CMSC 132:
Object-Oriented Programming II

Sorting

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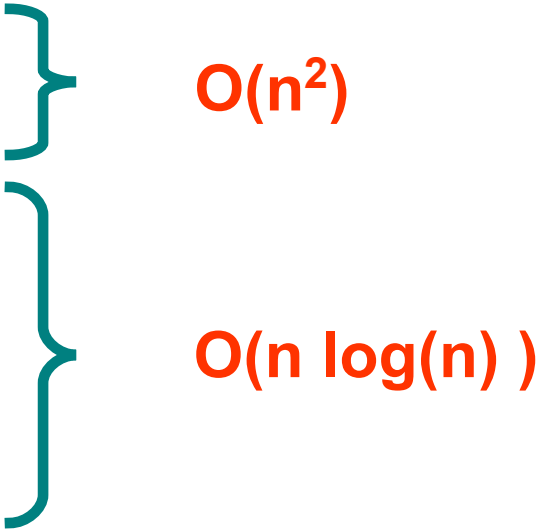
Overview

- Comparison sort
  - Bubble sort
  - Selection sort
  - Tree sort
  - Heap sort
  - Quick sort
  - Merge sort
- Linear sort
  - Counting sort
  - Bucket (bin) sort
  - Radix sort



Sorting

- **Goal**
  - Arrange elements in predetermined order
    - Based on key for each element
  - Derived from ability to compare two keys by size
- **Properties**
  - Stable ≅ relative order of equal keys unchanged
    - Stable: 3, 1, 4, 3, 3, 2 ≅ 1, 2, 3, 3, 3, 4
    - Unstable: 3, 1, 4, 3, 3, 2 ≅ 1, 2, 3, 3, 3, 4
  - In-place ≅ uses only constant additional space
  - External ≅ can efficiently sort large # of keys

Sorting

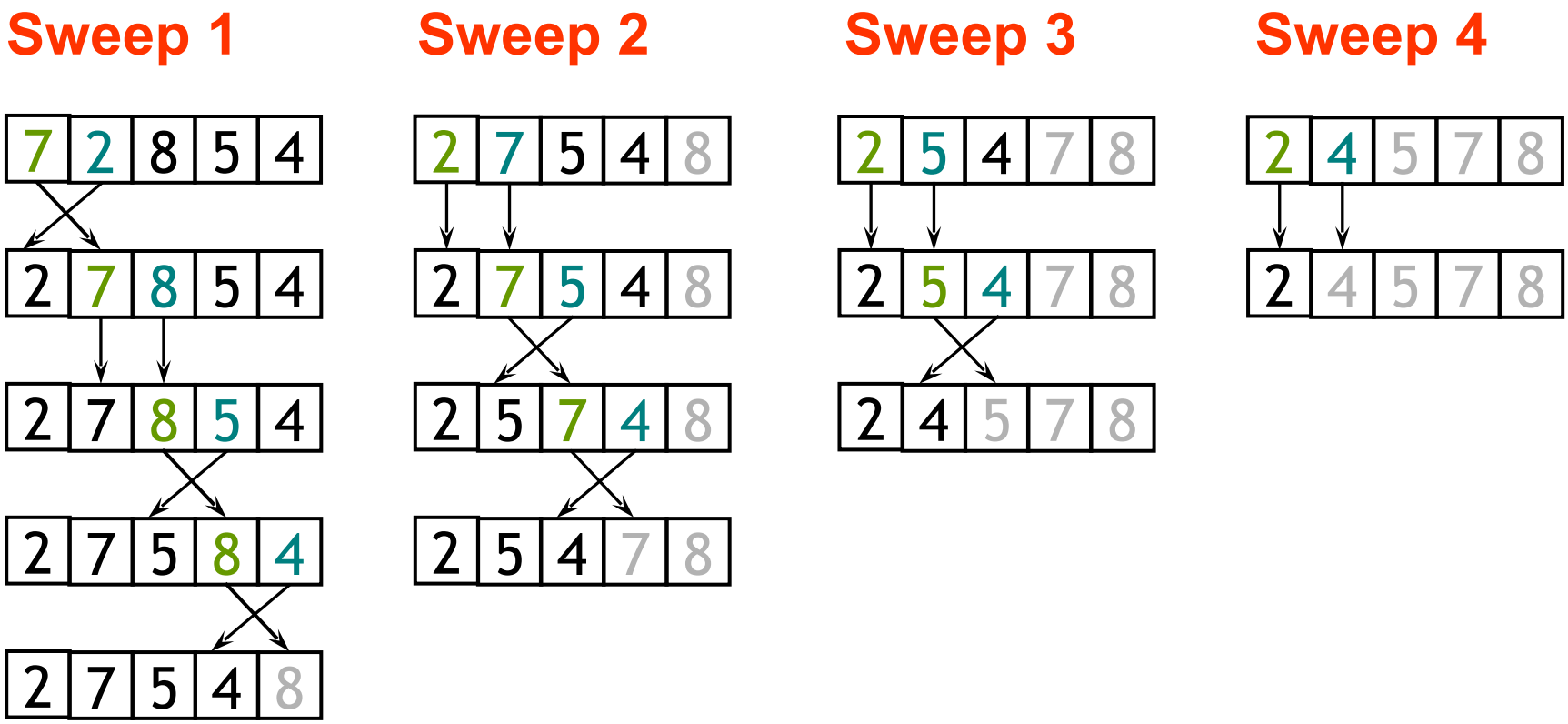
- Comparison sort
  - Only uses pairwise key comparisons
  - Proven lower bound of O(n log(n))
- Linear sort
  - Uses additional properties of keys

Bubble Sort

- **Approach**
  1. Iteratively sweep through shrinking portions of list
  2. Swap element x with its right neighbor if x is larger

- **Performance**
  - $O(n^2)$ average / worst case

Bubble Sort Example



Bubble Sort Code

```
void bubbleSort(int[ ] a) {
    int outer, inner;
    for (outer = a.length - 1; outer > 0; outer--)
        for (inner = 0; inner < outer; inner++)
          if (a[inner] > a[inner + 1])
              swap(a, inner, inner+1);
}

void swap(int a[ ], int x, int y) {
    int temp = a[x];
    a[x] = a[y];
    a[y] = temp;
}
```

Sweep through array

Swap with right neighbor if larger

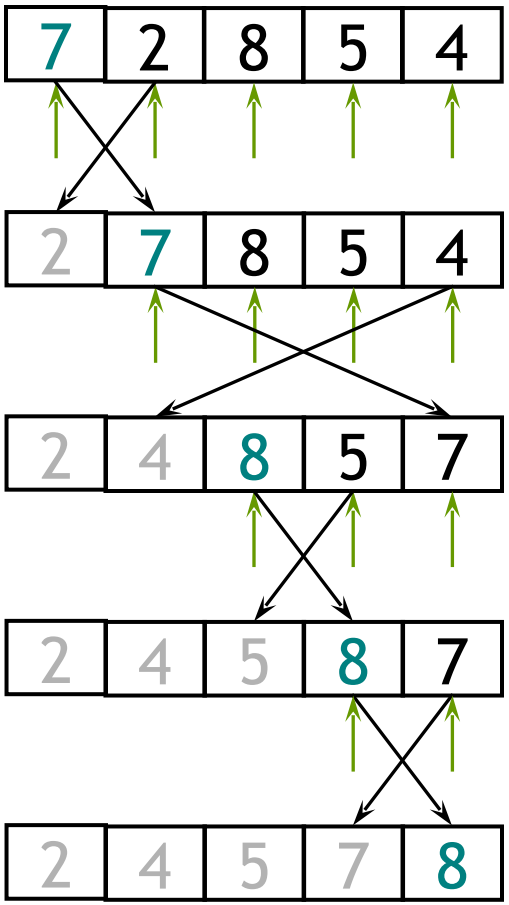
Swap array elements at positions x & y

Selection Sort

- **Approach**
  1. Iteratively sweep through shrinking portions of list
  2. Select smallest element found in each sweep
  3. Swap smallest element with front of current list

- **Performance**
  - $O(n^2)$ average / worst case

- **Example**



Selection Sort Code

```
void selectionSort(int[] a) {
        int outer, inner, min;
        for (outer = 0; outer < a.length - 1; outer++) {
            min = outer;
            for (inner = outer + 1; inner < a.length; inner++) {
                if (a[inner] < a[min]) {
                    min = inner;
                }
            }
            swap(a, outer, min);
        }
}
```

Sweep through array

Find smallest element

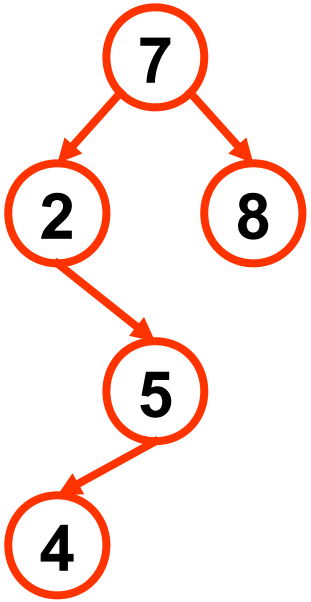
Swap with smallest element found

Tree Sort

- **Approach**
  1. Insert elements in binary search tree
  2. List elements using inorder traversal
- **Performance**
  - Binary search tree
    - O(n log(n)) average case
    - O(n^2) worst case
  - Balanced binary search tree
    - O(n log(n)) average / worst case

- **Example**

  Binary search tree



{ 7, 2, 8, 5, 4 }

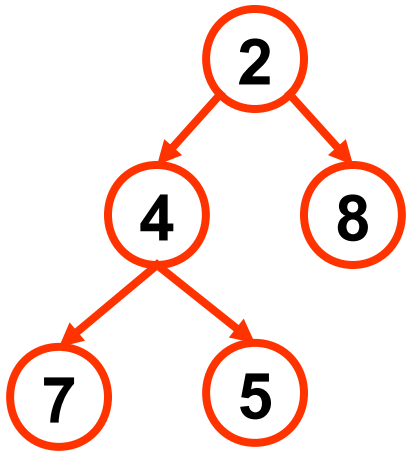
Heap Sort

- **Approach**
  1. Insert elements in heap
  2. Remove smallest element in heap, repeat
  3. List elements in order of removal from heap

- **Performance**
  - O(n log(n)) average / worst case

- **Example**

  Heap



{ 7, 2, 8, 5, 4 }

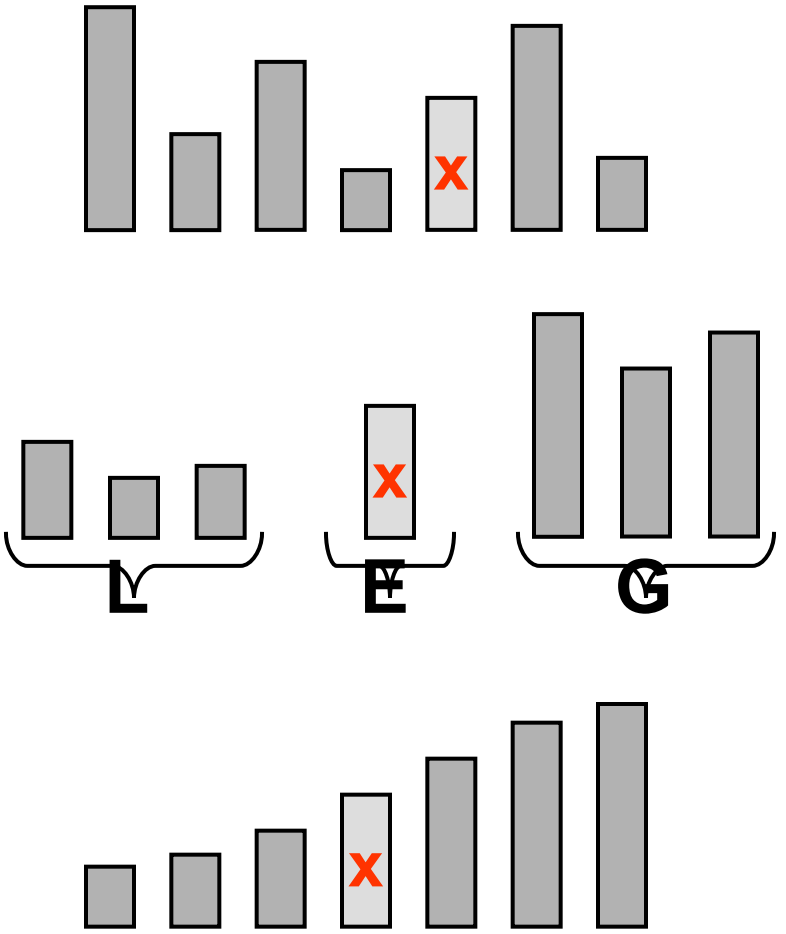
Quick Sort

- **Approach**
  1. Select pivot value (near median of list)
  2. Partition elements (into 2 lists) using pivot value
  3. Recursively sort both resulting lists
  4. Concatenate resulting lists
  - For efficiency pivot needs to partition list evenly

- **Performance**
  - O(n log(n)) average case
  - O(n^2) worst case

Quick Sort Algorithm

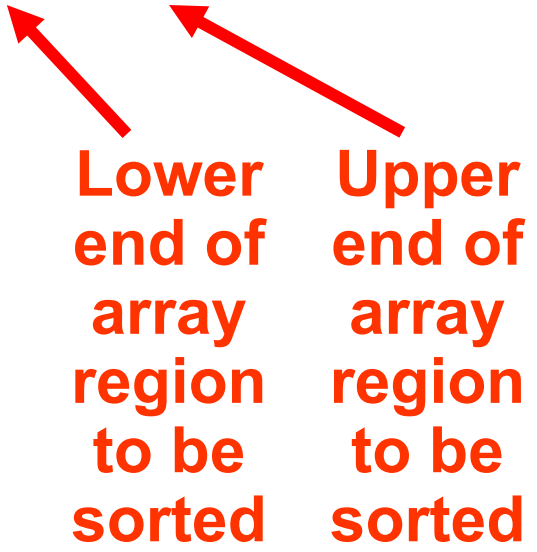
1. If list below size K
   - Sort w/ other algorithm
2. Else pick pivot x and partition S into
   - L elements < x
   - E elements = x
   - G elements > x
3. Quicksort L & G
4. Concatenate L, E & G
   - If not sorting in place



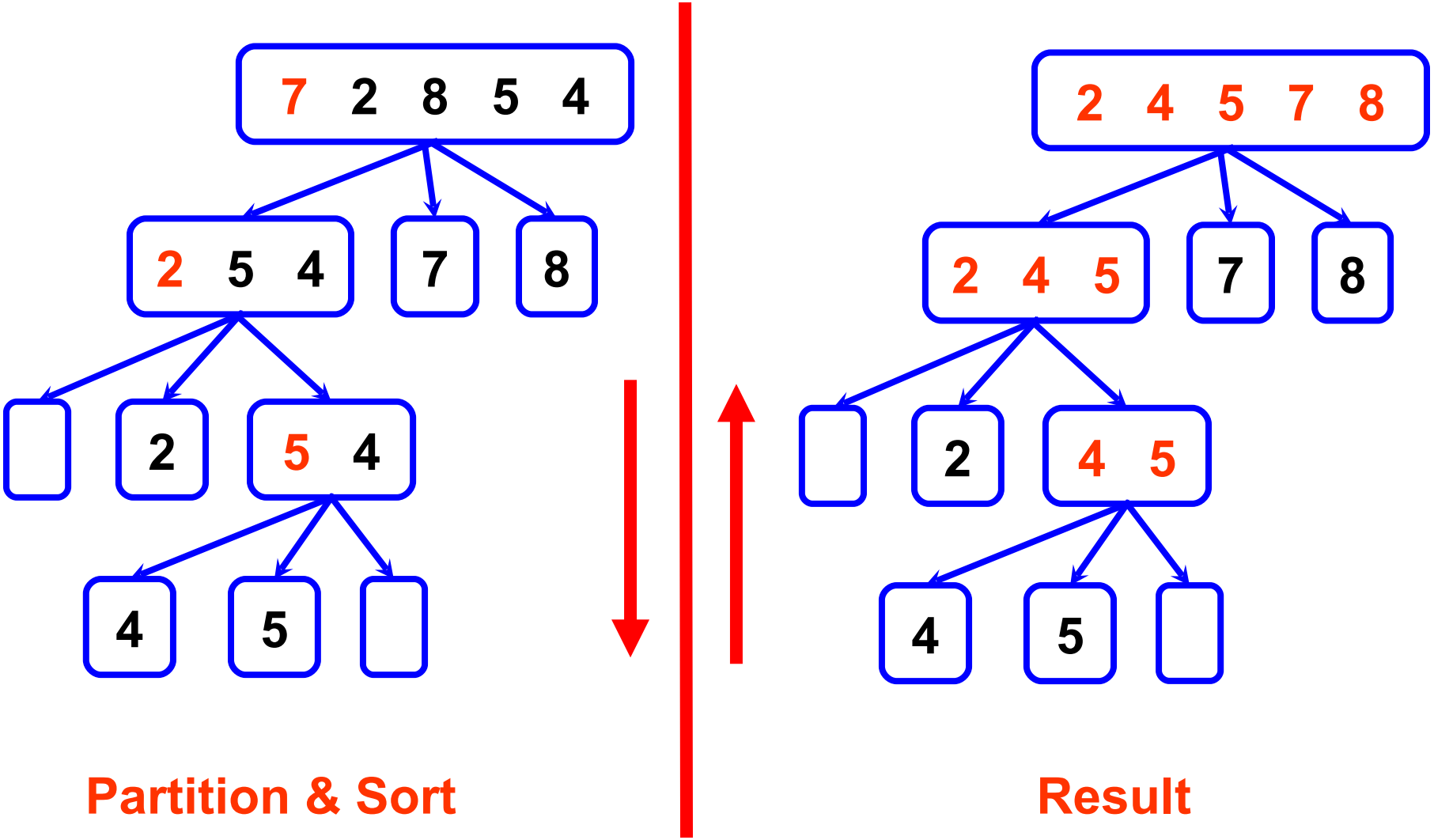
Quick Sort Code

```
void quickSort(int[] a, int x, int y) {
      int pivotIndex;
      if ((y - x) > 0) {
         pivotIndex = partionList(a, x, y);
         quickSort(a, x, pivotIndex - 1);
         quickSort(a, pivotIndex+1, y);
      }
}

int partionList(int[] a, int x, int y) {
       … // partitions list and returns index of pivot
}
```



Quick Sort Example



Quick Sort Code

```
int partitionList(int[] a, int x, int y) {
      int pivot = a[x];
      int left = x;
      int right = y;
      while (left < right) {
        while ((a[left] < pivot) && (left < right))
            left++;
        while (a[right] > pivot)
            right--;
         if (left < right)
            swap(a, left, right);
       }
       swap(a, x, right);
       return right;
}
```

Use first element as pivot

Partition elements in array relative to value of pivot

Place pivot in middle of partitioned array

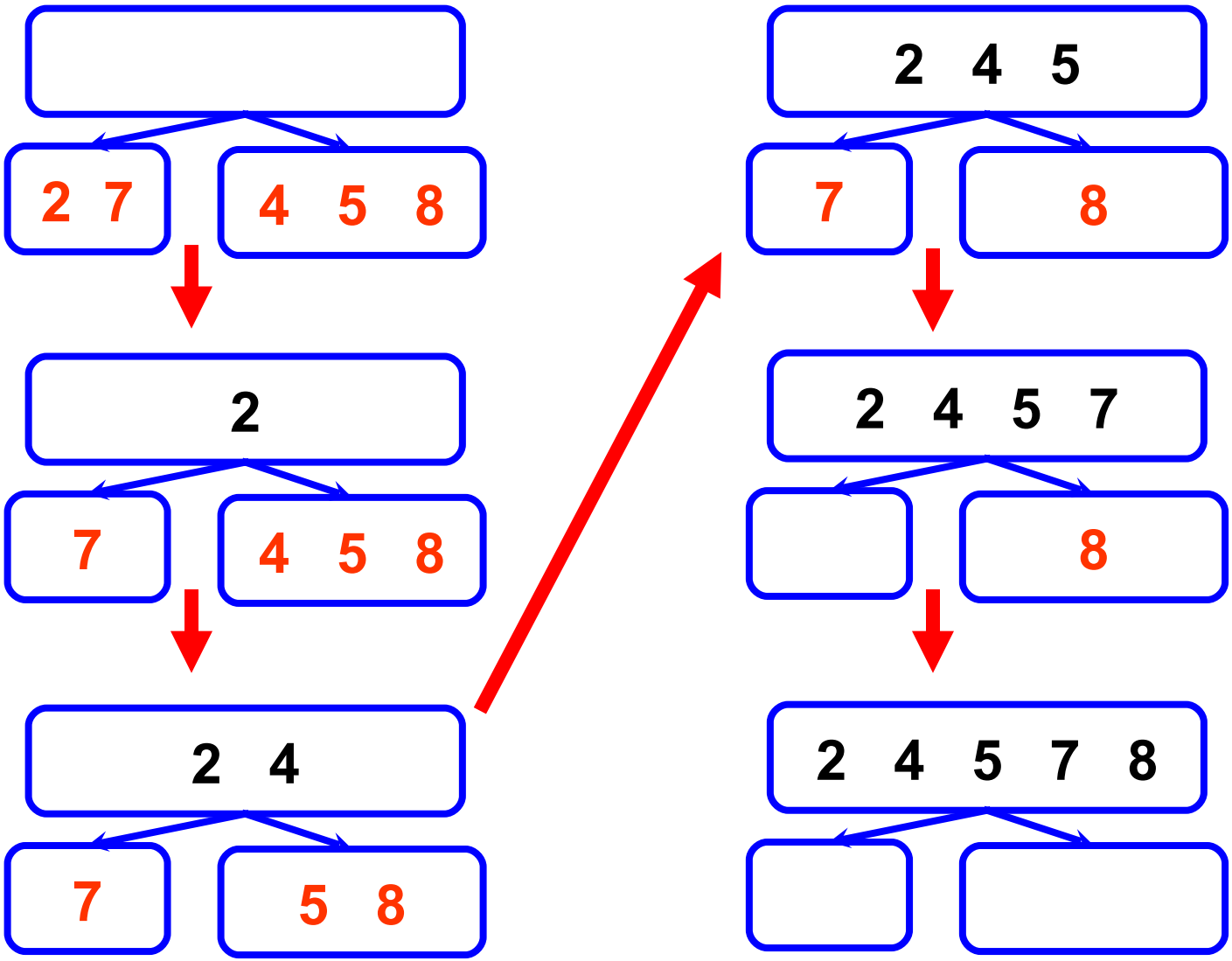
return index of pivot

Merge Sort

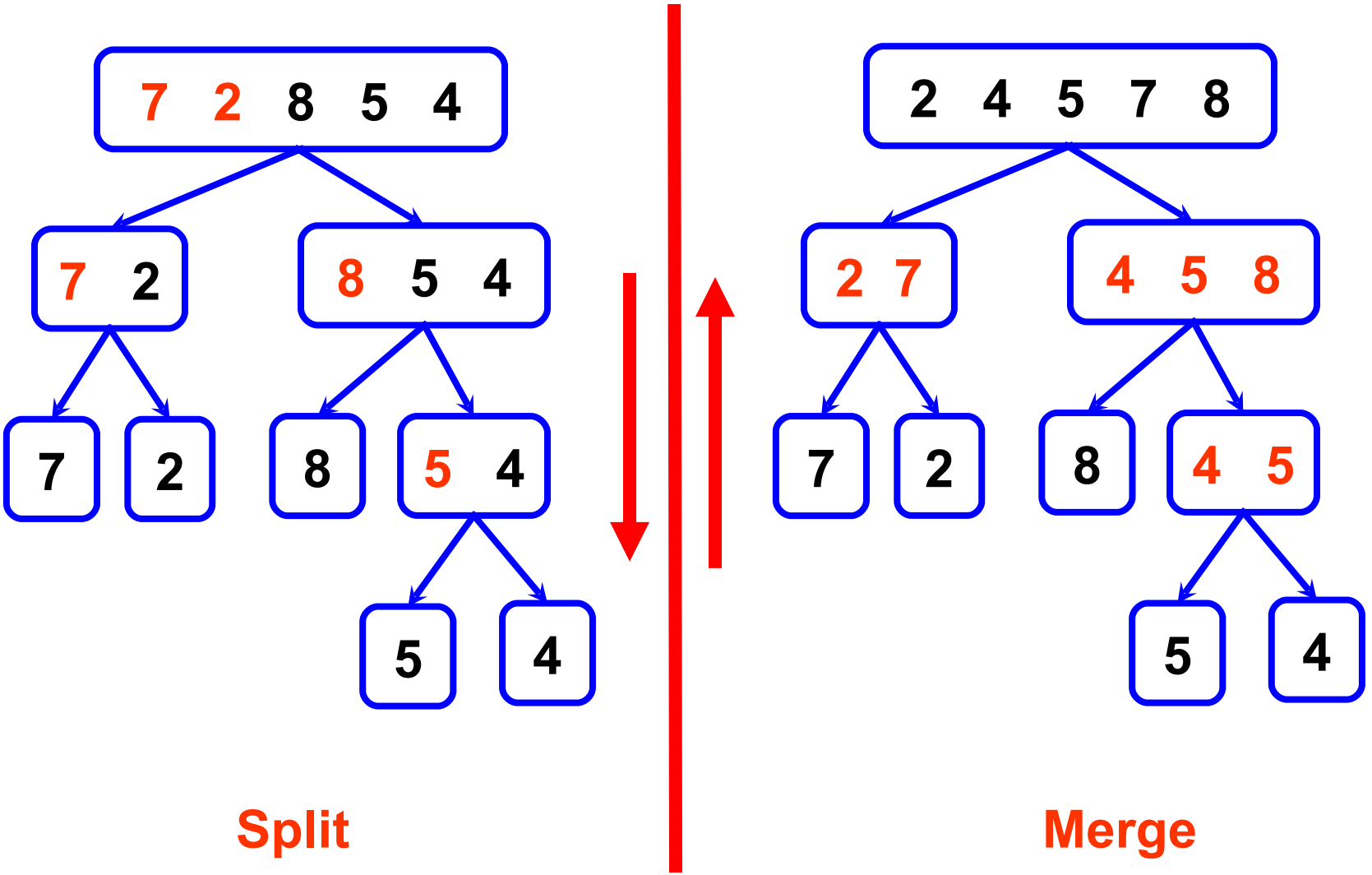
- **Approach**
  1. Partition list of elements into 2 lists
  2. Recursively sort both lists
  3. Given 2 sorted lists, merge into 1 sorted list
     a) Examine head of both lists
     b) Move smaller to end of new list

- **Performance**
  - O(n log(n)) average / worst case

Merge Example



Merge Sort Example



Merge Sort Code

```
void mergeSort(int[] a, int x, int y) {
      int mid = (x + y) / 2;
      if (y == x) return;
      mergeSort(a, x, mid);
      mergeSort(a, mid+1, y);
      merge(a, x, y, mid);
}
void merge(int[] a, int x, int y, int mid) {
      … // merges 2 adjacent sorted lists in array
}
```

Lower end of array region to be sorted

Upper end of array region to be sorted

Merge Sort Code

```
void merge (int[] a, int x, int y, int mid) {
    int size = y - x;
    int left = x;
    int right = mid+1;
    int[] tmp; int j;
    for (j = 0; j < size; j++) {
        if (left > mid) tmp[j] = a[right++];
        else if (right > y) || (a[left] < a[right])
            tmp[j] = a[left++];
        else tmp[j] = a[right++];
        }
    for (j = 0; j < size; j++)
        a[x+j] = tmp[j];
}
```

Upper end of 1st array region

Upper end of 2nd array region

Lower end of 1st array region

Copy smaller of two elements at head of 2 array regions to tmp buffer, then move on

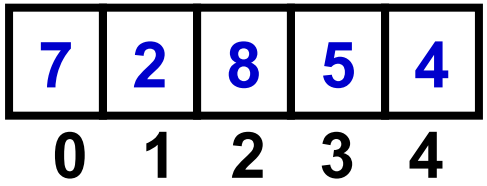
Copy merged array back

Counting Sort

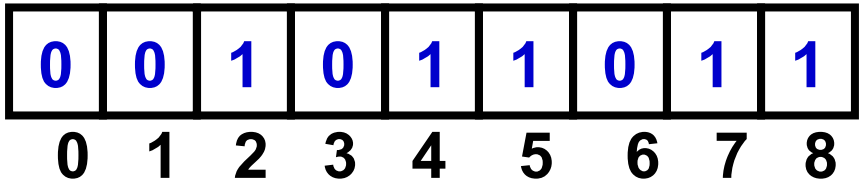
- **Approach**
  1. Sorts keys with values over range 0..k
  2. Count number of occurrences of each key
  3. Calculate # of keys ≅ each key
  4. Place keys in sorted location using # keys counted
     - If there are x keys ≅ key y
     - Put y in x^{th} position
     - Decrement x in case more instances of key y
- **Properties**
  - O(n + k) average / worst case

Counting Sort Example

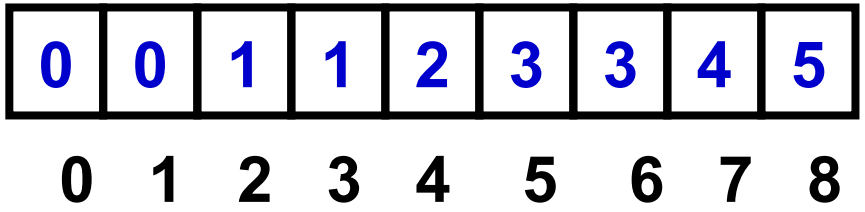
- Original list



- Count

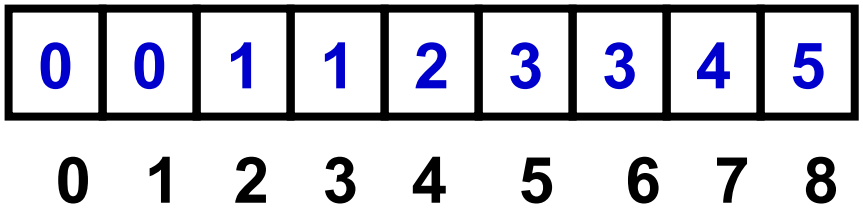


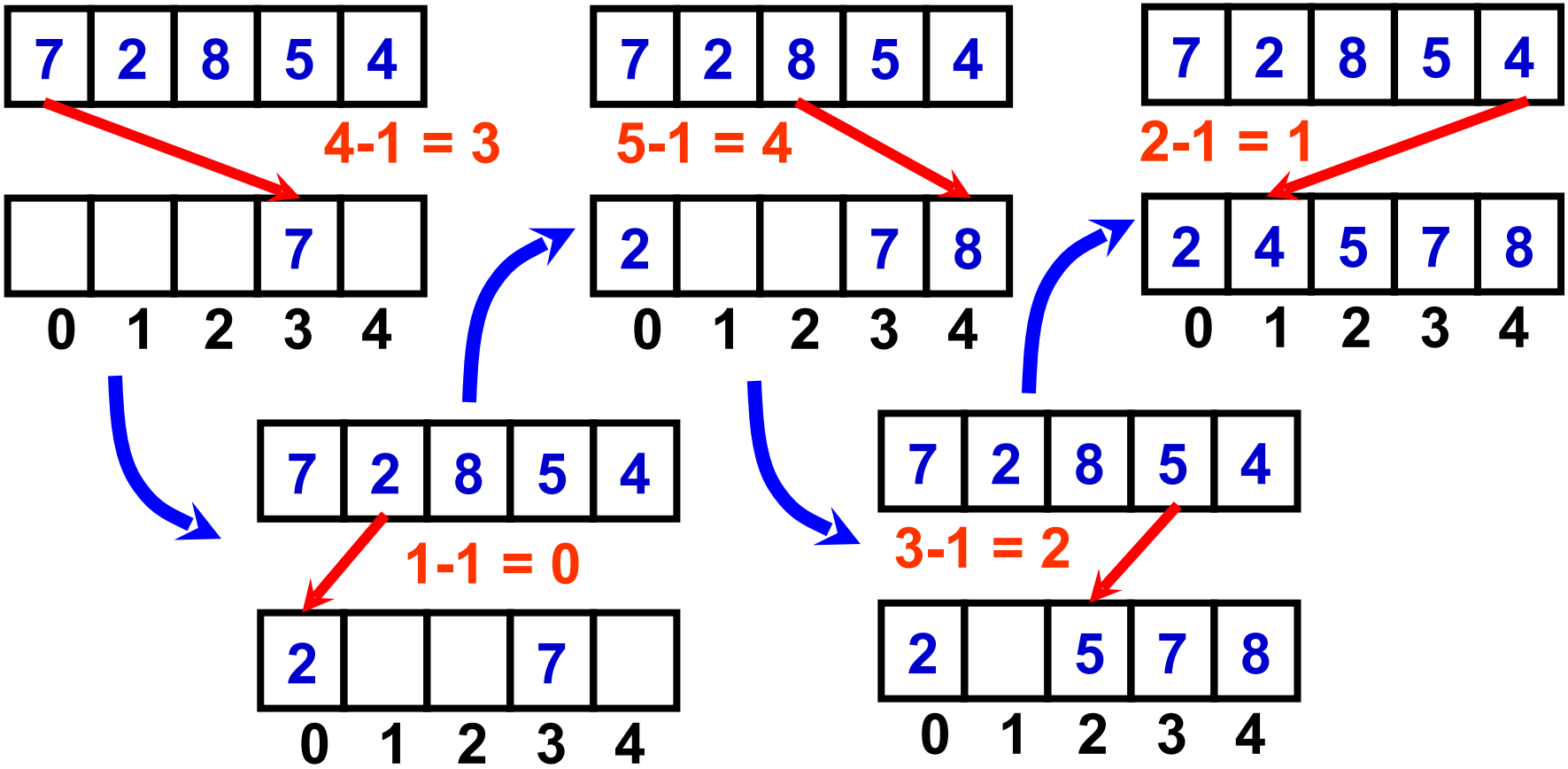
- Calculate # keys ≅ value



Counting Sort Example

- Assign locations





Counting Sort Code

```
void countSort(int[] a, int k) {  // keys have value 0…k
    int[] b; int[] c; int i;
    for (i = 0; i ≅  k; i++)          // initialize counts
        c[i] = 0;
    for (i = 0; i < a.size(); i++)  // count # keys
        c[a[i]]++;
    for (i = 1; i ≅  k; i++)          // calculate # keys ≅  value
    i
        c[i] = c[i] + c[i-1]
    for (i = a.size()-1; i > 0; i--) {
        b[c[a[i]]-1] = a[i];          // move key to location
        c[a[i]]--;                    // decrement # keys ≅  a[i]
    }
    for (i = 0; i < a.size(); i++)  // copy sorted list back to a
        a[i] = b[i];
}
```

Bucket (Bin) Sort

- **Approach**
  1. Divide key interval into k equal-sized subintervals
  2. Place elements from each subinterval into bucket
  3. Sort buckets (using other sorting algorithm)
  4. Concatenate buckets in order
- **Properties**
  - Pick large k so can sort n / k elements in O(1) time
  - O(n) average case
  - O(n^2) worst case
    - If most elements placed in same bucket and sorting buckets with O(n^2) algorithm

Bucket Sort Example

1. **Original list**
   - 623, 192, 144, 253, 152, 752, 552, 231
2. **Bucket based on 1st digit, then sort bucket**
   - 192, 144, 152 ≅ 144, 152, 192
   - 253, 231 ≅ 231, 253
   - 552 ≅ 552
   - 623 ≅ 623
   - 752 ≅ 752
3. **Concatenate buckets**
   - 144, 152, 192 | 231, 253 | 552 | 623 | 752

Radix Sort

- **Approach**
  1. Decompose key C into components C_1, C_2, … C_d
     - Component d is least significant
     - Each component has values over range 0..k
  2. For each key component i = d down to 1
     - Apply linear sort based on component C_i (sort must be stable)
  - Example key components
    - Letters (string), digits (number)
- **Properties**
  - O(d ≅ (n+k)) ≅ O(n) average / worst case

Radix Sort Example

1. **Original list**
   - 623, 192, 144, 253, 152, 752, 552, 231
2. **Sort on 3rd digit** (counting sort from 0-9)
   - 231, 192, 152, 752, 552, 623, 253, 144
3. **Sort on 2nd digit** (counting sort from 0-9)
   - 623, 231, 144, 152, 752, 552, 253, 192
4. **Sort on 1st digit** (counting sort from 0-9)
   - 144, 152, 192, 231, 253, 552, 623, 752

Compare with: counting sort from 144-752

Sorting Properties

Name	Compari-son Sort	Avg Case Complexity	Worst Case Complexity	In Place	Can be Stable
Bubble	☒	$O(n^2)$	$O(n^2)$	☒	☒
Selection	☒	$O(n^2)$	$O(n^2)$	☒	☒
Tree	☒	O(n log(n))	$O(n^2)$		
Heap	☒	O(n log(n))	O(n log(n))		
Quick	☒	O(n log(n))	$O(n^2)$	☒	
Merge	☒	O(n log(n))	O(n log(n))		☒
Counting		O(n)	O(n)		☒
Bucket		O(n)	$O(n^2)$		☒
Radix		O(n)	O(n)		☒

Sorting Summary

- Many different sorting algorithms
- Complexity and behavior varies
- Size and characteristics of data affect algorithm