Reasoning about Knowledge and Beliefs

Lecture 8

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- $\Diamond$ is the dual of $\Box$ for *necessity*, so $\neg\Diamond\varphi$ follows from $\Box\neg\varphi$.
- $\Box$ obeys the rule of Necessitation: if φ is a theorem, so is $\Box\varphi$.

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Since p was arbitrary, we have shown that *every truth is known*.

The Question

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There is a fairly large literature on knowability and related issues. See, e.g.:

J. Salerno. 2009. *New Essays on the Knowability Paradox*, OUP

J. van Benthem. 2004. "What One May Come to Know," *Analysis*.

P. Balbiani et al. 2008. "'Knowable' as 'Known after an Announcement,"' *Review of Symbolic Logic*.

Dynamic Epistemic Logic

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In the simplest case, we model an agent's acquisition of knowledge by the elimination of possibilities from an initial epistemic model.

Finding out that φ

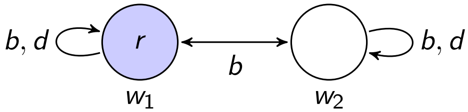
$$\mathcal{M} = \langle W, \{\sim_i\}_{i\in\mathcal{A}}, \{\preceq_i\}_{i\in\mathcal{A}}, V\rangle$$

$$\Big\Downarrow \textbf{ Find out that } \varphi$$

$$\mathcal{M}' = \langle W', \{\sim'_i\}_{i\in\mathcal{A}}, \{\preceq'_i\}_{i\in\mathcal{A}}, V|_{W'}\rangle$$

Example: College Park and Amsterdam

Recall the College Park agent who doesn't know whether it's raining in Amsterdam, whose epistemic state is represented by the model:



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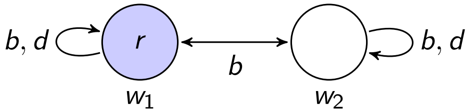

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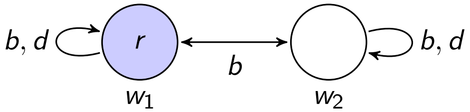

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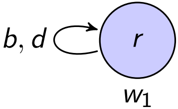

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Formally, $\mathcal{M}_{|\varphi} = \langle W_{|\varphi}, \{R_{a_{|\varphi}} \mid a \in \mathrm{Agt}\}, V_{|\varphi}\rangle$ is the model s.th.:

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In the single-agent case, this models the agent learning φ. In the multi-agent case, this models all agents *publicly* learning φ.

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Big Idea: we evaluate $[!\varphi]\psi$ and $\langle !\varphi\rangle\psi$ not by looking at *other worlds in the same model*, but rather by looking at a new model.

Public Announcement Logic

Suppose $\mathcal{M} = \langle W, \{\sim_i\}_{i\in\mathcal{A}}, \{\preceq_i\}_{i\in\mathcal{A}}, V\rangle$ is a multi-agent Kripke Model

$$\mathcal{M}, w \models [\psi]\varphi \text{ iff } \mathcal{M}, w \models \psi \text{ implies } \mathcal{M}|_\psi, w \models \varphi$$

where $\mathcal{M}|_\psi = \langle W', \{\sim'_i\}_{i\in\mathcal{A}}, \{\preceq'_i\}_{i\in\mathcal{A}}, V'\rangle$ with

- $W' = W \cap \{w \mid \mathcal{M}, w \models \psi\}$
- For each i, $\sim'_i = \sim_i \cap (W' \times W')$
- For each i, $\preceq'_i = \preceq_i \cap (W' \times W')$
- for all $p \in \mathsf{At}$, $V'(p) = V(p) \cap W'$

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$$[\psi]p \quad \leftrightarrow \quad (\psi \to p)$$

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\begin{array}{rcl}
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Theorem Every formula of Public Announcement Logic is equivalent to a formula of Epistemic Logic.

- $[q]Kq$

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- $Kp \rightarrow [q]Kp$

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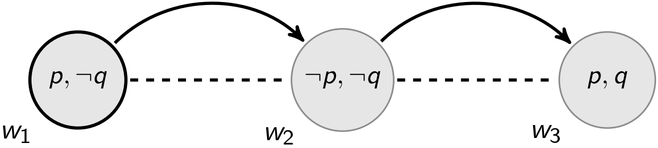

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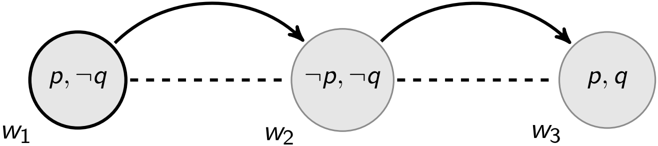

- $[\varphi]\varphi$

Public Announcement vs. Conditional Belief

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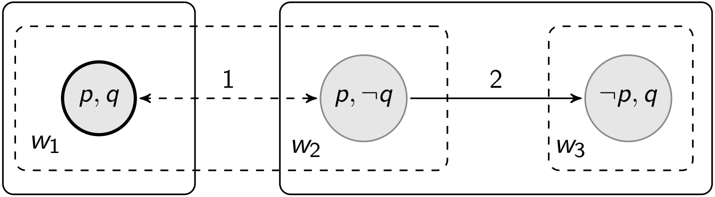

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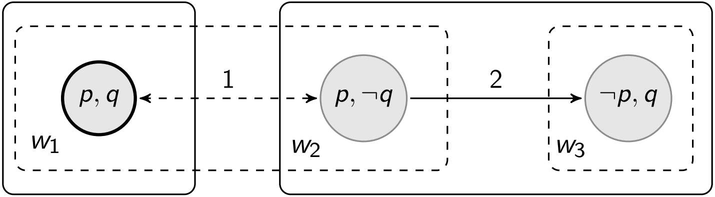

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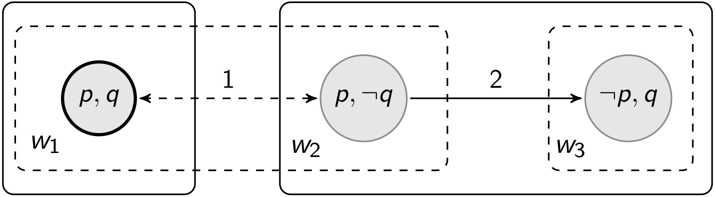

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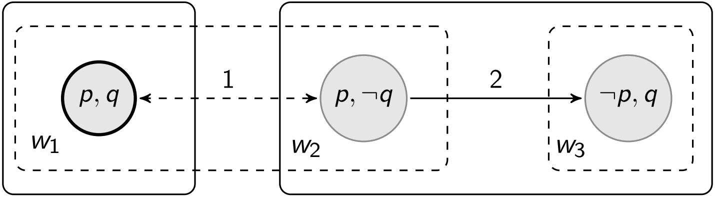

- $w_1 \models B_1 B_2 q$
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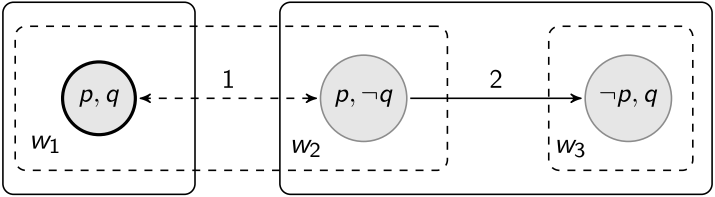
- $w_1 \models [p]\neg B_1 B_2 q$

Public Announcement vs. Conditional Belief

Are $[\varphi]B\psi$ and $B^{\varphi}\psi$ different? Yes!



- $w_1 \models B_1 B_2 q$
- $w_1 \models B_1^p B_2 q$
- $w_1 \models [p]\neg B_1 B_2 q$
- More generally, $B_i^p(p \wedge \neg K_i p)$ is satisfiable but $[p]B_i(p \wedge \neg K_i p)$ is not.

The Logic of Public Observation

- $[\varphi]K\psi \leftrightarrow (\varphi \to K(\varphi \to [\varphi]\psi))$

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- $[\varphi]K\psi \leftrightarrow (\varphi \rightarrow K(\varphi \rightarrow [\varphi]\psi))$

- $[\varphi][\preceq]\psi \leftrightarrow (\varphi \rightarrow [\preceq](\varphi \rightarrow [\varphi]\psi))$

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  $[\varphi]B\psi \leftrightarrow (\varphi \to B^{\varphi}[\varphi]\psi)$
  $[\varphi]B^{\alpha}\psi \leftrightarrow (\varphi \to B^{\varphi\wedge[\varphi]\alpha}[\varphi]\psi)$