Clear Thinking in an Uncertain World: Human Reasoning and its Foundations

Lecture 2

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But, what is **good thinking**?

- classical logic (modus ponens, modus tollens, etc.)
- non-monotonic/default logic
- closed-world reasoning
- induction (induction from examples)
- Abduction (inference to the best explanation)
- Bayesian inference
- case-based reasoning/reasoning by analogy
- fast and frugal heuristics
- ...

A Crash Course in Logic

Classical Logic

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- *Adjunction*: $P_1,\ P_2, \ldots P_n \vdash P_1 \wedge \cdots \wedge P_n$
- *Noncontradiction*: $P,\ \neg P \vdash Q$
- *Monotonicity 1*: $P \rightarrow Q \vdash (P \wedge R) \rightarrow Q$
- *Monotonicity 2*: If $P \vdash Q$ then $P, R \vdash Q$

Reminder: Issues from Lecture 1

- Cognitive limitations: rationality vs. genius
- Should we always make logical inferences?: Clutter avoidance
- Reasoning may lead to revising
- Foundational problems: Epistemic closure, natural language challenges

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2. P is true; $P \to Q$ is true; So, Q is true.

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A set of formulas is **inconsistent** if there is no way of making all of the formulas true

1. Ann recognizes that $\{P, Q, R\}$ are inconsistent
2. $\{P, Q, R\}$ are inconsistent

Rationality versus genius

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A, B, C imply D. Sam believes A, B and C. But some does nto realize that A, B, C imply D. In fact, it would take a genius to recognize that $A, B, C \vdash D$. And Sam, although a rational man, is far from a genius.

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From "It is raining in College Park" to "It is raining in College Park or Lily is at school" is a valid inference. In fact, there are infinitely many such trivial consequences (P, $P \vee Q$, $P \wedge P$, $P \rightarrow P$, $P \vee Q \vee R$, etc.), but these will just "clutter the mind".

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Also, if one "looses" the origination of this disjunctive belief, one may be mislead to think that there is a special reason to believe Lily is at school or there is a special connection between rain in College Park and Lily being at school.

Discovering a Contradiction

Sally believes A, B, C and has just come to realize that $A, B, C \vdash D$. Unfortunately, she also believes for very good reasons that D is false. So she now has reason to stop believing A, B or C, rather than a reason to believe D.

Reasoning May Lead to Revising

Modus Ponens: $P, P \rightarrow Q \vdash Q$

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Modus Ponens: $P, P \to Q \vdash Q$

Suppose that Ann believes that if she will attend Yale, then she will become an atheist. She also believes that she will attend Yale.

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She concludes that she will become an atheist.

But although MP gives Ann a reason to believe the conclusion, it does not decide that she will believe it. Instead of believing the conclusion, she may decide to drop her belief in the conditional.

Reasoning

"Reasoning is not the conscious rehearsal of argument; it is a process in which antecedent beliefs and intentions are minimally modified, by addition and subtraction, in the interests of explanatory coherence and the satisfaction of intrinsic desires." (G. Harman, pg. 56, "Practical Reasoning")

Foundational Problem: Epistemic Closure

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(1) The animal I am looking at is a zebra.

(2) If the animal I am looking at is a zebra, then it is not a mule cleverly disguised to look like a zebra.

(3) The animal I am looking at is not a mule cleverly disguised to look like a zebra.

S. Luper. *The Epistemic Closure Principle*. Stanford Encyclopedia of Philosophy: http://plato.stanford.edu/entries/closure-epistemic/.

Ordinary Language Challenges

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1. John goes drinking and John gets arrested.
2. John gets arrested and John goes drinking.

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$P \rightarrow Q \nvdash \neg P \rightarrow \neg Q$

1. If you tutor me in logic, I'll pay you \$50.
2. If you don't tutor me, I won't pay you \$50.

Ordinary Language Challenges: Gricean Implicature

> *He [the speaker] has said that p; there is no reason to suppose that he is not observing the maxims, or at least the Cooperative Principle; he could not be doing this unless he thought that q; he knows (and knows that I know that he knows) that I can see the supposition that he thinks that q is required....he intends me to think...that q; and so he has implicated q.*

Cooperative Principle: The speaker intends his contribution to be informative, warranted, relevant and well formed.

H. P. Grice. *Studies in the Way of Words*. Harvard University Press, 1989.

Domain independence, I

Secure argument: Human beings are sensitive to pain. Harry is a human being. So, Harry is sensitive to pain.

Generalization: X's are Y. A is an X. So A is Y.

Counterexample: Human beings are evenly distributed over the earth's surface. Harry is a human being. So, Harry is evenly distributed over the earth's surface.

Domain independence, I

Secure argument: There is a fire in my kitchen. My kitchen is in my house. Hence, there is a fire in my house.

Generalization: X is in Y. Y is in Z. So X is in Z.

Counterexample: There is a pain in my foot. My foot is in my shoe. Hence, there is a pain in my shoe.

Domain independence, II

All A are B.
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Therefore, all A are C.

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"How *not* to think about logical reasoning"

"This *schematic* character of inference patterns is identified with the "domain independence" or "topic neutrality" of logic generally, and many take it to be the principal interest of logic that its law seem independent of subject matter."

How to think about logical reasoning

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"We therefore view reasoning as consisting of two stages: first one has to establish the domain about which one reasons and its formal properties (what we will call "reasoning *to* an interpretation") and only after this initial step has been taken can one's reasoning be guided by formal laws (what we will call "reasoning *from* an interpretation")." (pg. 20)

The set of parameters characterizing a logic can be divided in three subsets:

1. Choice of formal language
2. Choice of a semantics for the formal language
3. Choice of a definition of valid arguments in the language

Classical Logic "Parameters"

1. *Syntax*: if φ, ψ are sentences, then so are $\neg\varphi$, $\varphi \wedge \psi$, $\varphi \vee \psi$, and $\varphi \rightarrow \psi$
2. *Semantics* (truth-functionality): the truth-value of a sentence is a function of the truth-values of its components only
3. *Semantics* (bivalence): sentences are either true or false, with nothing in-between
4. *consequence*: $\alpha_1 \ldots \alpha_n / \beta$ is valid iff β is true in all models of $\alpha_1, \ldots, \alpha_n$

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Domains to which classical logic is applicable must satisfy these four assumptions.

Truth-functionality without bivalence: “unknown”

“Is $2^{1257787} - 1$ prime?”

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Non-Truth-Functional Semantics

Intuitionistic logic:

1. $\varphi \wedge \psi$ means "I have a proof of both φ and ψ"
2. $\varphi \vee \psi$ means "I have a proof of φ or a proof of ψ"
3. $\varphi \rightarrow \psi$ means "I have a construction that transforms a proof of φ into a proof of ψ"
4. $\neg\varphi$ means "Any proof of φ leads to a contradiction"

Clearly, $\varphi \vee \neg\varphi$ is not valid.

An Intensional Logic: Deontic Logic

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Compare: $p \to q$ to $p \to Oq$.

"Common Sense" Reasoning

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(3) Tweety is a bird

So, (4) Tweety flies.

(3.1) Tweety is a penguin.

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$A \rightarrow B \vdash (A \wedge C) \rightarrow B$
'If you put sugar in the coffee, then it will taste good' can be true without 'If you put sugar and gasoline in the coffee, then it will taste good' being true.

Non-monotonic logic: What *should*/*do* I believe?

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Failure on monotonicity: B: Tweety is a bird; F: Tweety flies; P: Tweety is a penguin

$B \vdash F$ but $B, P \not\vdash F$.

Non-monotonic logic

$\varphi \mathrel{|\!\sim} \psi$ "If φ then *typically* (*mostly*, etc.) ψ"

Nonmonotonic Reasoning

Left logical equivalence: If $\vdash \varphi \leftrightarrow \psi$ and $\varphi \mid\sim \alpha$ then $\psi \mid\sim \alpha$

Right weakening: If $\vdash \alpha \rightarrow \beta$ and $\varphi \mid\sim \alpha$ then $\varphi \mid\sim \beta$

And: If $\varphi \mid\sim \alpha$ and $\varphi \mid\sim \beta$ then $\varphi \mid\sim (\alpha \wedge \beta)$

Or: If $\varphi \mid\sim \alpha$ and $\psi \mid\sim \alpha$ then $(\varphi \vee \psi) \mid\sim \alpha$

Monotonicity

Monotonicity: $\varphi \mathrel{|\!\sim} \alpha$ then $\varphi \wedge \psi \mathrel{|\!\sim} \alpha$

C: coffee in the cup, T: the liquid tastes good; O: oil is in the cup

$C \mathrel{|\!\sim} T$ but $C \wedge O \mathrel{|\!\not\sim} T$
But note that $O \mathrel{|\!\not\sim} T$

Cautious Monotonicity: If $\varphi \mathrel{|\!\sim} \alpha$ and $\varphi \mathrel{|\!\sim} \beta$ then $\varphi \wedge \alpha \mathrel{|\!\sim} \beta$

Rational Monotonicity: If $\varphi \mathrel{|\!\sim} \alpha$ and $\varphi \mathrel{|\!\not\sim} \neg\beta$, then $\varphi \wedge \beta \mathrel{|\!\sim} \alpha$

Closed-world reasoning

Negation as failure
Suppose you are interested in whether there are any direct flights from Amsterdam to Cleveland, Ohio.

After searching online at a number of relevant sites (Expedia, Orbitz, KLM, etc.), you do not find any. You conclude that there are *no direct flights between Amsterdam and Cleveland*.

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- Reasoning rarely happens in real life: we have developed "fast and frugal algorithms" which allow us to take quick decisions which are optimal given constraints of time and energy.

Concluding Remarks

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J. Hintikka. *Inquiry as Inquiry*. Kluwer Academic Publishers, 1999.

Next: More on logic: read Chapter 2 of Stenning and van Lambalgen