg., obligations)

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Time

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Many variations

- discrete or continuous
- branching or linear
- point based or interval based

See, for example,

Antony Galton. *Temporal Logic*. Stanford Encyclopedia of Philosophy: http://plato.stanford.edu/entries/logic-temporal/.

I. Hodkinson and M. Reynolds. *Temporal Logic*. Handbook of Modal Logic, 2008.

Models of Time

$\mathcal{T} = \langle T, <, V \rangle$ where

- T is a set of **time points** (or **moments**),
- $< \subseteq T \times T$ is the **precedence relation**: $s < t$ means "time point s precedes time point t (or s occurs earlier than t)" and
- $V : \mathsf{At} \to \wp(T)$ is a valuation function (describing when the atomic propositions are true).

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Examples: $\langle \mathbb{N}, < \rangle$, $\langle \mathbb{Z}, < \rangle$, $\langle \mathbb{Q}, < \rangle$, $\langle \mathbb{R}, < \rangle$

Other properties of $<$

- **Linearity**: for all $s, t \in T$, $s < t$ or $s = t$ of $t < s$
- **Past-linear**: for all $s, x, y \in T$, if $x < s$ and $y < s$, then either $x < y$ or $x = y$ or $y < x$
- **Denseness** for all $s, t \in T$, if $s < t$ then there is a $z \in T$ such that $s < z$ and $z < t$
- **Discreteness**: for all $s, t \in T$, if $s < t$ then there is a z such that ($s < z$ and there is no u such that $s < u$ and $u < z$)

Priorean Temporal Logic

$\mathcal{L}_t$ be defined by the following grammar

$$p \mid \neg\varphi \mid \varphi \wedge \psi \mid G\varphi \mid H\varphi$$

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$G\varphi$: "φ is **going to** become true"

$H\varphi$: "φ **has been** true"

$F\varphi := \neg G \neg \varphi$: "$\varphi$ is true in the future"

$P\varphi := \neg H \neg \varphi$: "$\varphi$ was true some time in the past"

$\mathcal{M} = \langle T, <, V \rangle$

- $\mathcal{M}, t \models p$ iff $t \in V(p)$
- $\mathcal{M}, t \models \neg\varphi$ iff $\mathcal{M}, t \not\models \varphi$
- $\mathcal{M}, t \models \varphi \wedge \psi$ iff $\mathcal{M}, t \models \varphi$ and $\mathcal{M}, t \models \psi$
- $\mathcal{M}, t \models G\varphi$ iff for all $s \in T$, if $t < s$ then $\mathcal{M}, s \models \varphi$
- $\mathcal{M}, t \models H\varphi$ iff for all $s \in T$, if $s < t$ then$\mathcal{M}, s \models \varphi$

- $\mathcal{M}, t \models F\varphi$ iff there is $s \in T$ such that $t < s$ and $\mathcal{M}, s \models \varphi$
- $\mathcal{M}, t \models P\varphi$ iff there is $s \in T$ such that $s < t$ and $\mathcal{M}, s \models \varphi$

Frame Correspondence

- $H\bot \vee PH\bot$ is valid

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- $(F\top \wedge \varphi \wedge H\varphi) \to FH\varphi$ is valid

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- $(F\top \wedge \varphi \wedge H\varphi) \rightarrow FH\varphi$ is valid iff the flow of time is discrete

Basic Temporal Logic

All classical propositional tautologies

Distribution
$G(\varphi \to \psi) \to (G\varphi \to G\psi)$
$G(\varphi \to \psi) \to (G\varphi \to G\psi)$

Converse
$\varphi \to GP\varphi$
$\varphi \to HF\varphi$

Transitivity: $G\varphi \to GG\varphi$

Modus Ponens: from φ and $\varphi \to \psi$ infer ψ
Temporal Generalization: from φ infer $F\varphi$; from φ infer $G\varphi$

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Theorem. The above logic is sound and complete with respect to the class of all flows of time

Logic of Linear Time

Theorem. The above logic with the linearity axioms is sound and complete with respect to the class of all **linear** flows of time

- $PF\varphi \to (P\varphi \vee \varphi \vee F\varphi)$
- $FP\varphi \to (F\varphi \vee \varphi \vee P\varphi)$

Other Languages: Since and Until

$\mathcal{M}, t \models \varphi U \psi$ iff $\mathcal{M}, s \models \psi$ for some s such that $t < s$ and $\mathcal{M}, u \models \varphi$ for all u with $t < u < s$

$\mathcal{M}, t \models \varphi S \psi$ iff $\mathcal{M}, s \models \psi$ for some s such that $s < t$ and $\mathcal{M}, u \models \varphi$ for all u with $s < u < t$

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Theorem (Kamp). Over the class of linear, continuous orderings, every temporal operator can be defined using the above modalities

Branching Time

Each moment $t \in T$ can be decided into the $Past(t) = \{s \in T \mid s < t\}$ and the $Future(t) = \{s \in T \mid t < s\}$ ("A-series")

Typically, it is assumed that the past is linear, but the future may be branching.

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$F\varphi$: "it will be the case that φ"

φ will be the case "in the case in the actual course of events" or "no matter what course of events"

Branching Time Logics

A **branch** b in $\langle T, < \rangle$ is a maximal linearly ordered subset of T

$s \in T$ is **on a branch** b **of** T provided $s \in b$ (we also say "b is a branch going through t").

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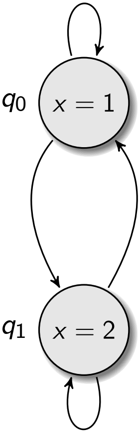
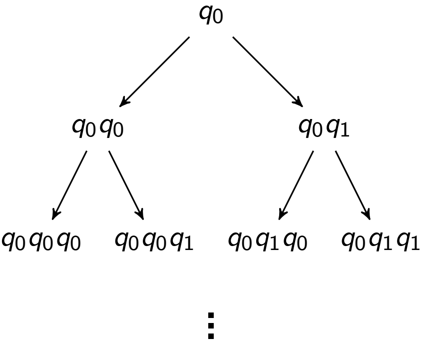
- $\mathcal{M}, t, b \models G\varphi$ iff for all $s \in T$, if s is on b and $t < s$ then $\mathcal{M}, s, b \models \varphi$
- $\mathcal{M}, t, b \models H\varphi$ iff for all $s \in T$, if s is on b and $s < t$ then $\mathcal{M}, s, b \models \varphi$
- $\mathcal{M}, t, b \models \forall\varphi$ iff $\mathcal{M}, s, c \models \varphi$ for all branches c through t

Computational vs. Behavioral Structures





Temporal Logics

Temporal Logics

- *Linear Time Temporal Logic*: Reasoning about computation paths:

  $F\varphi$: φ is true some time in *the* future.

A. Pnuelli. *A Temporal Logic of Programs*. in *Proc. 18th IEEE Symposium on Foundations of Computer Science* (1977).

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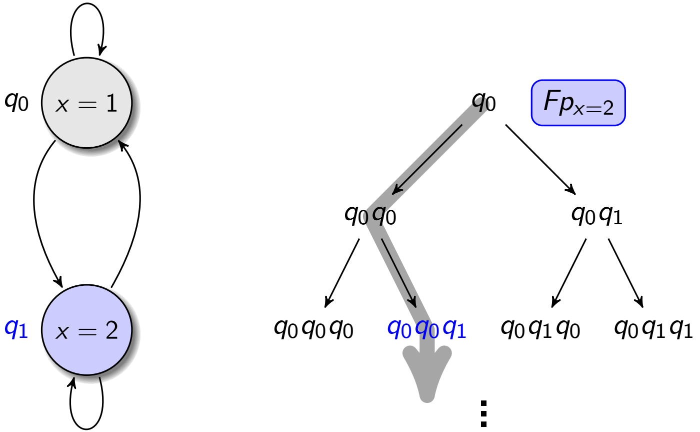
- *Branching Time Temporal Logic*: Allows quantification over paths:

$\exists F\varphi$: there is a path in which φ is eventually true.

E. M. Clarke and E. A. Emerson. *Design and Synthesis of Synchronization Skeletons using Branching-time Temproal-logic Specifications*. In *Proceedings Workshop on Logic of Programs*, LNCS (1981).

Temporal Logics



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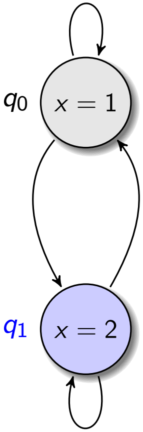

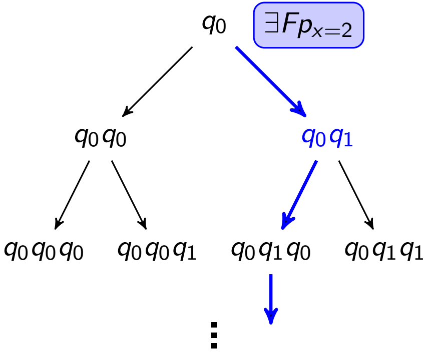

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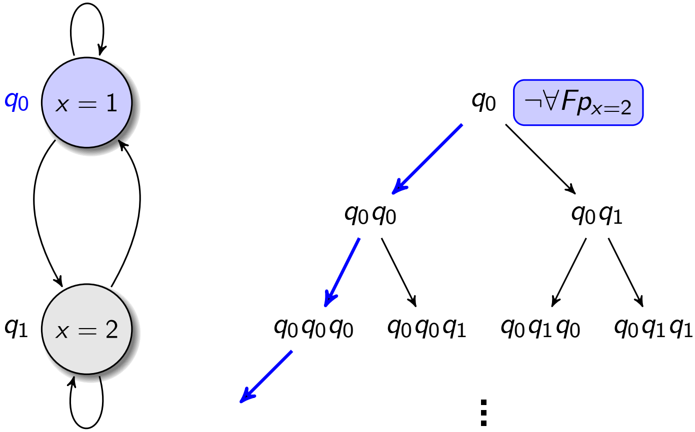

Interval Values

J. Allen and G. Ferguson. *Actions and Events in Interval Temporal Logics*. Journal of Logic and Computation, 1994.

J. Halpern and Y. Shoham. *A Propositional Modal Logic of Time Intervals*. Journal of the ACM, 38:4, pp. 935 - 962, 1991.

J. van Benthem. *Logics of Time*. Kluwer, 1991.

Interval Temporal Logics

Let $\mathcal{T} = \langle T, < \rangle$ be a frame and $I(\mathcal{T}) = \{[a, b] \mid a, b \in T$ and $a \leq b\}$ be the set of intervals over T

Models are $\mathcal{M} = \langle I(\mathcal{T}), \{R_X\}, V \rangle$ where $R_X \subseteq I(\mathcal{T}) \times I(\mathcal{T})$ and $V : \mathrm{At} \to \wp(I(\mathcal{T}))$.

Interval Temporal Logics

Let $\mathcal{T} = \langle T, < \rangle$ be a frame and $I(\mathcal{T}) = \{[a, b] \mid a, b \in T$ and $a \leq b\}$ be the set of intervals over T

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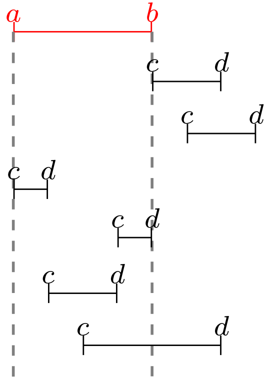
- $\mathcal{M}, [a, b] \models p$ iff $[a, b] \in V(p)$
- $\mathcal{M}, [a, b] \models \mathrm{pt}$ iff $a = b$
- $\mathcal{M}, [a, b] \models \langle X \rangle \varphi$ iff there is an interval $[c, d]$ such that $[a, b] R_X [c, d]$ and $\mathcal{M}, [c, d] \models \varphi$

$\langle A \rangle$	$[a,b]R_A[c,d] \Leftrightarrow b = c$
$\langle L \rangle$	$[a,b]R_L[c,d] \Leftrightarrow b < c$
$\langle B \rangle$	$[a,b]R_B[c,d] \Leftrightarrow a = c, d < b$
$\langle E \rangle$	$[a,b]R_E[c,d] \Leftrightarrow b = d, a < c$
$\langle D \rangle$	$[a,b]R_D[c,d] \Leftrightarrow a < c, d < b$
$\langle O \rangle$	$[a,b]R_O[c,d] \Leftrightarrow a < c < b < d$



High Undecidability!

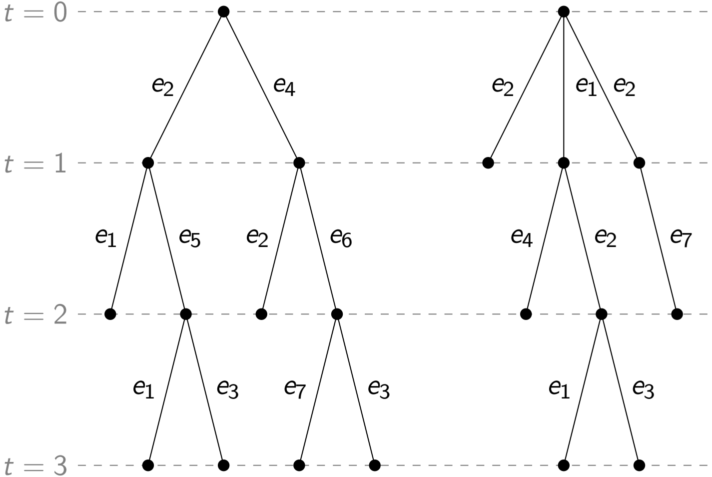
D. Bersolin et al.. *The dark side of interval temporal logic: sharpening the undecidability border*. 2011.

Epistemic Temporal Logic

R. Parikh and R. Ramanujam. *A Knowledge Based Semantics of Messages*. *Journal of Logic, Language and Information*, 12: 453 – 467, 1985, 2003.

FHMV. Reasoning about Knowledge. MIT Press, 1995.

The 'Playground'



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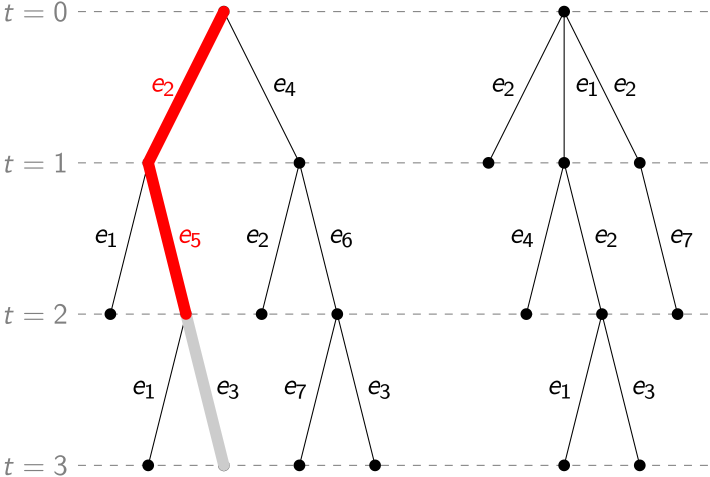

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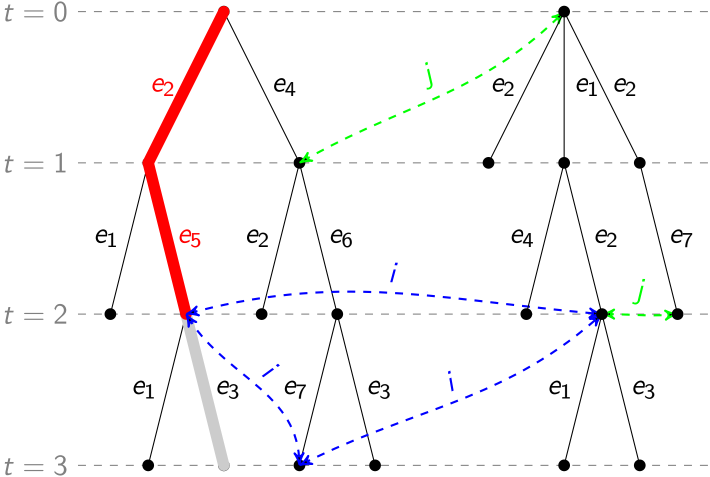

Formal Languages

- $P\varphi$ (φ is true *sometime* in the past),
- $F\varphi$ (φ is true *sometime* in the future),
- $Y\varphi$ (φ is true at *the* previous moment),
- $N\varphi$ (φ is true at *the* next moment),
- $N_e\varphi$ (φ is true after event e)
- $K_i\varphi$ (agent i knows φ) and
- $C_B\varphi$ (the group $B \subseteq \mathcal{A}$ commonly knows φ).

History-based Models

An ETL **model** is a structure $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}, V\rangle$ where $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}\rangle$ is an ETL frame and

$V : \mathsf{At} \to 2^{\mathsf{finite}(\mathcal{H})}$ is a valuation function.

Formulas are interpreted at pairs H, t:

$$H, t \models \varphi$$

Truth in a Model

- $H, t \models P\varphi$ iff there exists $t' \leq t$ such that $H, t' \models \varphi$
- $H, t \models F\varphi$ iff there exists $t' \geq t$ such that $H, t' \models \varphi$
- $H, t \models N\varphi$ iff $H, t+1 \models \varphi$
- $H, t \models Y\varphi$ iff $t > 1$ and $H, t-1 \models \varphi$
- $H, t \models K_i\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_i H'_m$ then $H', m \models \varphi$
- $H, t \models C\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_* H'_m$ then $H', m \models \varphi$.

where $\sim_*$ is the reflexive transitive closure of the union of the $\sim_i$.

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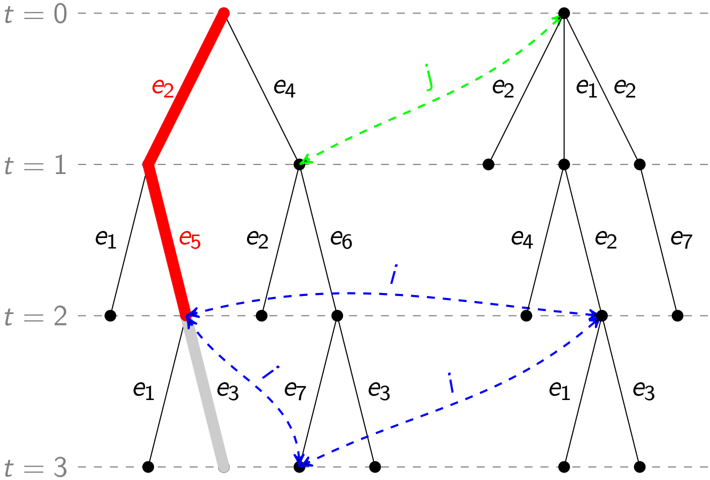

t = 0
t = 1
t = 2
t = 3
e2
e4
j
e2
e1
e2
e1
e5
e2
e6
e4
e2
e7
i
e1
e3
e7
e3
i
e1
e3

An Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

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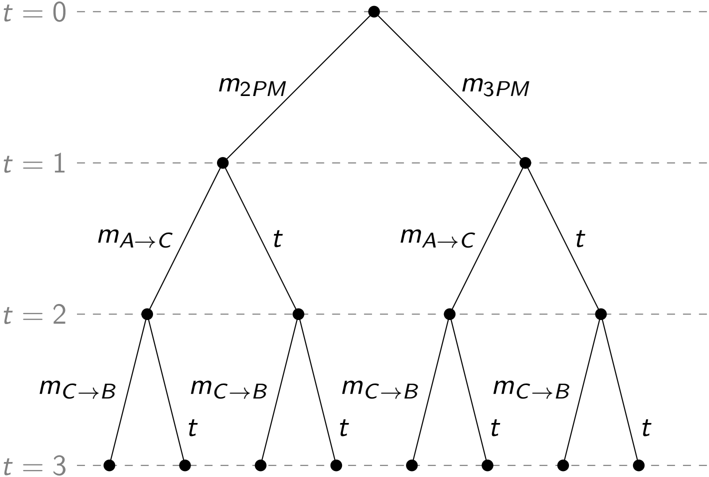
Is this procedure correct?


$t = 0$
$t = 1$
$t = 2$
$t = 3$
m_{2PM}
m_{3PM}
$m_{A \to C}$
t
$m_{A \to C}$
t
$m_{C \to B}$
t
$m_{C \to B}$
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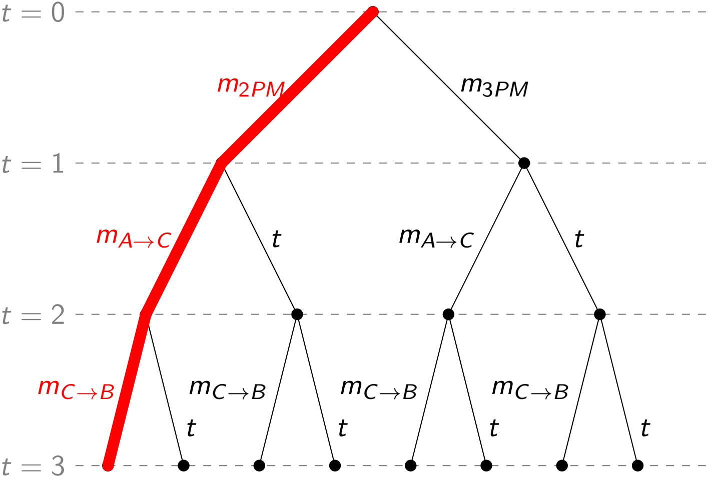
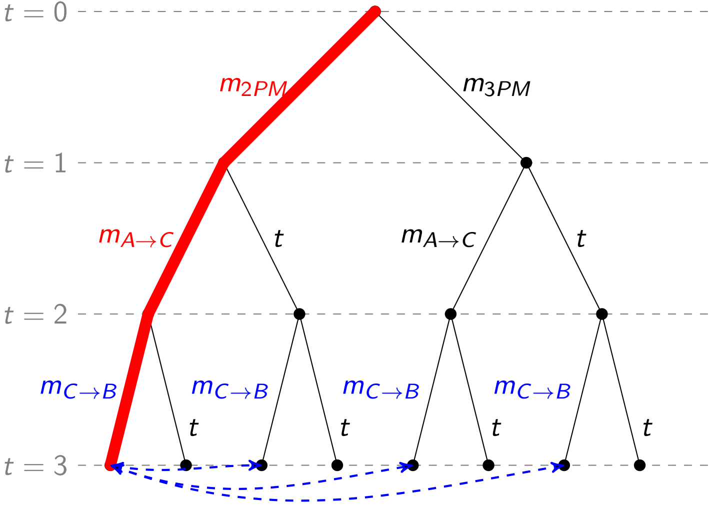
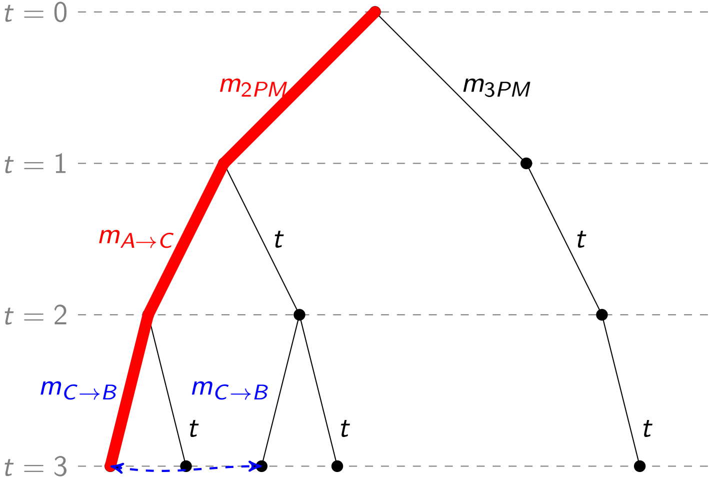
$m_{C \to B}$
t
$m_{C \to B}$
t



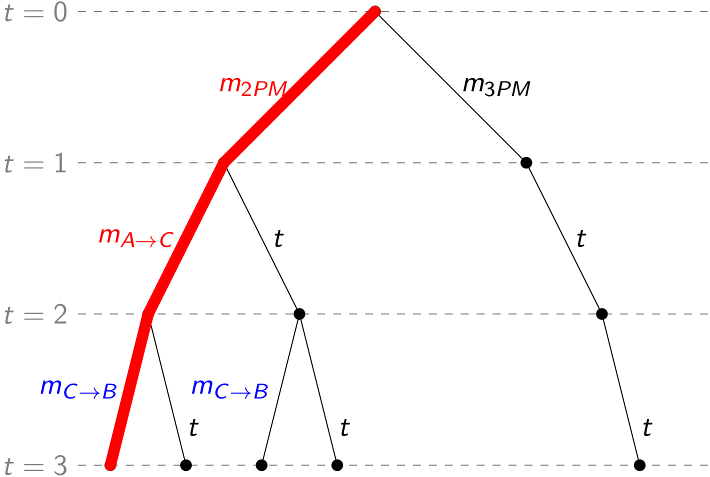
$$H, 3 \models \varphi$$



Bob's uncertainty: $H, 3 \models \neg K_B P_{2PM}$

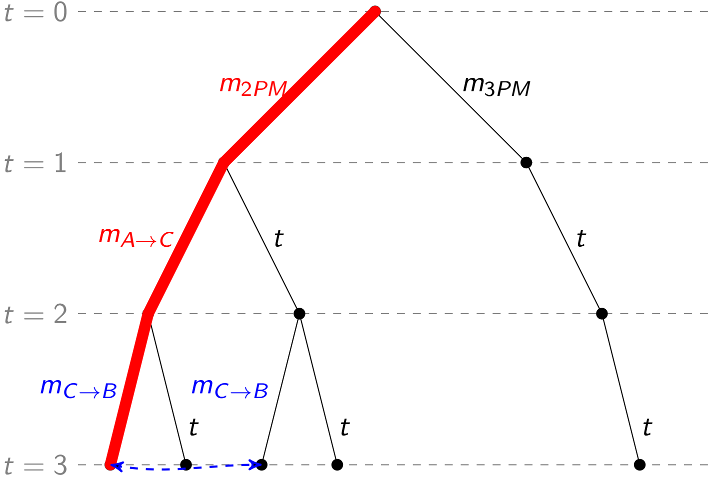


Bob's uncertainty + 'Protocol information': $H, 3 \models K_B P_{2PM}$



Bob's uncertainty + 'Protocol information':

$H, 3 \models \neg K_B K_A K_B P_{2PM}$



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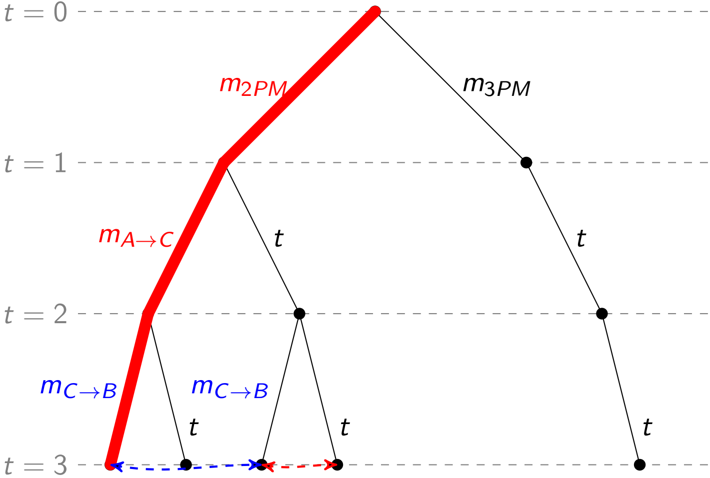

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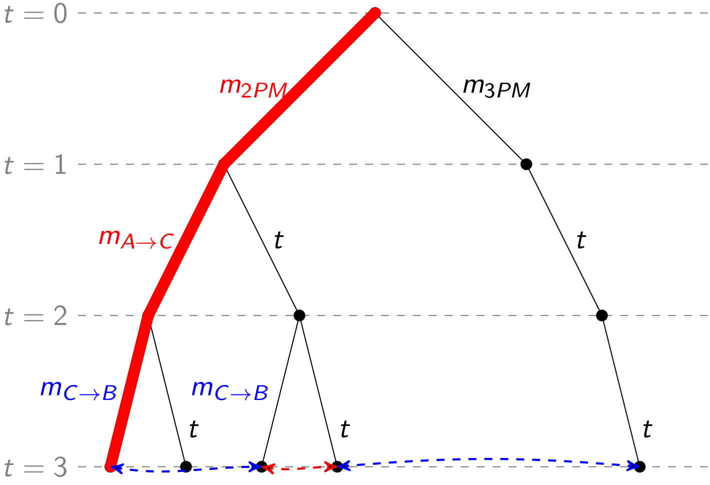

Bob's uncertainty + 'Protocol information':

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Parameters of the Logical Framework

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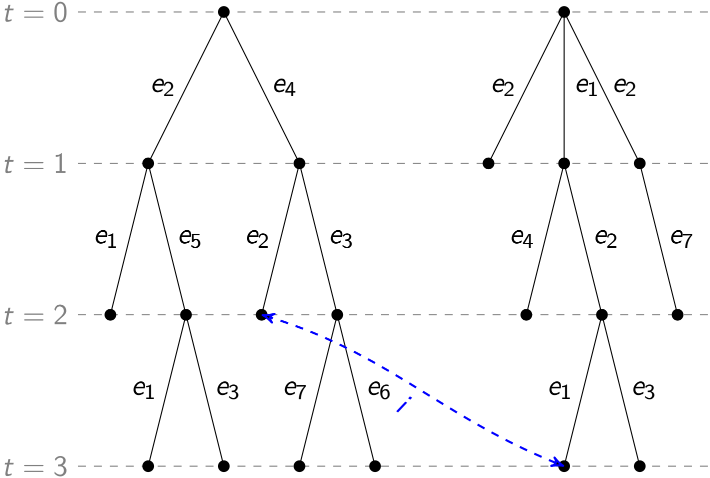
3. Conditions on the reasoning abilities of the agents. Do the agents satisfy perfect recall? No miracles? Do they agents' know what time it is?

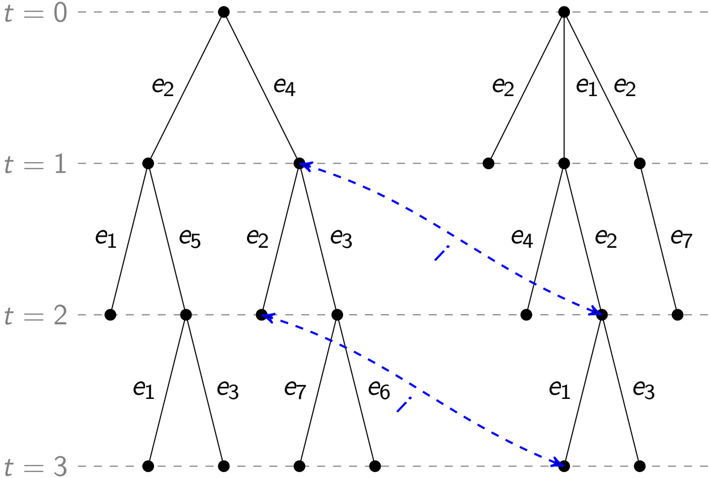
Agent Oriented Properties:

- **No Miracles**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $H \sim_i H'$ then $He \sim_i H'e$.
- **Perfect Recall**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $He \sim_i H'e$ then $H \sim_i H'$.
- **Synchronous**: For all finite histories $H, H' \in \mathcal{H}$, if $H \sim_i H'$ then $\mathrm{len}(H) = \mathrm{len}(H')$.

Perfect Recall



Perfect Recall



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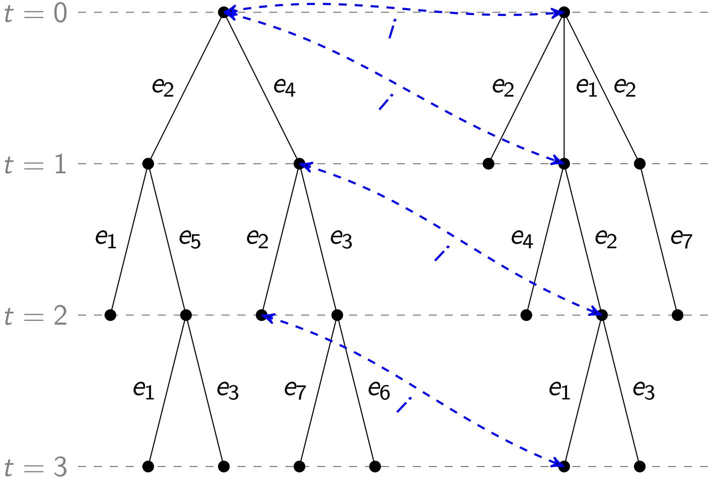
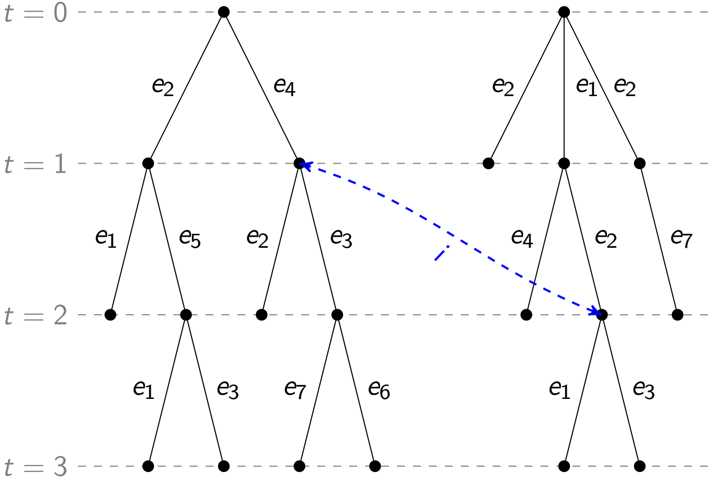
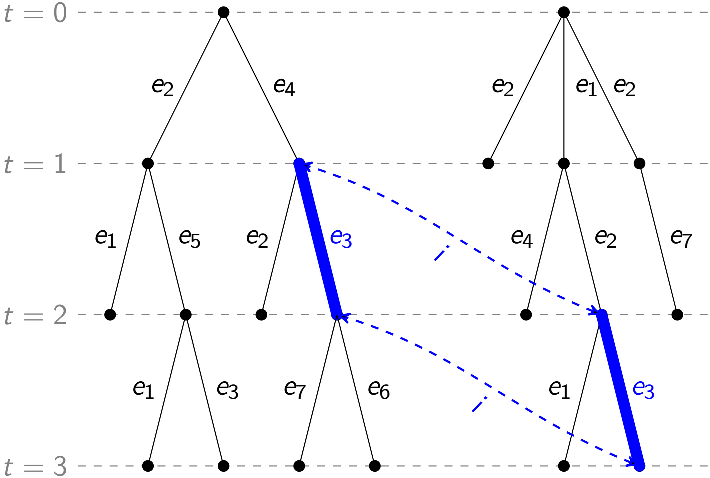


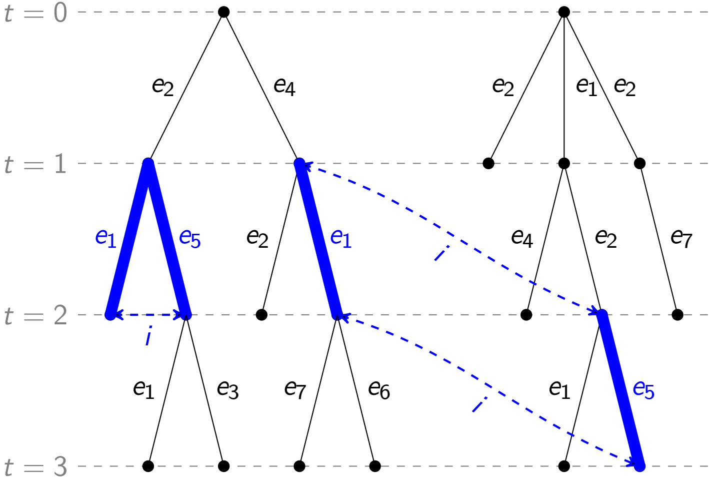
No Miracles



No Miracles



No Miracles



Ideal Agents

Assume there are two agents

Theorem

The logic of ideal *agents with respect to a language with common knowledge and future is highly undecidable (for example, by assuming perfect recall).*

J. Halpern and M. Vardi.. *The Complexity of Reasoning abut Knowledge and Time*. *J. Computer and Systems Sciences*, 38, 1989.

J. van Benthem and EP. *The Tree of Knowledge in Action*. Proceedings of AiML, 2006.

Constrained Public Announcement

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3. $\langle A \rangle K_i P \leftrightarrow \langle A \rangle \top \wedge K_i (A \rightarrow \langle A \rangle P)$

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4. $\langle A \rangle K_i P \leftrightarrow \langle A \rangle \top \wedge K_i (\langle A \rangle \top \to \langle A \rangle P)$