Reasoning about Knowledge and Beliefs

Lecture 9

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Conceptions of Belief

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Eric Schwitzgebel. *Belief*. In The Stanford Encyclopedia of Philosophy.

Franz Huber. *Formal Theories of Belief*. In The Stanford Encyclopedia of Philosophy.

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D. Christensen. *Putting Logic in its Place*. Oxford University Press.

H. Leitgeb. *The Lockean Thesis Revisited*. Working Paper, 2010.

Epistemic-Probability Models

Adding Probabilities

Epistemic-Probability Model: $\mathcal{M} = \langle W, \{\sim_i\}_{i\in\mathcal{A}}, \{P_i\}_{i\in\mathcal{A}}, V\rangle$ where each $\sim_i$ is an equivalence relation on W is an epistemic model and $P_i : W \to \Delta(W)$ assigns to each state a probability measure over W, and V is a valuation function.

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Write p_i^w for the i's probability measure at state w. We make two natural assumptions:

1. For all $v \in W$, if $p_i^w(v) > 0$ then $p_i^w = p_i^v$; and
2. For all $v \notin [w]_i$, $p_i^w(v) = 0$.

Common Prior

Epistemic-Probabilistic Models: $\mathcal{M} = \langle W, \{\sim_i\}_{i\in\mathcal{A}}, p, V\rangle$

Common Prior: $p : W \to [0,1]$ is a probability measure (assume W finite)

Truth: $\mathcal{M}, w \models \varphi$ is defined as follows:

- $\mathcal{M}, w \models p$ iff $w \in V(p)$ (with $p \in \mathrm{At}$)
- $\mathcal{M}, w \models \neg\varphi$ if $\mathcal{M}, w \not\models \varphi$
- $\mathcal{M}, w \models \varphi \wedge \psi$ if $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$
- $\mathcal{M}, w \models K_i\varphi$ if for each $v \in W$, if $w \sim_i v$, then $\mathcal{M}, v \models \varphi$
- $\mathcal{M}, w \models B^r\varphi$ iff $p([\![\varphi]\!] \mid [w]_i) = \frac{p([\![\varphi]\!] \cap [w]_i)}{p([w]_i)} \geq r$

- $B_i^0 \varphi$
- $B_i^1 \top$
- $B_i^q(\varphi \wedge \psi) \wedge B_i^p(\varphi \wedge \neg\psi) \rightarrow B_i^{q+p}\varphi, \quad q + p \leq 1$
- $\neg B_i^q(\varphi \wedge \psi) \wedge \neg B_i^p(\varphi \wedge \neg\psi) \rightarrow \neg B_i^{q+p}\varphi, \quad q + p \leq 1$
- $B_i^q \varphi \rightarrow \neg B_i^p \neg\varphi, \quad q + p > 1$

Archimedian Rule: If $\psi \rightarrow B_i^p \varphi$ is valid for each $p < q$, then $\psi \rightarrow B_i^q \varphi$ is valid.

- $K_i\varphi \rightarrow B_i^q\varphi$
- $B_i^q\varphi \rightarrow K_iB_i^q\varphi$
- $\neg B_i^q\varphi \rightarrow K_i\neg B_i^q\varphi$
- if $\varphi \rightarrow \psi$ is valid then so is $B_i^q\varphi \rightarrow B_i^q\psi$

- $K_i\varphi \to B_i^q\varphi$
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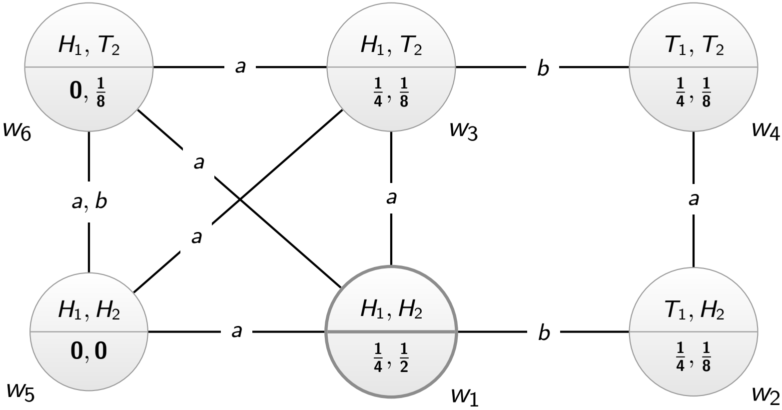
- if $\varphi \to \psi$ is valid then so is $B_i^q\varphi \to B_i^q\psi$

$$\pi_i([\![\varphi]\!]_{\mathcal{M}} \mid [\![B_i^q\varphi]\!]_{\mathcal{M}}) \geq q$$


H_1, T_2
$0, \frac{1}{8}$
w_6
a
H_1, T_2
$\frac{1}{4}, \frac{1}{8}$
w_3
b
T_1, T_2
$\frac{1}{4}, \frac{1}{8}$
w_4
a, b
a
a
a
a
H_1, H_2
$0, 0$
w_5
a
H_1, H_2
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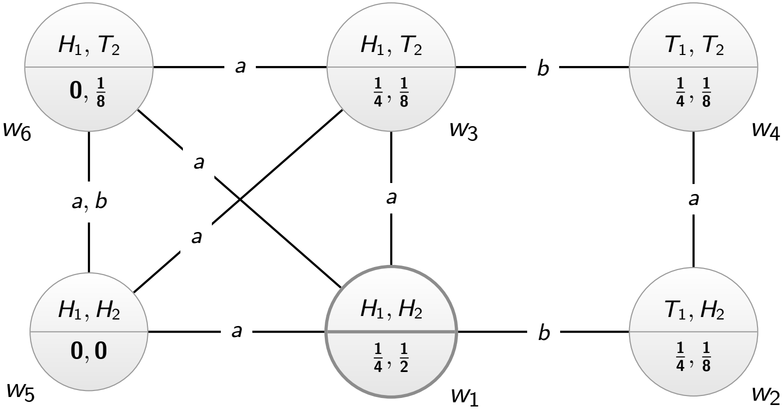
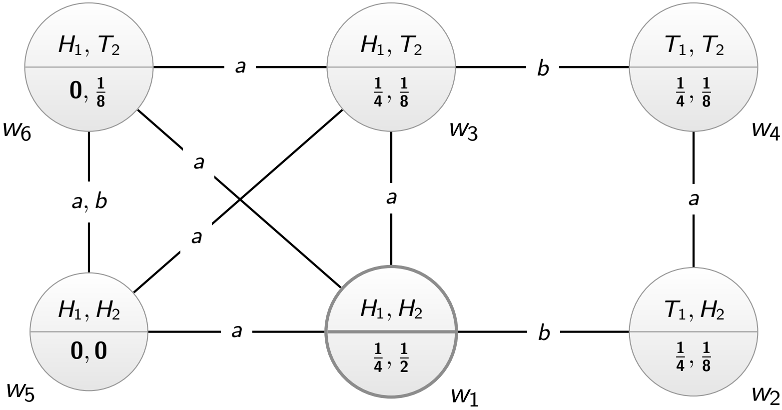
w_2



- $\mathcal{M}, w_1 \models \neg K_a H_2 \wedge \neg K_a T_2 \wedge B_a^{\frac{1}{2}} H_2 \wedge B_a^{\frac{1}{2}} T_2$



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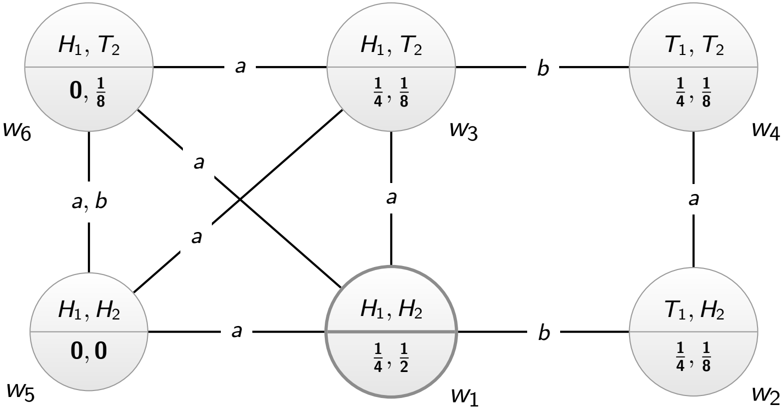

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- $\mathcal{M}, w_1 \models \neg K_a(K_b H_2 \vee K_b T_2) \wedge B_a^1(K_b H_2 \vee K_b T_2)$