# Reasoning about Knowledge and Beliefs

Lecture 1

Eric Pacuit

University of Maryland, College Park

pacuit.org
epacuit@umd.edu

September 4, 2013

# Practicalities

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- Course website on ELMS
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- Office Hours: Tuesdays 1-2 PM
- Office: Skinner 1103A

# Practicalities: Grading

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1. Participation & short quizzes (30%)
   - Quizzes are available online (multiple choice, true/false, short answers)
   - It is your job to monitor the due dates and take the quizzes!
   - No make-ups!

2. 2-3 Problem Sets (30%)
   - Short essay questions
   - Short proofs

3. Final Exam (40%)
   - In-class exam given during finals week

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Contemporary research papers published in academic journals and recent books (consult the schedule for details).

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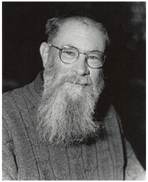
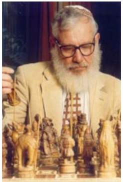
1. W. Holliday, Epistemic Logic and Epistemology, *Handbook of Formal Philosophy*, Springer, forthcoming
2. E. Pacuit, Dynamic Epistemic Logic I: Modeling Knowledge and Belief, *Philosophy Compass*, 2013
3. E. Pacuit, Dynamic Epistemic Logic II: Logics of Information Change, *Philosophy Compass*, 2013
4. R. Sorensen, Epistemic Paradoxes, Stanford Encyclopedia of Philosophy, 2011

# Foundations of Epistemic Logic



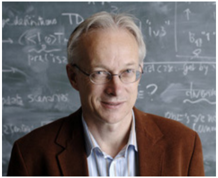
David Lewis

Jakko Hintikka



Robert Aumann

Larry Moss



Johan van Benthem

Alexandru Baltag

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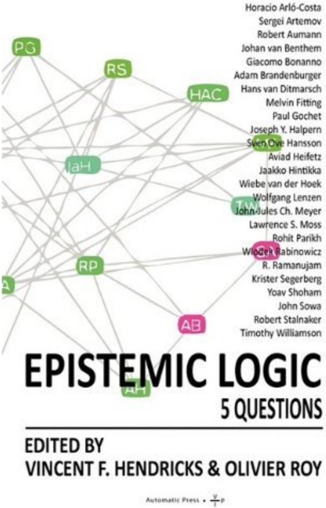

# Ten Puzzles and Paradoxes

1. Surprise Exam
2. The Knower
3. Logical Omniscience/Knowledge Closure
4. Lottery Paradox & Preface Paradox
5. Margin of Error Paradox
6. Fitch's Paradox
7. Aumann's Agreeing to Disagree Theorem
8. Brandenburger-Keisler Paradox
9. Absent-Minded Driver
10. Common Knowledge of Rationality and Backwards Induction

## A Word about the Word ‘Paradox’

What is a paradox? Roughly: an apparently good argument from apparently true premises to an apparently false/absurd conclusion.

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W.V. Quine. *The Ways of Paradox*. The Ways of Paradox and Other Essays. Cambridge: Harvard University Press, 1966.

Three introductory examples

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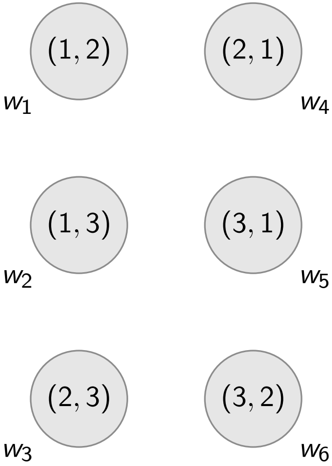

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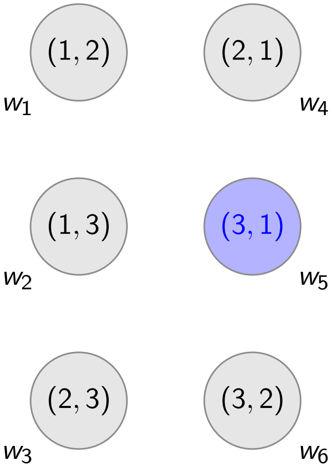

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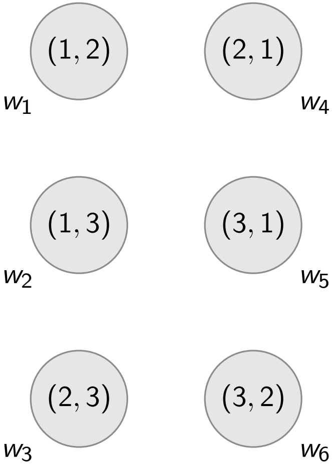

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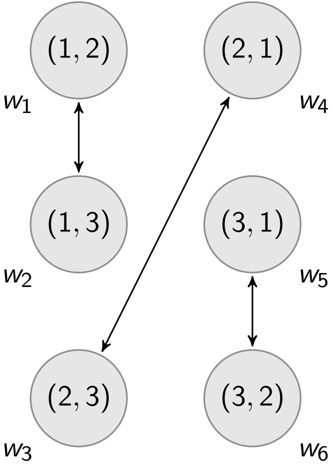

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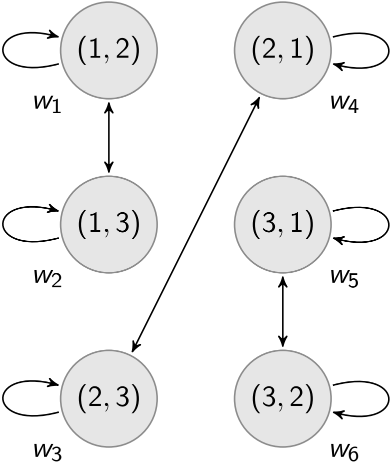

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Suppose $H_i$ is intended to mean "Ann has card $i$"

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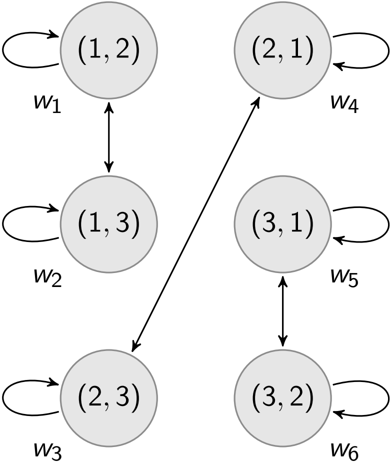
Eg., $V(H_1) = \{w_1, w_2\}$



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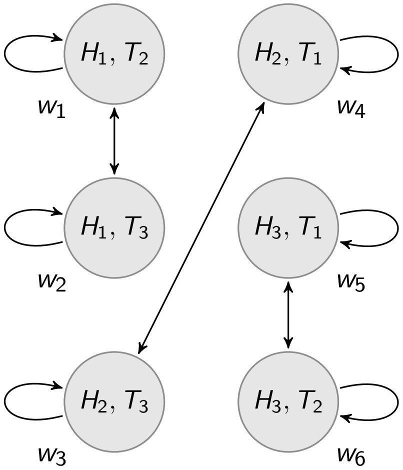

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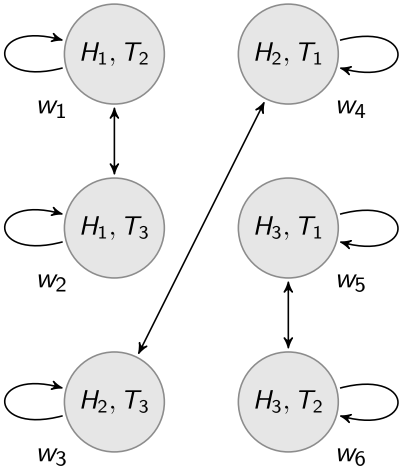

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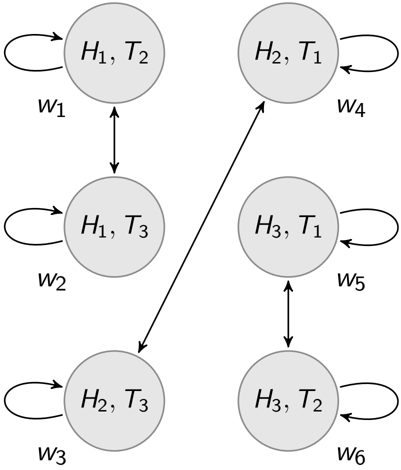

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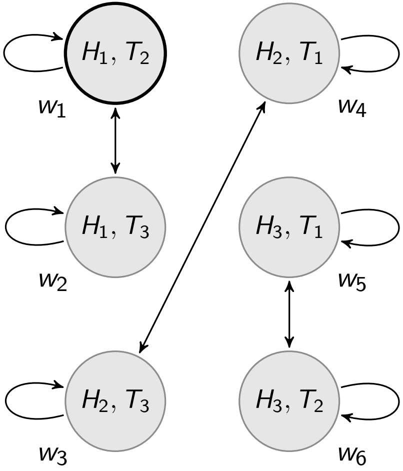

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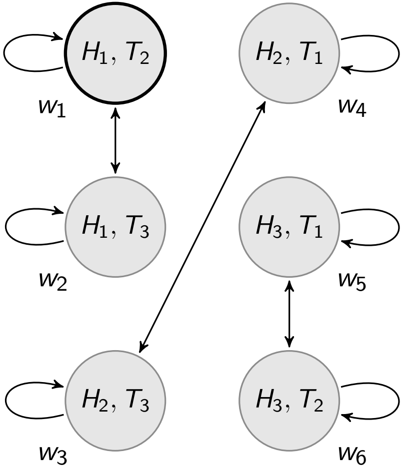

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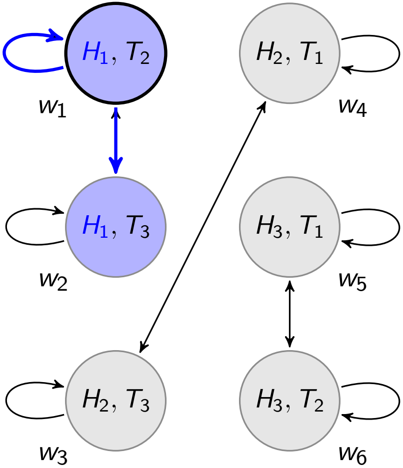

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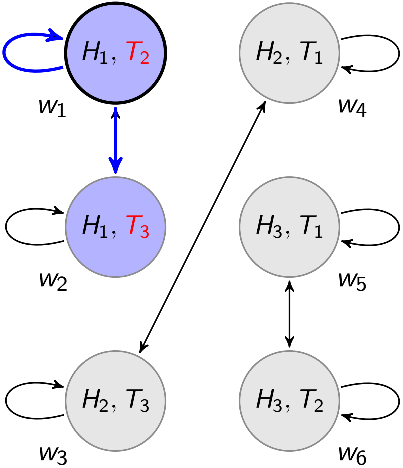

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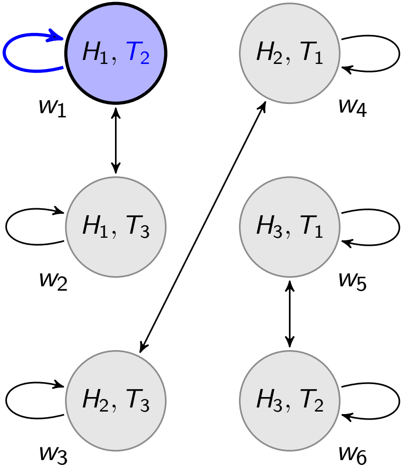

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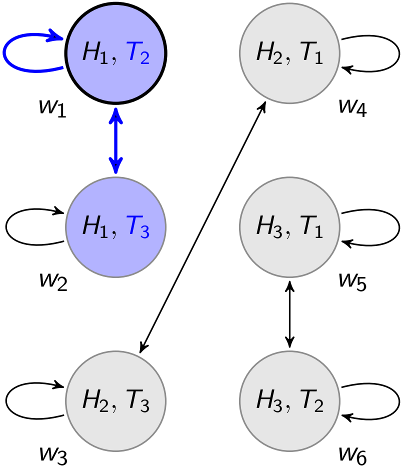

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- $K_a K_b \varphi$: "Ann knows that Bob knows $\varphi$"
- $K_a(K_b\varphi \vee K_b\neg\varphi)$: "Ann knows that Bob knows whether $\varphi$
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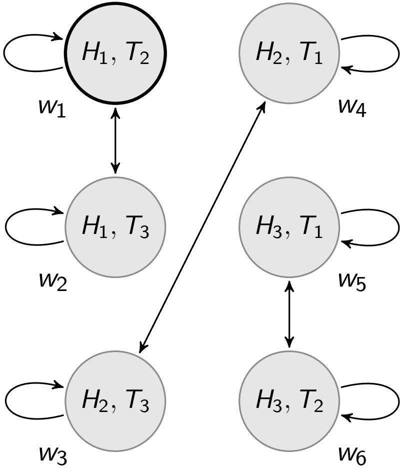

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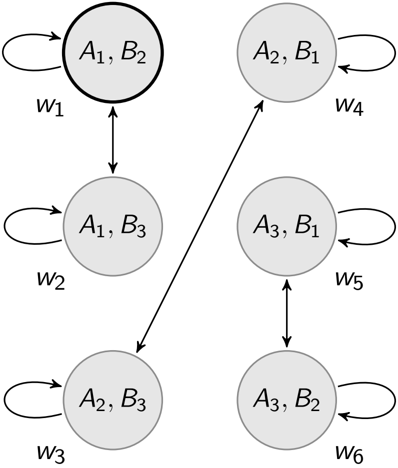

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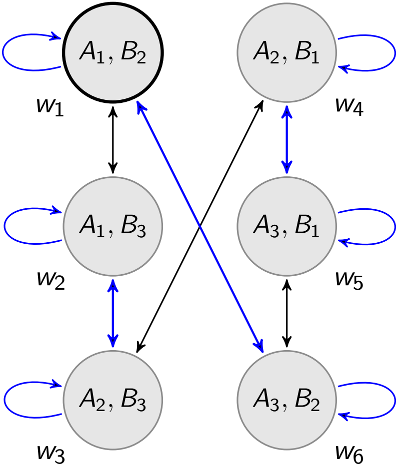

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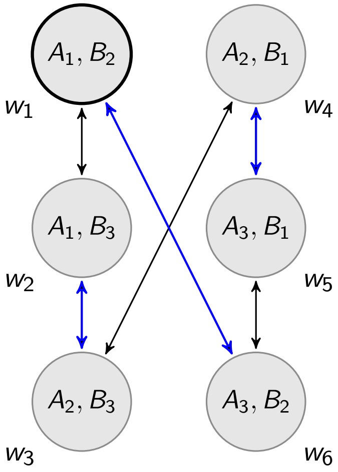

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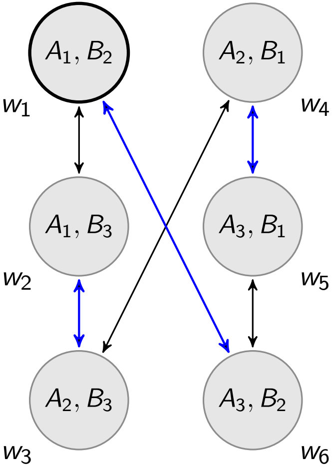

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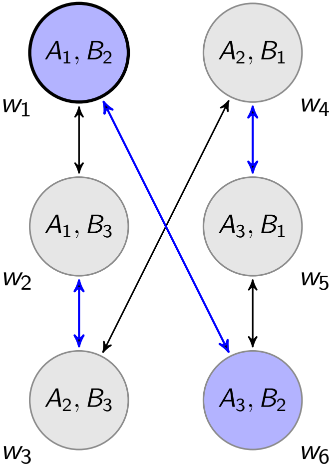

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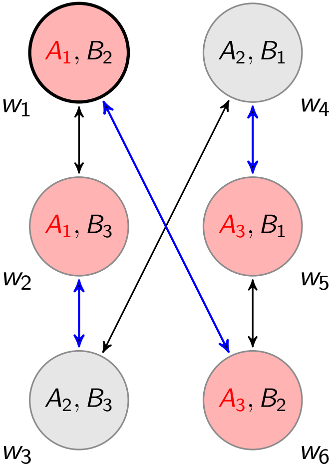

# College Park and Amsterdam

Let $K_c$ stand for **agent** $c$ **knows that** and $K_a$ stand for **agent** $a$ **knows that**. Suppose agent $c$, who lives in College Park, knows that agent $a$ lives in Amsterdam. Let $r$ stand for 'it's raining in Amsterdam'. Although $c$ doesn't know whether it's raining in Amsterdam, $c$ knows that $a$ knows whether it's raining there:

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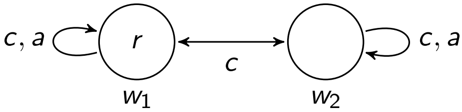
The following picture depicts a situation in which this is true, where an arrow represents *compatibility with one's knowledge*:



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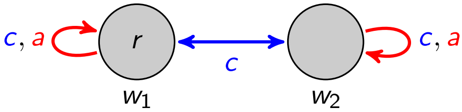

Now suppose that agent $c$ doesn't know whether agent $a$ has left Amsterdam for a vacation. (Let $v$ stand for '$a$ has left Amsterdam on vacation'.) Agent $c$ knows that if $a$ is not on vacation, then $a$ knows whether it's raining in Amsterdam; but if $a$ is on vacation, then $a$ won't bother to follow the weather.

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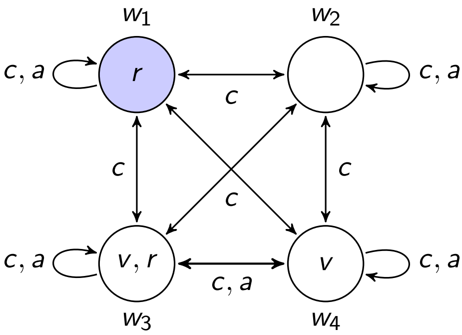

The Muddy Children Puzzle

Three children are outside playing. Two of them get mud on their forehead. They cannot see or feel the mud on their own foreheads, but can see who is dirty.

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Then the children are repeatedly asked "do you know if you have mud on your forehead?"

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# Muddy Children

Assume:

- There are three children: Ann, Bob and Charles.
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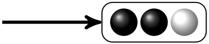
state-of-affairs



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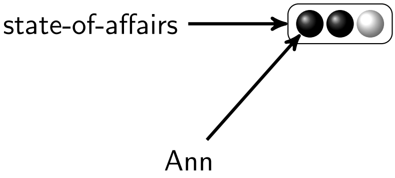

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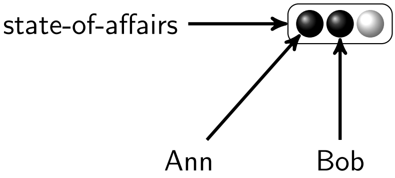

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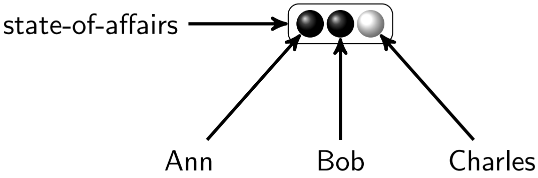

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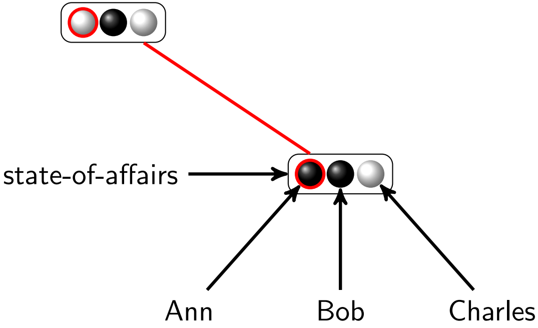

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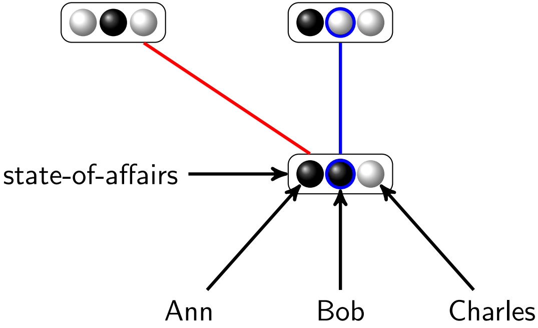

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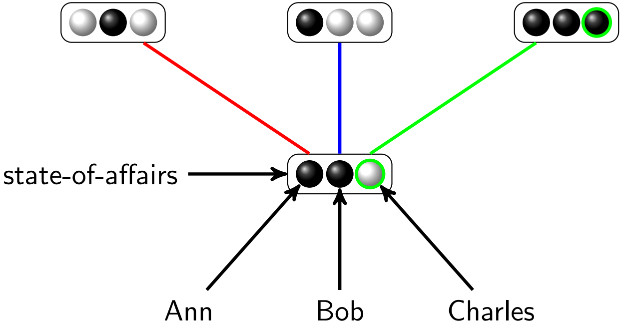

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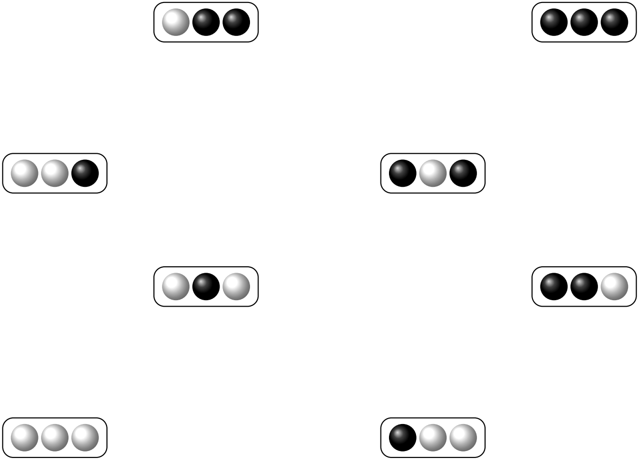
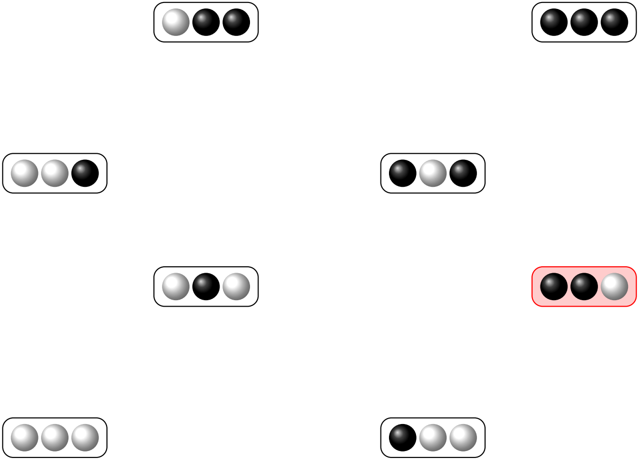


The 8 possible situations

# Muddy Children



The actual situation

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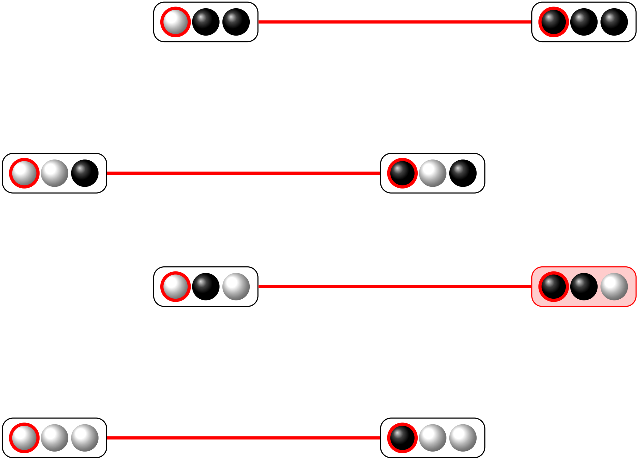
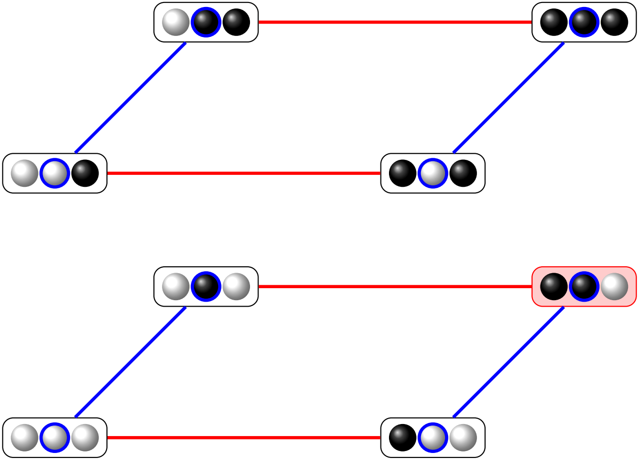


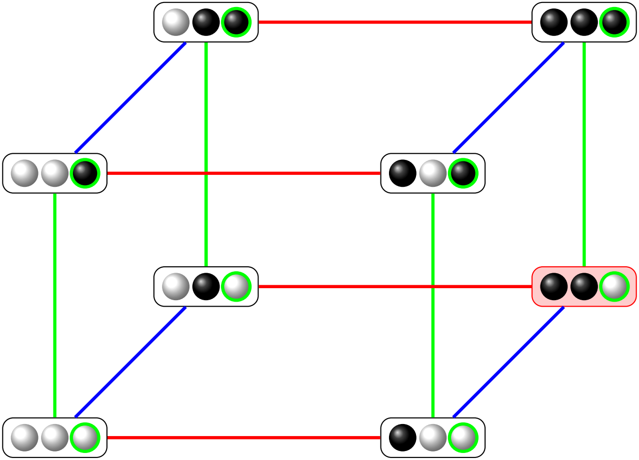
Ann's uncertainty

# Muddy Children



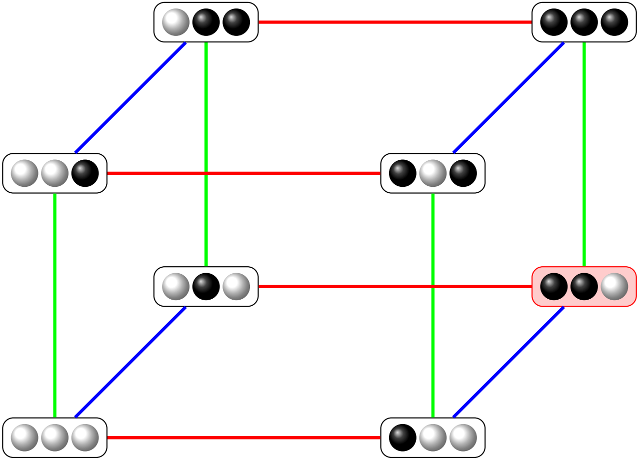
Bob's uncertainty

# Muddy Children



Charles' uncertainty

# Muddy Children



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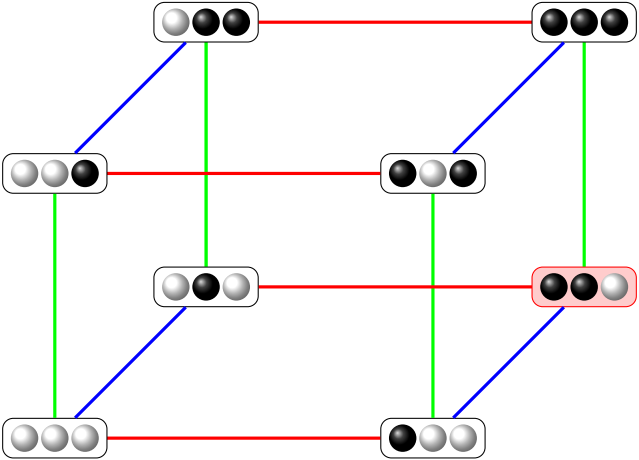
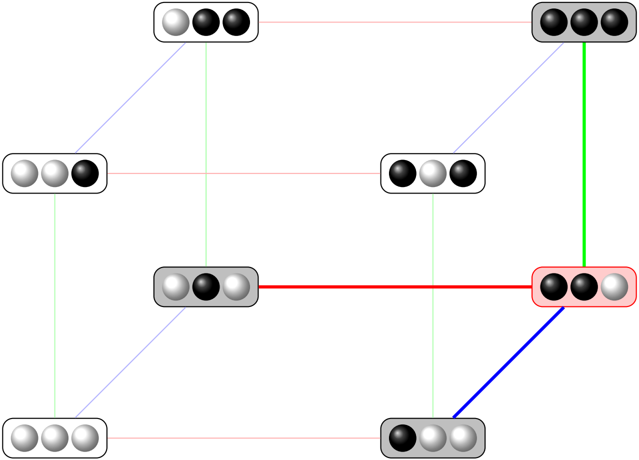


None of the children know if they are muddy

# Muddy Children



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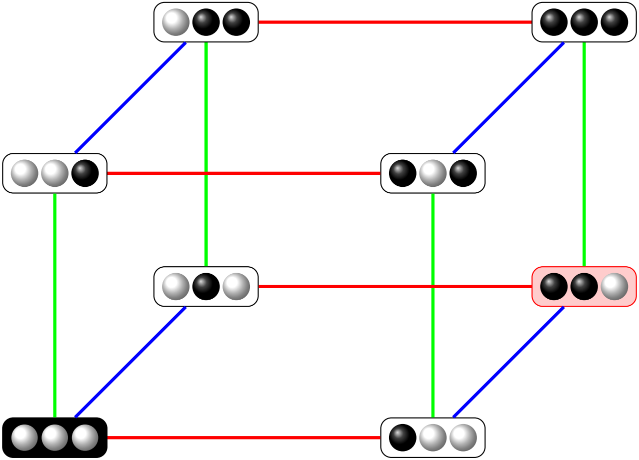
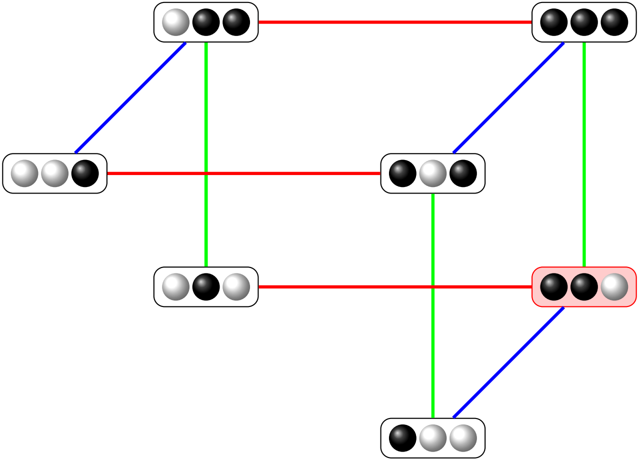


"At least one has mud on their forehead."

# Muddy Children



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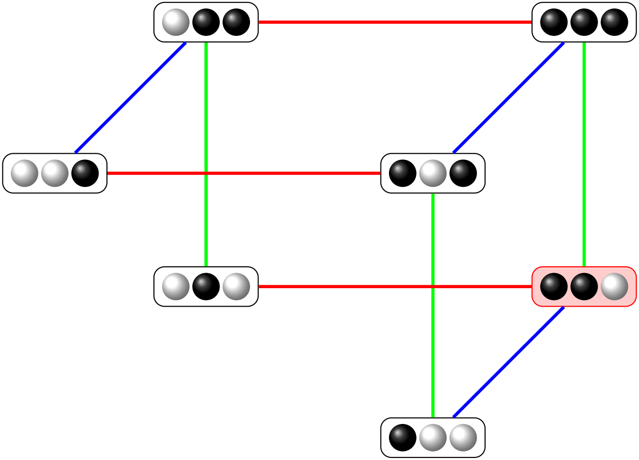

"Who has mud on their forehead?"

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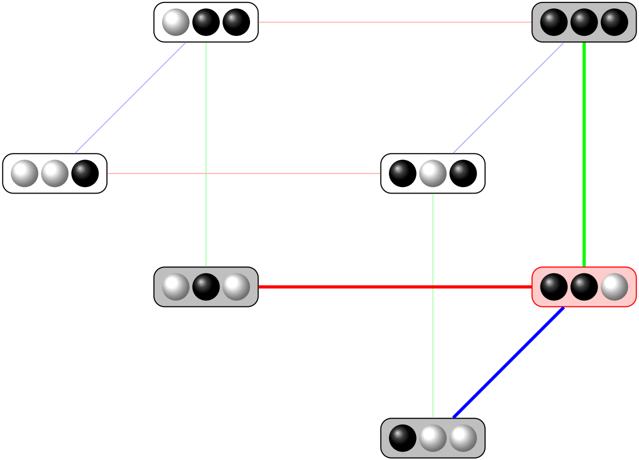

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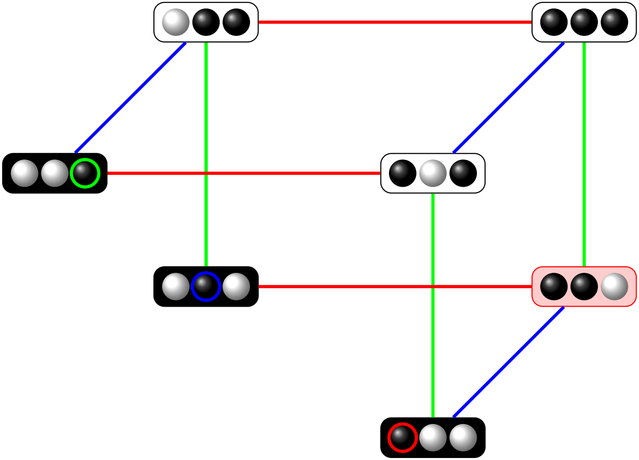

No one steps forward.

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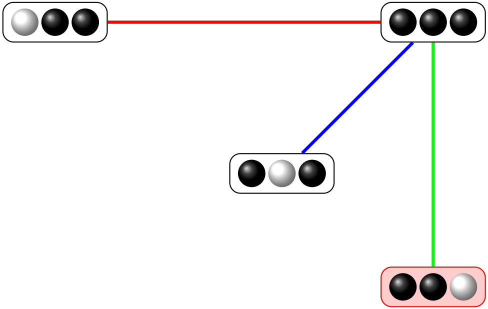

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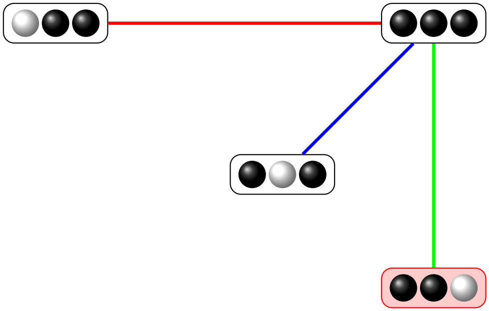
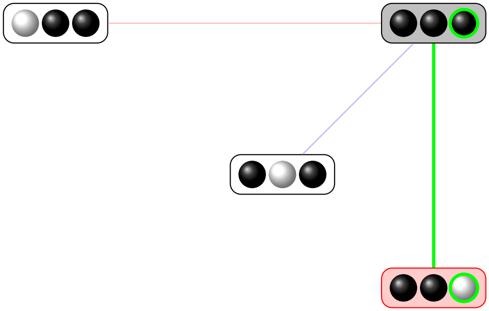


“Who has mud on their forehead?”

# Muddy Children



Charles does not know he is clean.

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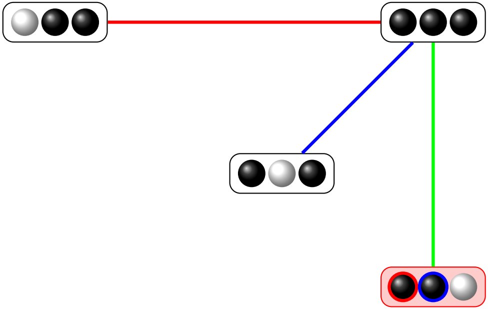
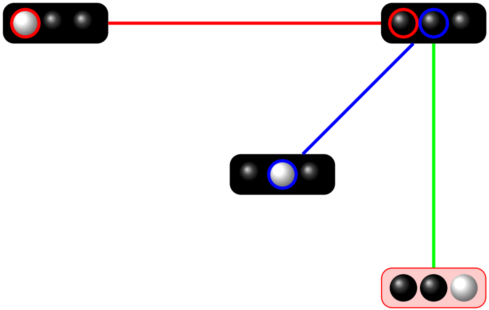


Ann and Bob step forward.

# Muddy Children



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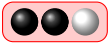

Now, Charles knows he is clean.