# Clear Thinking in an Uncertain World: Human Reasoning and its Foundations 

Lecture 12

Eric Pacuit<br>Department of Philosophy<br>University of Maryland, College Park<br>pacuit.org<br>epacuit@umd.edu

November 18, 2013

# Two Puzzles about Rationality and Coordination 

1. The Prisoner's Dilemma
2. Newcomb's Paradox

## Game Situations

1. a group of self-interested agents (players) involved in some interdependent decision problem

## Game Situations

## Bob <br> $L \quad R$ <br> 

1. a group of self-interested agents (players) involved in some interdependent decision problem

## Game Situations

## Bob <br> $L \quad R$ <br> 10 <br> $0 \quad 1$

1. a group of self-interested agents (players) involved in some interdependent decision problem

## Game Situations

$$
\begin{array}{ccc} 
& L & R \\
& 11 & 00 \\
& U & 1
\end{array}
$$

1. a group of self-interested agents (players) involved in some interdependent decision problem

## Game Situations



1. a group of self-interested agents (players) involved in some interdependent decision problem

## Game Situations



1. a group of self-interested agents (players) involved in some interdependent decision problem

## Game Situations



What should Ann (Bob) do?

## Game Situations



What should Ann (Bob) do?

## Just Enough Game Theory

"Game theory is a bag of analytical tools designed to help us understand the phenomena that we observe when decision-makers interact."

Osborne and Rubinstein. Introduction to Game Theory. MIT Press .

## Just Enough Game Theory

"Game theory is a bag of analytical tools designed to help us understand the phenomena that we observe when decision-makers interact."

Osborne and Rubinstein. Introduction to Game Theory. MIT Press .

A game is a description of strategic interaction that includes

- actions the players can take
- description of the players' interests (i.e., preferences),
- description of the "structure" of the decision problem


## Just Enough Game Theory

> "Game theory is a bag of analytical tools designed to help us understand the phenomena that we observe when decision-makers interact."

Osborne and Rubinstein. Introduction to Game Theory. MIT Press .

A game is a description of strategic interaction that includes

- actions the players can take
- description of the players' interests (i.e., preferences),
- description of the "structure" of the decision problem

It does not specify the actions that the players do take.

A solution concept is a systematic description of the outcomes that may emerge in a family of games.

This is the starting point for most of game theory and includes many variants: Nash equilibrium, backwards inductions, or iterated dominance of various kinds.

These are usually thought of as the embodiment of "rational behavior" in some way and used to analyze game situations.

A solution concept is a systematic description of the outcomes that may emerge in a family of games.

This is the starting point for most of game theory and includes many variants: Nash equilibrium, backwards inductions, or iterated dominance of various kinds.

These are usually thought of as the embodiment of "rational behavior" in some way and used to analyze game situations.

For this course, solution concepts are more of an endpoint.

Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they only care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.

Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they only care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.


Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they only care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.


Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they only care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.




What should Ann do?


What should Ann do? Bob best choice in Ann's worst choice


What should Ann do? maximize over each row and choose the maximum value

## Bob <br> TB TS <br> 

What should Bob do? minimize over each column and choose the maximum value

## Zero-Sum Games

Von Neumann Minmax Theorem. In any finite, two-player, zero-sum game, there is always at least one minmax solution.


What is a rational choice for Ann (Bob)?


What is a rational choice for Ann (Bob)? Flip a coin!


What is a rational choice for Ann (Bob)?


What is a rational choice for Ann (Bob)? Play a different game!

## Prisoner's Dilemma

Two people commit a crime.

## Prisoner's Dilemma

Two people commit a crime. The are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing.

## Prisoner's Dilemma

Two people commit a crime. The are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing. The police offer the following deal. Each one of them can confess and get credit for it.

## Prisoner's Dilemma

Two people commit a crime. The are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing. The police offer the following deal. Each one of them can confess and get credit for it. If only one confesses, he becomes a state witness and not only is he not punished, he gets a reward.

## Prisoner's Dilemma

Two people commit a crime. The are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing. The police offer the following deal. Each one of them can confess and get credit for it. If only one confesses, he becomes a state witness and not only is he not punished, he gets a reward. If both confess, they will be punished but will get reduced sentences for helping the police.

## Prisoner's Dilemma

Two people commit a crime. The are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing. The police offer the following deal. Each one of them can confess and get credit for it. If only one confesses, he becomes a state witness and not only is he not punished, he gets a reward. If both confess, they will be punished but will get reduced sentences for helping the police. If neither confesses, the police honestly admit that there is no way to convict them, and they are set free.

## Prisoner's Dilemma

## Two options: Confess (C), Don't Confess (D)

## Prisoner's Dilemma

## Two options: Confess (C), Don't Confess (D)

Possible outcomes:

## Prisoner's Dilemma

## Two options: Confess (C), Don't Confess (D)

Possible outcomes: We both confess ( $C, C$ ),

## Prisoner's Dilemma

## Two options: Confess (C), Don't Confess (D)

Possible outcomes: We both confess ( $C, C$ ), I confess but my partner doesn't ( $C, D$ ),

## Prisoner's Dilemma

Two options: Confess ( $C$ ), Don't Confess ( $D$ )

Possible outcomes: We both confess $(C, C)$, I confess but my partner doesn't $(C, D)$, My partner confesses but I don't $(D, C)$,

## Prisoner's Dilemma

## Two options: Confess (C), Don't Confess (D)

Possible outcomes: We both confess ( $C, C$ ), I confess but my partner doesn't $(C, D)$, My partner confesses but I don't $(D, C)$, neither of us confess $(D, D)$.

## Prisoner's Dilemma



## Prisoner's Dilemma



Ann's preferences

## Prisoner's Dilemma



Bob's preferences

## Prisoner's Dilemma



What should Ann (Bob) do?

## Dominance Reasoning



## Dominance Reasoning



## Dominance Reasoning



## Prisoner's Dilemma



What should Ann (Bob) do?

## Prisoner's Dilemma



What should Ann (Bob) do? Dominance reasoning

## Prisoner's Dilemma



What should Ann (Bob) do? Dominance reasoning

## Prisoner's Dilemma



What should Ann (Bob) do? Dominance reasoning is not Pareto!

## Prisoner's Dilemma



What should Ann (Bob) do? Think as a group!

## Prisoner's Dilemma



What should Ann (Bob) do? Play against your mirror image!

## Prisoner's Dilemma



What should Ann (Bob) do? Play against your mirror image!

## Prisoner's Dilemma



What should Ann (Bob) do? Change the game (eg., Symbolic Utilities)


What should/will Ann (Bob) do?


Assurance Game

What should/will Ann (Bob) do?


What should/will Ann (Bob) do?

## Nozick: Symbolic Utility

"Yet the symbolic value of an act is not determined solely by that act.

## Nozick: Symbolic Utility

"Yet the symbolic value of an act is not determined solely by that act. The act's meaning can depend upon what other acts are available with what payoffs and what acts also are available to the other party or parties.

## Nozick: Symbolic Utility

"Yet the symbolic value of an act is not determined solely by that act. The act's meaning can depend upon what other acts are available with what payoffs and what acts also are available to the other party or parties. What the act symbolizes is something it symbolizes when done in that particular situation, in preference to those particular alternatives.

## Nozick: Symbolic Utility

"Yet the symbolic value of an act is not determined solely by that act. The act's meaning can depend upon what other acts are available with what payoffs and what acts also are available to the other party or parties. What the act symbolizes is something it symbolizes when done in that particular situation, in preference to those particular alternatives. If an act symbolizes "being a cooperative person," it will have that meaning not simply because it has the two possible payoffs it does

## Nozick: Symbolic Utility

"Yet the symbolic value of an act is not determined solely by that act. The act's meaning can depend upon what other acts are available with what payoffs and what acts also are available to the other party or parties. What the act symbolizes is something it symbolizes when done in that particular situation, in preference to those particular alternatives. If an act symbolizes "being a cooperative person," it will have that meaning not simply because it has the two possible payoffs it does but also because it occupies a particular position within the two-person matrix - that is, being a dominated action that (when joined with the other person's dominated action) yield a higher payoff to each than does the combination of dominated actions. " (pg. 55)
R. Nozick. The Nature of Rationality. Princeton University Press, 1993.


Prisoner's Dilemma

What should/will Ann (Bob) do?


Prisoner's Dilemma


What should/will Ann (Bob) do?


What should/will Ann (Bob) do?


Prisoner's Dilemma


What should/will Ann (Bob) do?


Prisoner's Dilemma


What should/will Ann (Bob) do?


Prisoner's Dilemma


What should/will Ann (Bob) do?


Prisoner's Dilemma


What should/will Ann (Bob) do?


Prisoner's Dilemma


What should/will Ann (Bob) do?


Prisoner's Dilemma


What should/will Ann (Bob) do?
"Game theorists think it just plain wrong to claim that the Prisoners' Dilemma embodies the essence of the problem of human cooperation.
"Game theorists think it just plain wrong to claim that the Prisoners' Dilemma embodies the essence of the problem of human cooperation. On the contrary, it represents a situation in which the dice are as loaded against the emergence of cooperation as they could possibly be. If the great game of life played by the human species were the Prisoner's Dilemma, we wouldn't have evolved as social animals!
"Game theorists think it just plain wrong to claim that the Prisoners' Dilemma embodies the essence of the problem of human cooperation. On the contrary, it represents a situation in which the dice are as loaded against the emergence of cooperation as they could possibly be. If the great game of life played by the human species were the Prisoner's Dilemma, we wouldn't have evolved as social animals! .... No paradox of rationality exists. Rational players don't cooperate in the Prisoners' Dilemma, because the conditions necessary for rational cooperation are absent in this game."
(pg. 63)
K. Binmore. Natural Justice. Oxford University Press, 2005.

## Newcomb's Paradox

Two boxes in front of you, $A$ and $B$.

Box $A$ contains $\$ 1,000$ and box $B$ contains either $\$ 1,000,000$ or nothing.

## Newcomb's Paradox

Two boxes in front of you, $A$ and $B$.

Box $A$ contains $\$ 1,000$ and box $B$ contains either $\$ 1,000,000$ or nothing.

Your choice: either open both boxes, or else just open B. (You can keep whatever is inside any box you open, but you may not keep what is inside a box you do not open).

## Newcomb's Paradox



A very powerful being, who has been invariably accurate in his predictions about your behavior in the past, has already acted in the following way:

1. If he has predicted that you will open just box $B$, he has in addition put $\$ 1,000,000$ in box $B$
2. If he has predicted you will open both boxes, he has put nothing in box $B$.
What should you do?
R. Nozick. Newcomb's Problem and Two Principles of Choice. 1969.

## Newcomb's Paradox

|  | $\mathrm{B}=1 \mathrm{M}$ | $\mathrm{B}=0$ |
| :---: | :---: | :---: |
| 1 Box | 1 M | 0 |
| 2 Boxes | $1 \mathrm{M}+1000$ | 1000 |



## Newcomb's Paradox

|  | $\mathrm{B}=1 \mathrm{M}$ | $\mathrm{B}=0$ |
| :---: | :---: | :---: |
| 1 Box | 1 M | 0 |
| 2 Boxes | $1 \mathrm{M}+1000$ | 1000 |


|  | $\mathrm{B}=1 \mathrm{M}$ | $\mathrm{B}=0$ |
| :---: | :---: | :---: |
| 1 Box | $h$ | $1-h$ |
| 2 Boxes | $1-h$ | $h$ |

## Newcomb's Paradox

J. Collins. Newcomb's Problem. International Encyclopedia of Social and Behavorial Sciences, 1999.

## Newcomb's Paradox

There is a conflict between maximizing your expected value (1-box choice) and dominance reasoning (2-box choice).

## Newcomb's Paradox

There is a conflict between maximizing your expected value (1-box choice) and dominance reasoning (2-box choice).

Dominance reasoning is appropriate only when probability of outcome is independent of choice.

## Newcomb's Paradox

There is a conflict between maximizing your expected value (1-box choice) and dominance reasoning (2-box choice).

Dominance reasoning is appropriate only when probability of outcome is independent of choice. (A nasty nephew wants inheritance from his rich Aunt.

## Newcomb's Paradox

There is a conflict between maximizing your expected value (1-box choice) and dominance reasoning (2-box choice).

Dominance reasoning is appropriate only when probability of outcome is independent of choice. (A nasty nephew wants inheritance from his rich Aunt. The nephew wants the inheritance, but other things being equal, does not want to apologize.

## Newcomb's Paradox

There is a conflict between maximizing your expected value (1-box choice) and dominance reasoning (2-box choice).

Dominance reasoning is appropriate only when probability of outcome is independent of choice. (A nasty nephew wants inheritance from his rich Aunt. The nephew wants the inheritance, but other things being equal, does not want to apologize. Does dominance give the nephew a reason to not apologize?

## Newcomb's Paradox

There is a conflict between maximizing your expected value (1-box choice) and dominance reasoning (2-box choice).

Dominance reasoning is appropriate only when probability of outcome is independent of choice. (A nasty nephew wants inheritance from his rich Aunt. The nephew wants the inheritance, but other things being equal, does not want to apologize. Does dominance give the nephew a reason to not apologize? Whether or not the nephew is cut from the will may depend on whether or not he apologizes.)

## Newcomb's Paradox

There is a conflict between maximizing your expected value (1-box choice) and dominance reasoning (2-box choice).

Dominance reasoning is appropriate only when probability of outcome is independent of choice. (A nasty nephew wants inheritance from his rich Aunt. The nephew wants the inheritance, but other things being equal, does not want to apologize. Does dominance give the nephew a reason to not apologize? Whether or not the nephew is cut from the will may depend on whether or not he apologizes.)

What the Predictor did yesterday is probabilistically dependent on the choice today, but causally independent of today's choice.
$V(A)=\sum_{w} V(w) \cdot P_{A}(w)$
(the expected value of act $A$ is a probability weighted average of the values of the ways $w$ in which $A$ might turn out to be true)
$V(A)=\sum_{w} V(w) \cdot P_{A}(w)$
(the expected value of act $A$ is a probability weighted average of the values of the ways $w$ in which $A$ might turn out to be true)

Orthodox Bayesian Decision Theory: $P_{A}(w):=P(w \mid A)$
(Probability of $w$ given $A$ is chosen)

Causal Decision theory: $P_{A}(w)=P(A \square \rightarrow w)$ (Probability of if $A$ were chosen then $w$ would be true)

Suppose 99\% confidence in predictors reliability.
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
$M$ : receive $\$ 1,000,000$
L: receive \$1,001,000

Suppose 99\% confidence in predictors reliability.
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
$M$ : receive $\$ 1,000,000$
L: receive $\$ 1,001,000$
$V\left(B_{1}\right)=V(M) P\left(M \mid B_{1}\right)+V(N) P\left(N \mid B_{1}\right)$

Suppose 99\% confidence in predictors reliability.
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
M: receive \$1,000,000
L: receive \$1,001,000
$V\left(B_{1}\right)=V(M) P\left(M \mid B_{1}\right)+V(N) P\left(N \mid B_{1}\right)=$
$1000000 \cdot 0.99+0 \cdot 0.01$

Suppose 99\% confidence in predictors reliability.
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
M: receive \$1,000,000
L: receive \$1,001,000
$V\left(B_{1}\right)=V(M) P\left(M \mid B_{1}\right)+V(N) P\left(N \mid B_{1}\right)=$ $1000000 \cdot 0.99+0 \cdot 0.01=990,000$

Suppose 99\% confidence in predictors reliability.
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
$M$ : receive $\$ 1,000,000$
L: receive \$1,001,000
$V\left(B_{1}\right)=V(M) P\left(M \mid B_{1}\right)+V(N) P\left(N \mid B_{1}\right)=$
$1000000 \cdot 0.99+0 \cdot 0.01=990,000$
$V\left(B_{2}\right)=V(L) P\left(L \mid B_{2}\right)+V(K) P\left(K \mid B_{2}\right)$

Suppose 99\% confidence in predictors reliability.
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
$M$ : receive $\$ 1,000,000$
L: receive \$1,001,000
$V\left(B_{1}\right)=V(M) P\left(M \mid B_{1}\right)+V(N) P\left(N \mid B_{1}\right)=$
$1000000 \cdot 0.99+0 \cdot 0.01=990,000$
$V\left(B_{2}\right)=V(L) P\left(L \mid B_{2}\right)+V(K) P\left(K \mid B_{2}\right)=$
$1001000 \cdot 0.01+1000 \cdot 0.99$

Suppose 99\% confidence in predictors reliability.
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
$M$ : receive $\$ 1,000,000$
L: receive \$1,001,000
$V\left(B_{1}\right)=V(M) P\left(M \mid B_{1}\right)+V(N) P\left(N \mid B_{1}\right)=$
$1000000 \cdot 0.99+0 \cdot 0.01=990,000$
$V\left(B_{2}\right)=V(L) P\left(L \mid B_{2}\right)+V(K) P\left(K \mid B_{2}\right)=$
$1001000 \cdot 0.01+1000 \cdot 0.99=11,000$

Let $\mu$ be the assigned to the conditional $B_{1} \square \rightarrow M$ (and $B_{2} \square \rightarrow L$ ) (both conditional are true iff the Predictor put $\$ 1,000,000$ in box $B$ yesterday).
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
M: receive \$1,000,000
L: receive $\$ 1,001,000$

Let $\mu$ be the assigned to the conditional $B_{1} \square \rightarrow M$ (and $B_{2} \square \rightarrow L$ ) (both conditional are true iff the Predictor put $\$ 1,000,000$ in box $B$ yesterday).
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
M: receive \$1,000,000
L: receive $\$ 1,001,000$
$V\left(B_{1}\right)=V(M) P\left(B_{1} \square \rightarrow M\right)+V(N) P\left(B_{1} \square \rightarrow N\right)$

Let $\mu$ be the assigned to the conditional $B_{1} \square \rightarrow M$ (and $B_{2} \square \rightarrow L$ ) (both conditional are true iff the Predictor put $\$ 1,000,000$ in box $B$ yesterday).
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
M: receive \$1,000,000
L: receive $\$ 1,001,000$
$V\left(B_{1}\right)=V(M) P\left(B_{1} \square \rightarrow M\right)+V(N) P\left(B_{1} \square \rightarrow N\right)=$
$1000000 \cdot \mu+0 \cdot 1-\mu$

Let $\mu$ be the assigned to the conditional $B_{1} \square \rightarrow M$ (and $B_{2} \square \rightarrow L$ ) (both conditional are true iff the Predictor put $\$ 1,000,000$ in box $B$ yesterday).
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
$M$ : receive $\$ 1,000,000$
L: receive $\$ 1,001,000$
$V\left(B_{1}\right)=V(M) P\left(B_{1} \square \rightarrow M\right)+V(N) P\left(B_{1} \square \rightarrow N\right)=$
$1000000 \cdot \mu+0 \cdot 1-\mu=1000000 \mu$

Let $\mu$ be the assigned to the conditional $B_{1} \square \rightarrow M$ (and $B_{2} \square \rightarrow L$ ) (both conditional are true iff the Predictor put $\$ 1,000,000$ in box $B$ yesterday).
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
$M$ : receive $\$ 1,000,000$
L: receive $\$ 1,001,000$
$V\left(B_{1}\right)=V(M) P\left(B_{1} \square \rightarrow M\right)+V(N) P\left(B_{1} \square \rightarrow N\right)=$
$1000000 \cdot \mu+0 \cdot 1-\mu=1000000 \mu$
$V\left(B_{2}\right)=V(L) P\left(B_{2} \square \rightarrow L\right)+V(K) P\left(B_{2} \square \rightarrow K\right)$

Let $\mu$ be the assigned to the conditional $B_{1} \square \rightarrow M$ (and $B_{2} \square \rightarrow L$ ) (both conditional are true iff the Predictor put $\$ 1,000,000$ in box $B$ yesterday).
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
$M$ : receive $\$ 1,000,000$
L: receive $\$ 1,001,000$
$V\left(B_{1}\right)=V(M) P\left(B_{1} \square \rightarrow M\right)+V(N) P\left(B_{1} \square \rightarrow N\right)=$
$1000000 \cdot \mu+0 \cdot 1-\mu=1000000 \mu$
$V\left(B_{2}\right)=V(L) P\left(B_{2} \square \rightarrow L\right)+V(K) P\left(B_{2} \square \rightarrow K\right)=$
$1001000 \cdot \mu+1000 \cdot 1-\mu$

Let $\mu$ be the assigned to the conditional $B_{1} \square \rightarrow M$ (and $B_{2} \square \rightarrow L$ ) (both conditional are true iff the Predictor put $\$ 1,000,000$ in box $B$ yesterday).
$B_{1}$ : one-box (open box $B$ )
$B_{2}$ : two-box choice (open both $A$ and $B$ )
$N$ : receive nothing
$K$ : receive $\$ 1,000$
$M$ : receive $\$ 1,000,000$
L: receive $\$ 1,001,000$
$V\left(B_{1}\right)=V(M) P\left(B_{1} \square \rightarrow M\right)+V(N) P\left(B_{1} \square \rightarrow N\right)=$
$1000000 \cdot \mu+0 \cdot 1-\mu=1000000 \mu$
$V\left(B_{2}\right)=V(L) P\left(B_{2} \square \rightarrow L\right)+V(K) P\left(B_{2} \square \rightarrow K\right)=$
$1001000 \cdot \mu+1000 \cdot 1-\mu=1000000 \mu+1000$
D. Lewis. Prisoner's Dilemma Is a Newcomb Problem. Philosophy and Public Affairs, 8, pgs. 235-240, 1979.
S. Brams. Newcomb's Problem and Prisoners' Dilemma. The Journal of Conflict Resolution, 19:4, pgs. 596-612, 1975.
S. Hurley. Newcomb's Problem, Prisoner's Dilemma and Collective Action. Synthese 86, pgs. 173-196, 1991.

