CMSC 132:
Object-Oriented Programming II

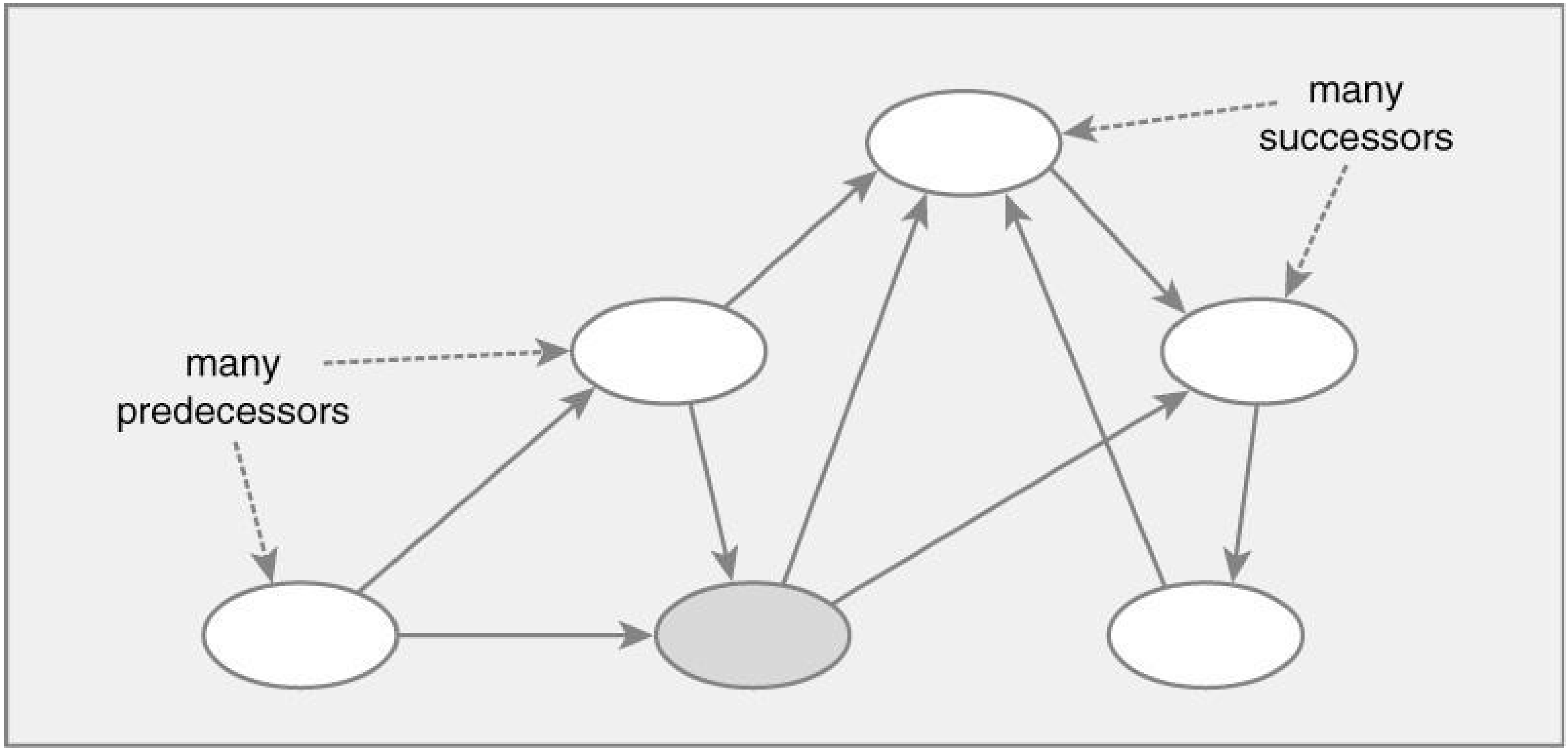
Graphs & Graph Traversal

Department of Computer Science

University of Maryland, College Park

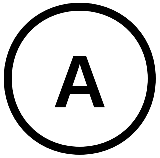
Graph Data Structures

- **Many-to-many relationship between elements**
  - **Each element has multiple predecessors**
  - **Each element has multiple successors**

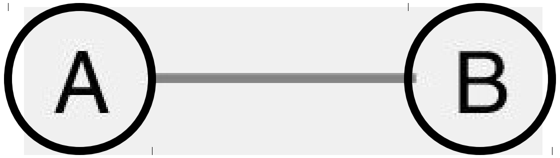


Graph Definitions

- **Node**
  - **Element of graph**
  - **State**
    - **List of adjacent/neighbor/successor nodes**

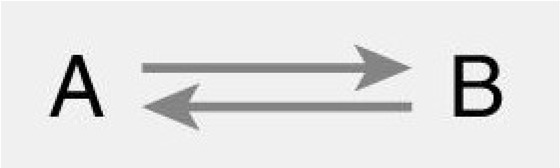


- **Edge**
  - **Connection between two nodes**
  - **State**
    - **Endpoints of edge**

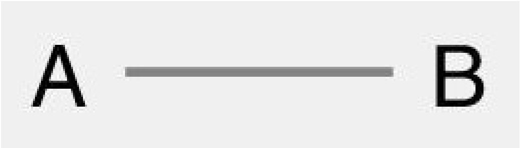


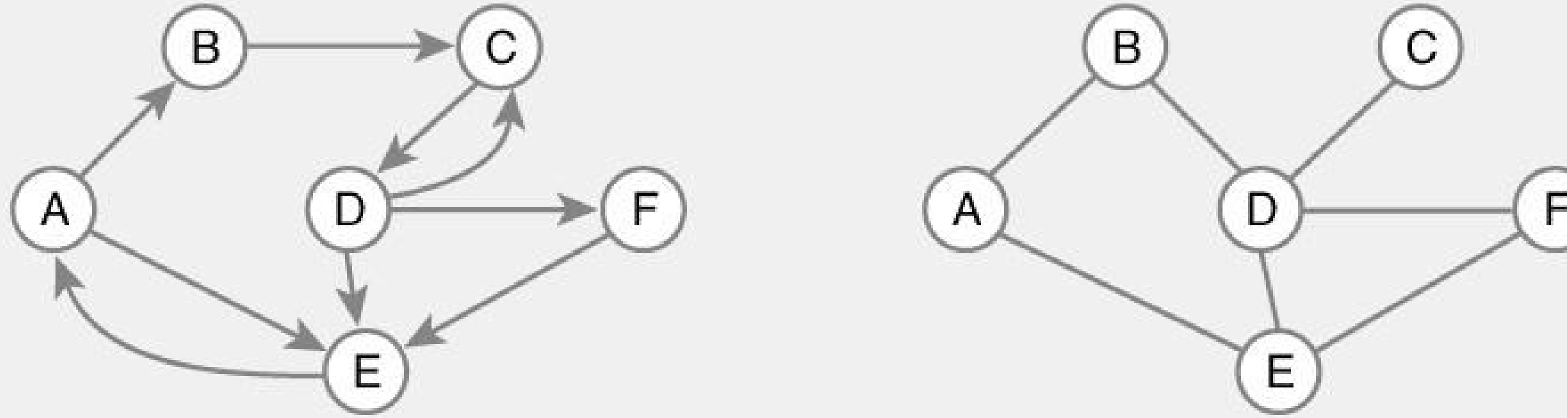
Graph Definitions

- **Directed graph**
  - **Directed edges**



- **Undirected graph**
  - **Undirected edges**

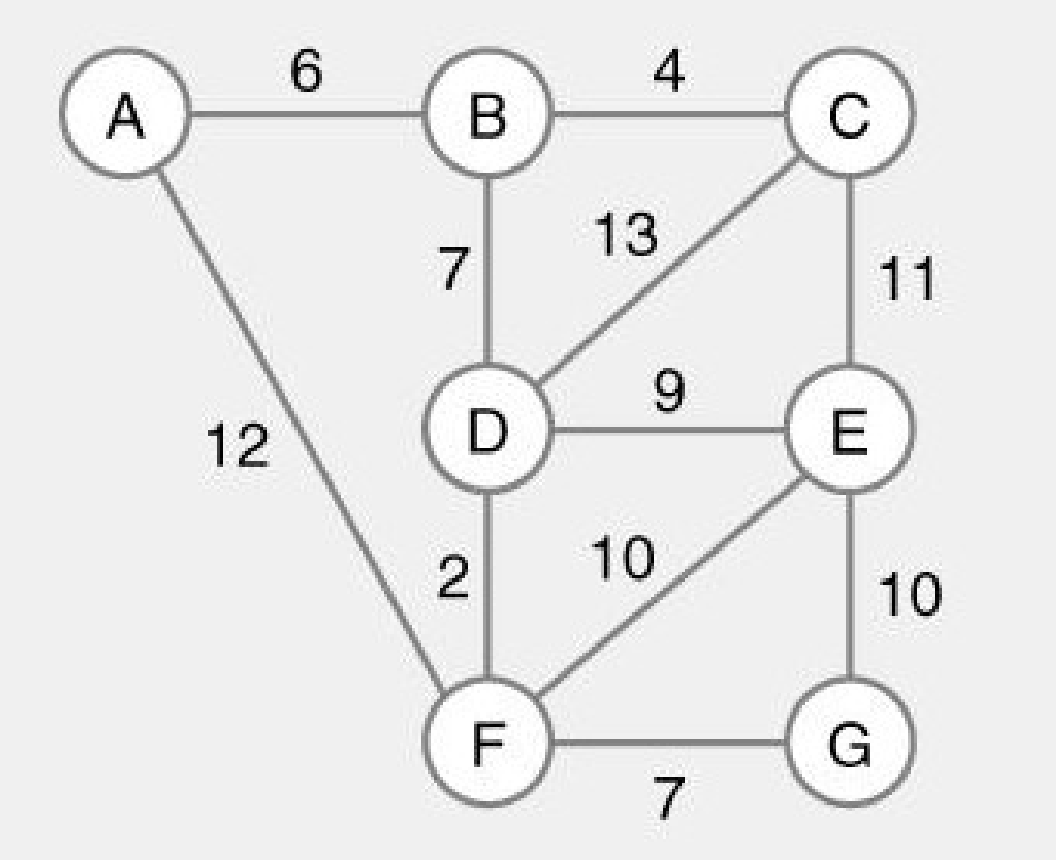




(a) Directed graph (b) Undirected graph

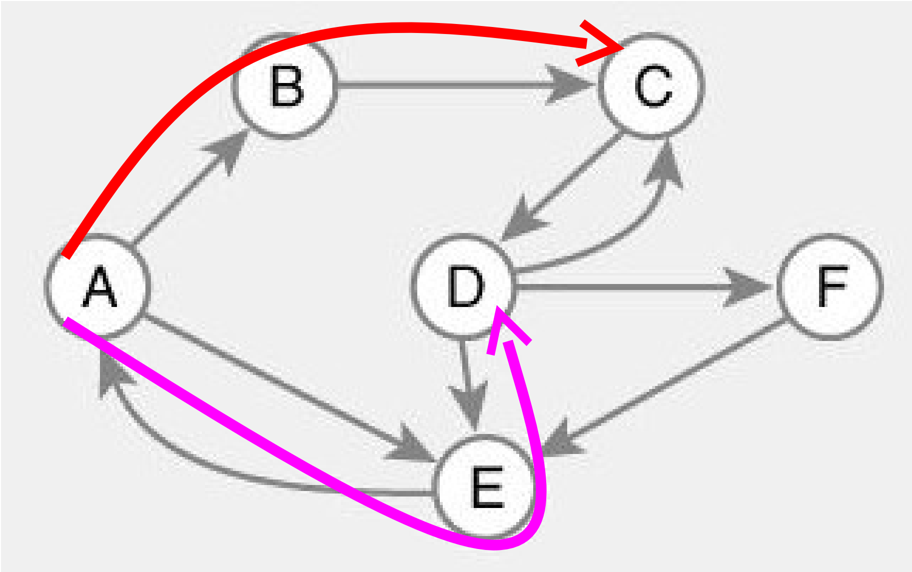
Graph Definitions

- **Weighted graph**
  - **Weight (cost) associated with each edge**



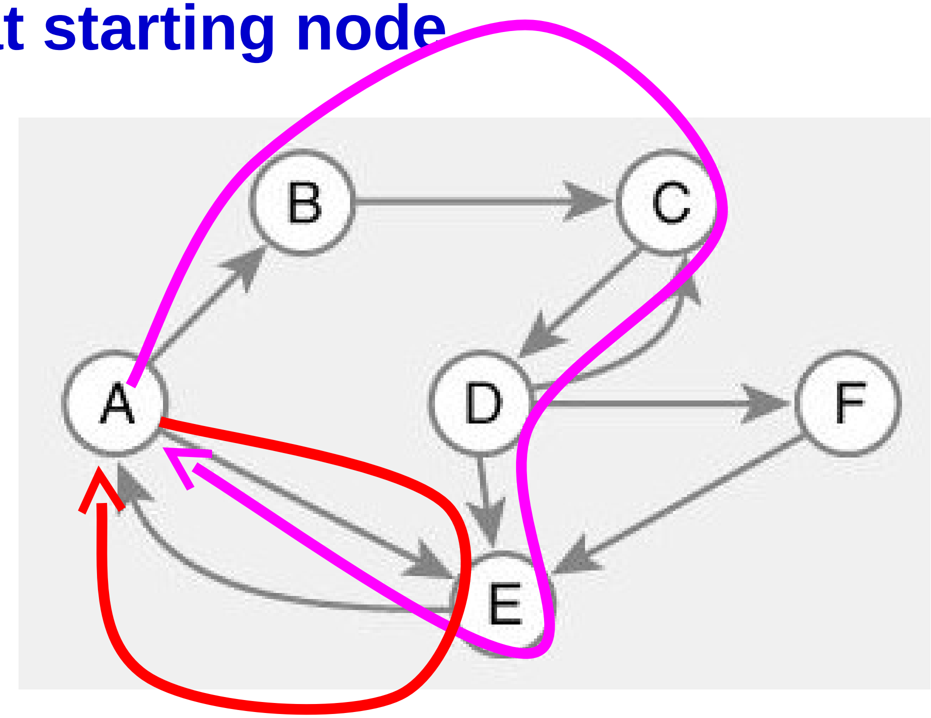
Graph Definitions

- **Path**
  - **Sequence of nodes n_1, n_2, … n_k**
  - **Edge exists between each pair of nodes n_i , n_{i+1}**
  - **Example**
    - **A, B, C is a path**
    - **A, E, D is not a path**



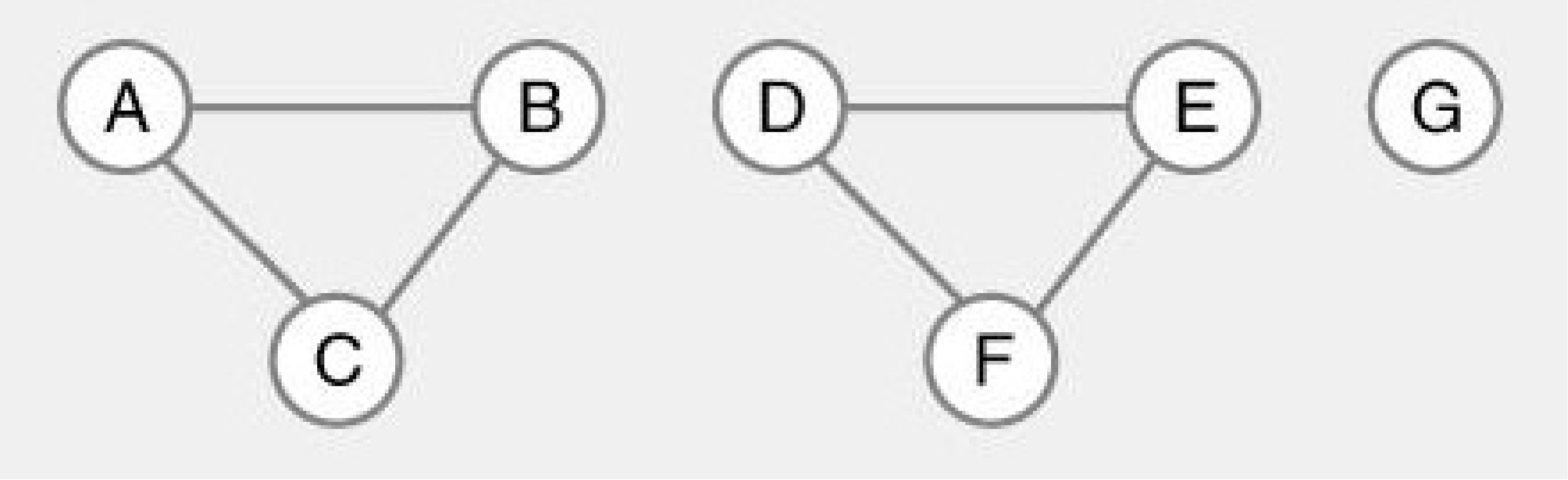
Graph Definitions

- Cycle
  - Path that ends back at starting node
  - Example
    - A, E, A
    - A, B, C, D, E, A
- Simple path
  - No cycles in path
- Acyclic graph
  - No cycles in graph



Graph Definitions

- Reachable
  - Path exists between nodes
- Connected graph
  - Every node is reachable from some node in graph



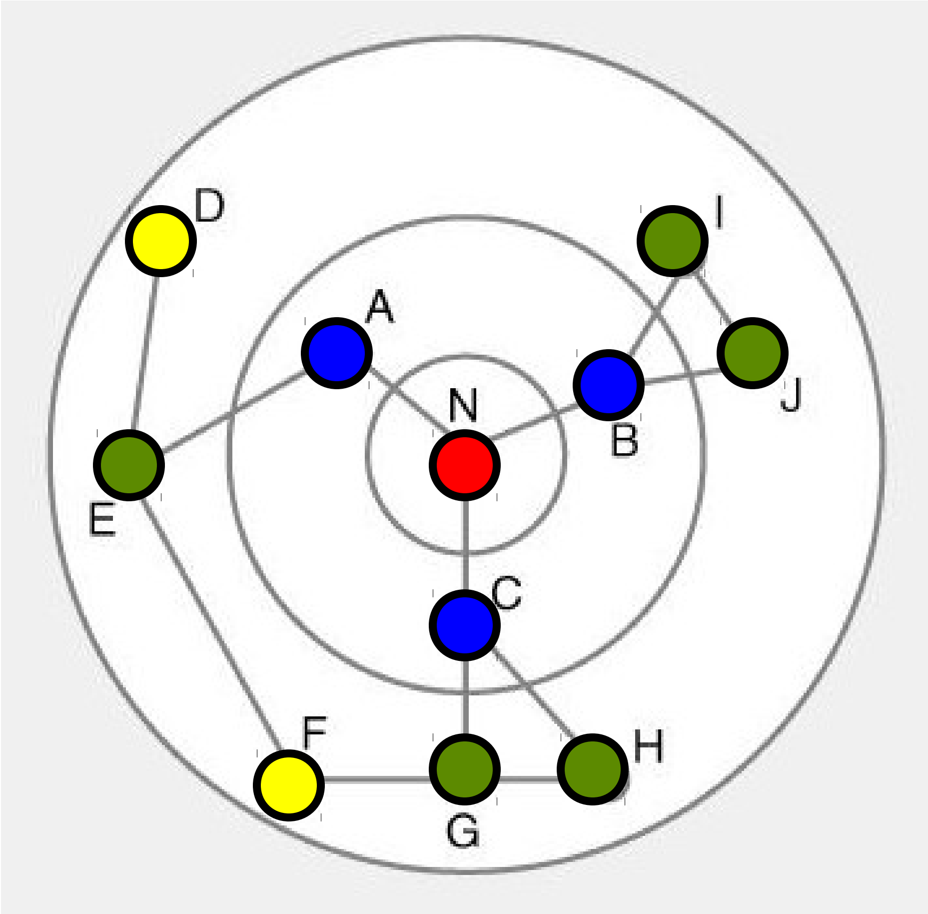
Unconnected graphs

Graph Operations

- Traversal (search)
  - Visit each node in graph exactly once
  - Usually perform computation at each node
  - Two approaches
    - Breadth first search (BFS)
    - Depth first search (DFS)

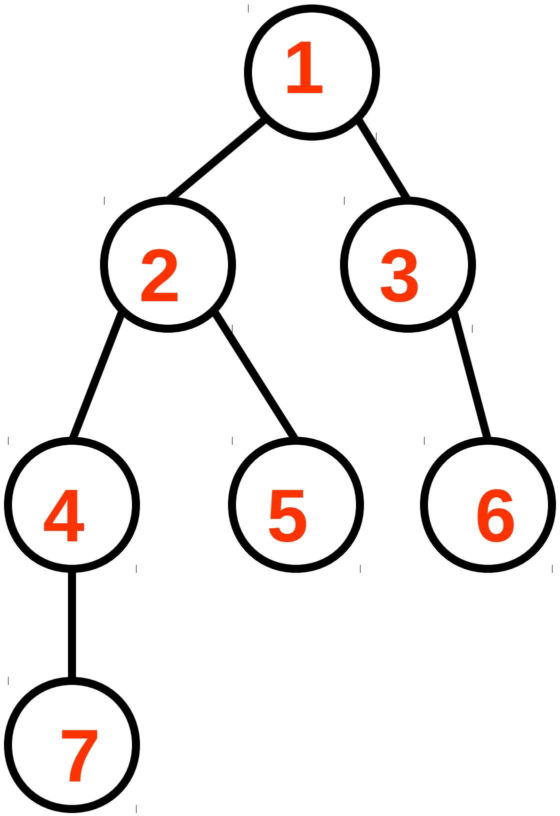
Breadth-first Search (BFS)

- **Approach**
  - **Visit all neighbors of node first**
  - **View as series of expanding circles**
  - **Keep list of nodes to visit in queue**
- **Example traversal**
  1. **n**
  2. **a, c, b**
  3. **e, g, h, i, j**
  4. **d, f**

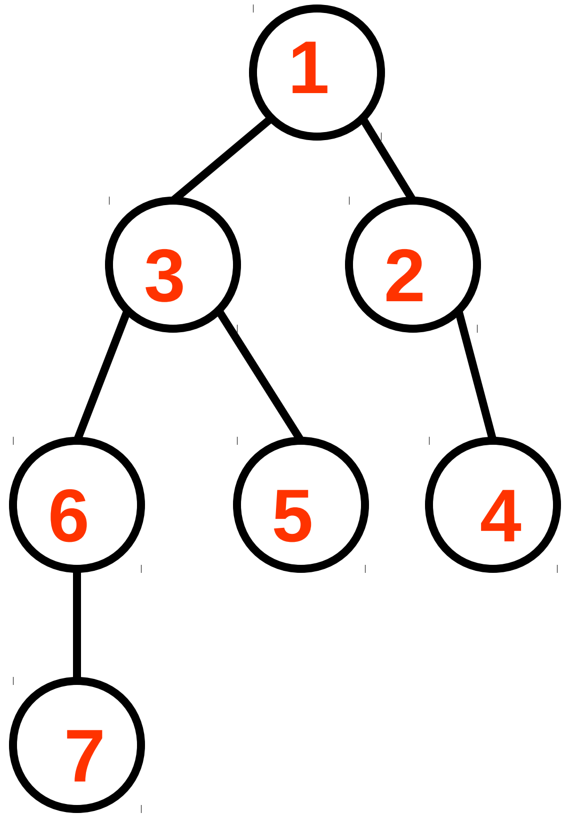


Breadth-first Tree Traversal

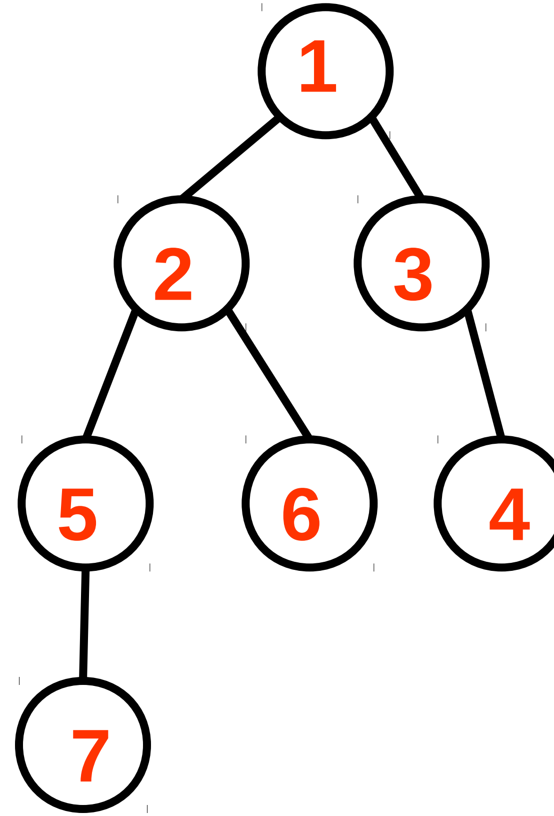
- Example traversals starting from 1



Left to right



Right to left



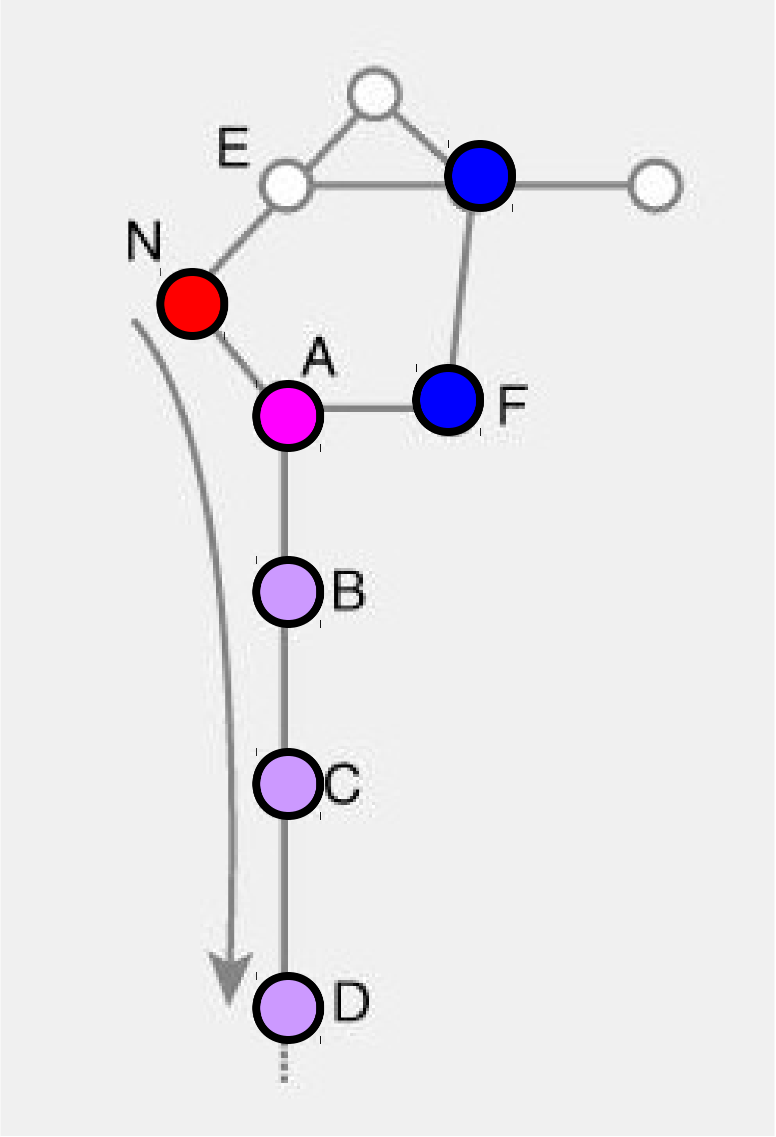
Random

Traversals Orders

- Order of successors
  - For tree
    - Can order children nodes from left to right
  - For graph
    - Left to right doesn't make much sense
    - Each node just has a set of successors and predecessors; there is no order among edges
- For breadth first search
  - Visit all nodes at distance k from starting point
  - Before visiting any nodes at (minimum) distance k+1 from starting point

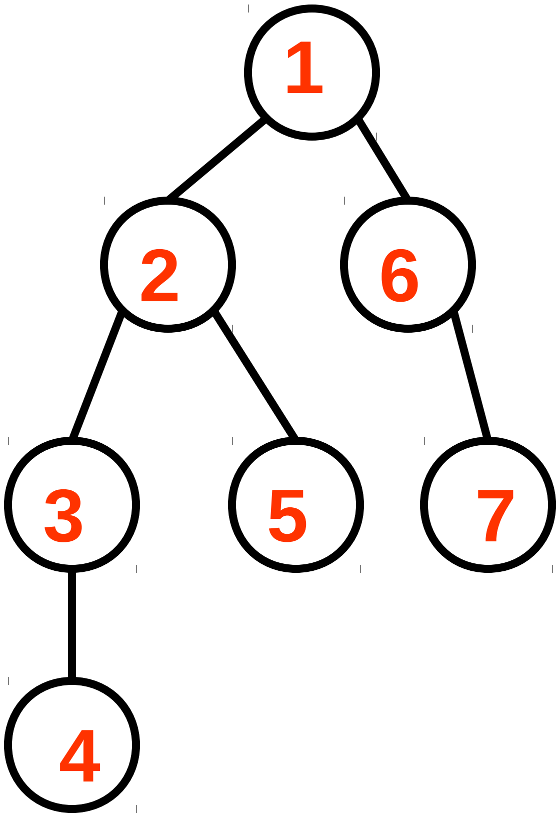
Depth-first Search (DFS)

- **Approach**
  - **Visit all nodes on path first**
  - **Backtrack when path ends**
  - **Keep list of nodes to visit in a stack**
- **Example traversal**
  1. **N**
  2. **A**
  3. **B, C, D, …**
  4. **F…**

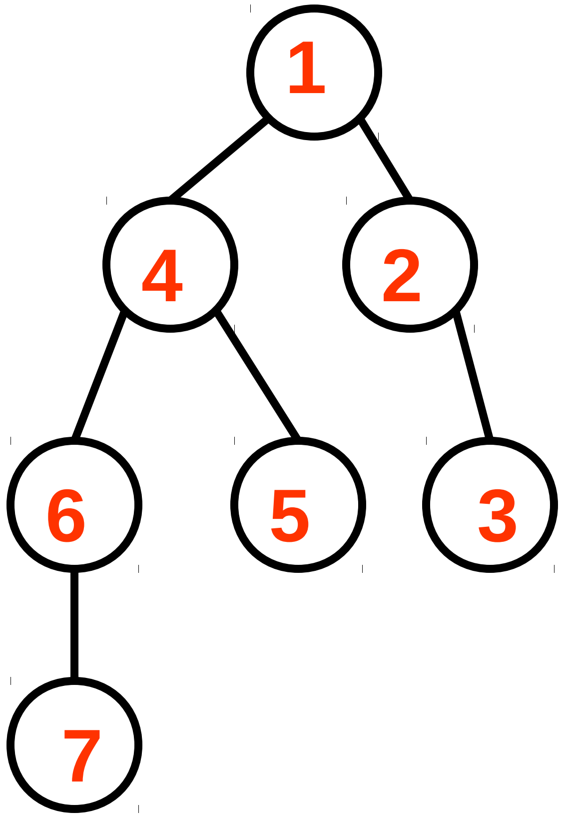


Depth-first Tree Traversal

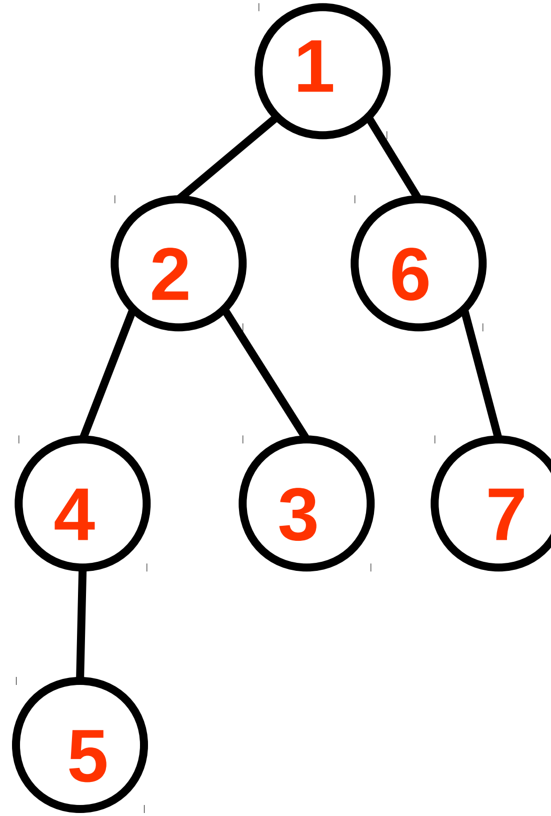
- **Example traversals from 1 (preorder)**



Left to right



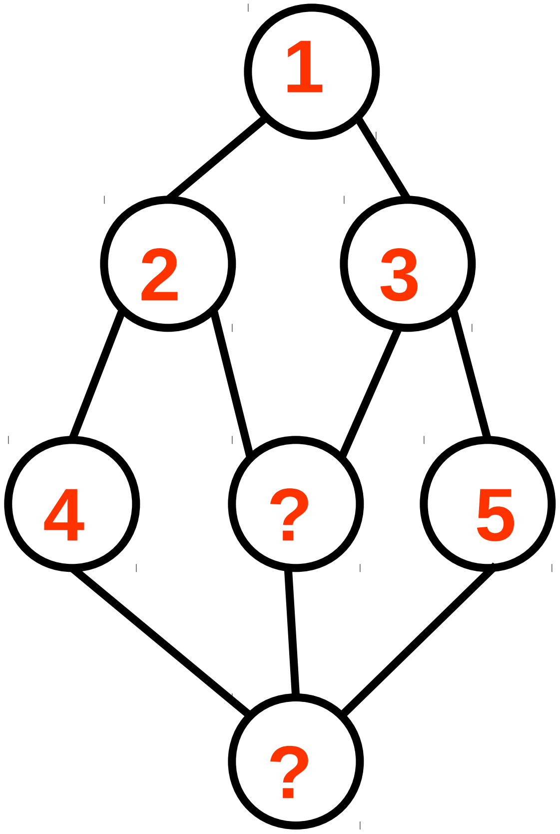
Right to left



Random

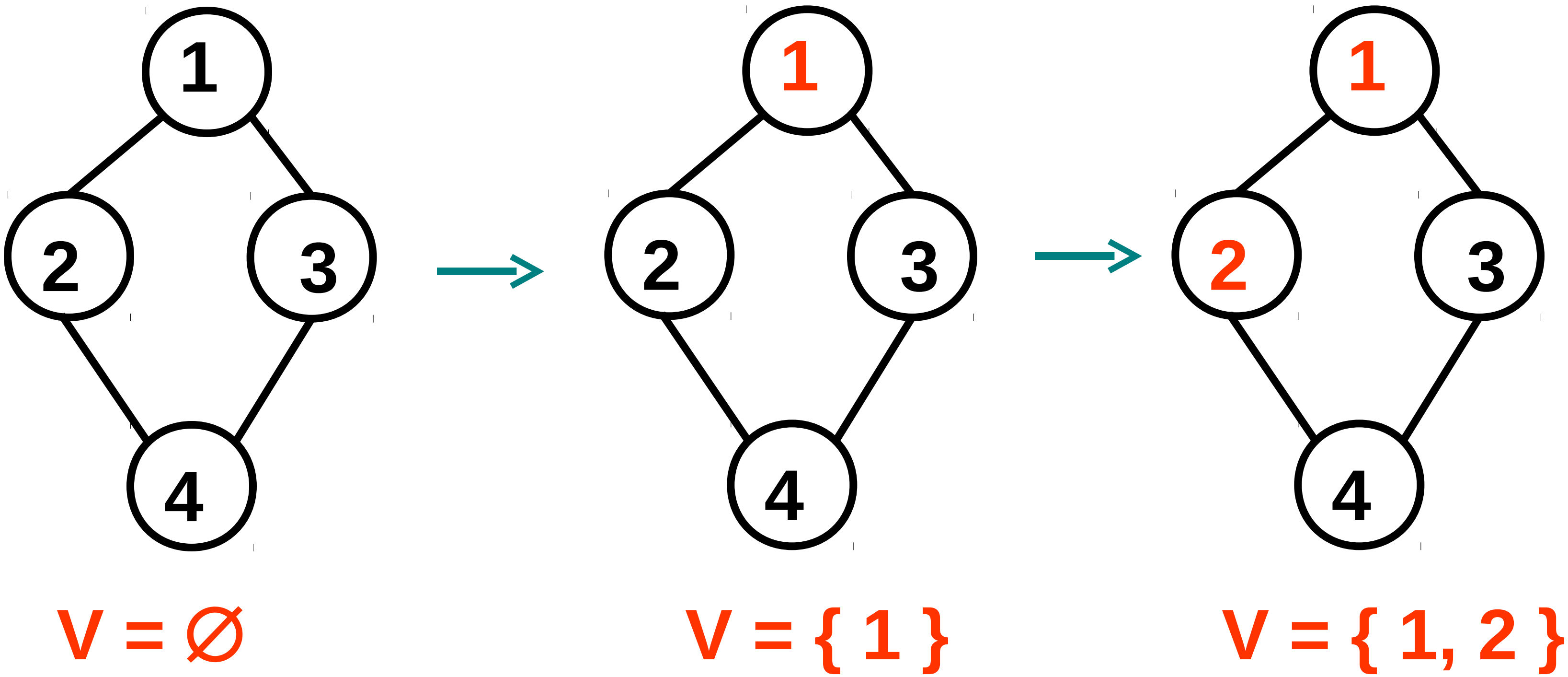
Traversal Algorithms

- **Issue**
  - How to avoid revisiting nodes
  - Infinite loop if cycles present
- **Approaches**
  - Record set of visited nodes
  - Mark nodes as visited



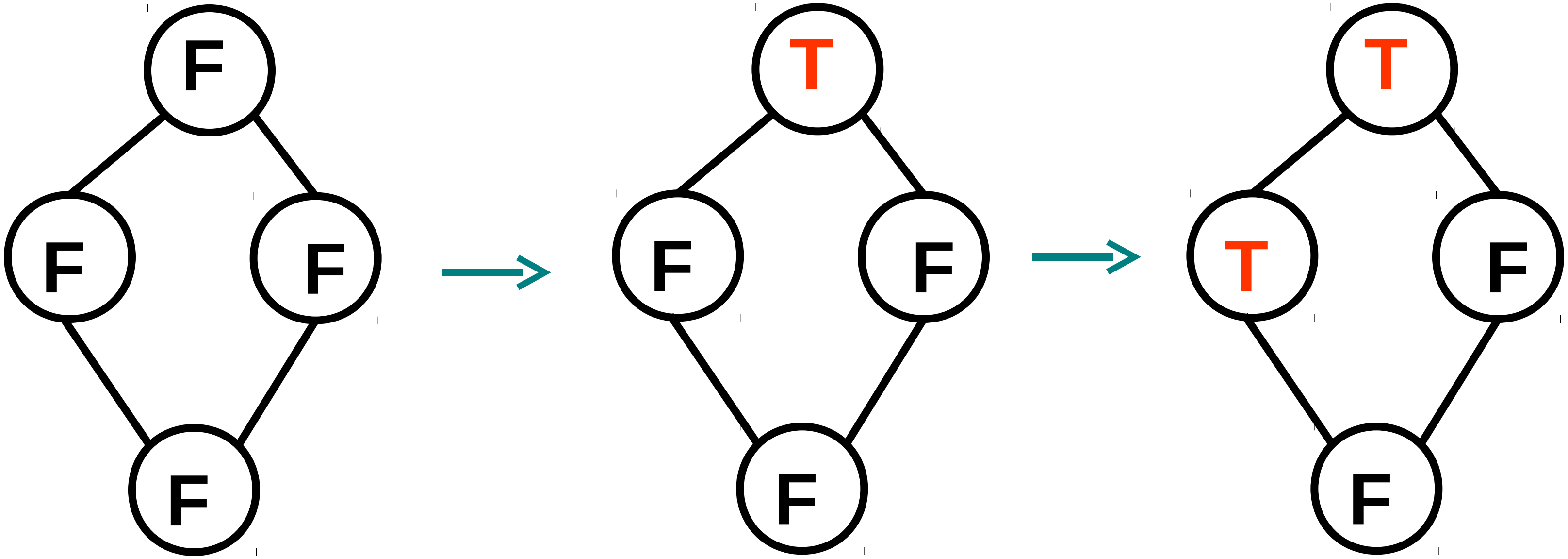
Traversal – Avoid Revisiting Nodes

- Record set of visited nodes
  - Initialize { Visited } to empty set
  - Add to { Visited } as nodes is visited
  - Skip nodes already in { Visited }



Traversal – Avoid Revisiting Nodes

- **Mark nodes as visited**
  - **Initialize tag on all nodes (to False)**
  - **Set tag (to True) as node is visited**
  - **Skip nodes with tag = True**



General Traversal Algorithm

```
{ Visited } = ∅
{ Discovered } = { 1st node }
while ( { Discovered } ≠ ∅ )
    take node X out of { Discovered }
    if X not in { Visited }
        add X to { Visited }
        for each successor Y of X
            if ( Y is not in { Visited } )
                add Y to { Discovered }
```

Traversal Algorithm Using Tags

```
for all nodes X
    set X.tag = False
{ Discovered } = { 1st node }
while ( { Discovered } ≠ ∅ )
    take node X out of { Discovered }
    if (X.tag = False)
        set X.tag = True
        for each successor Y of X
            if (Y.tag = False)
                add Y to { Discovered }
```

Traversal Algorithm with Queue

```
for all nodes X
     X.tag = False
put 1st node in Queue
while ( Queue not empty )
    take node X out of Queue
    if (X.tag = False)
        set X.tag = True
        for each successor Y of X
            if (Y.tag = False)
                put Y in Queue
```

Traversal Algorithm with Stack

```
for all nodes X
    X.tag = False
put 1st node in Stack
while (Stack not empty )
    pop X off Stack
    if (X.tag = False)
        set X.tag = True
        for each successor Y of X
            if (Y.tag = False)
                push Y onto Stack
```

BFS vs. DFS Traversal

- Implement { Discovered } as Queue
  - First in, first out
  - Traverse nodes breadth first
- Implement { Discovered } as Stack
  - First in, last out
  - Traverse nodes depth first

Recursive Traversal Algorithm

```
Traverse( )
    for all nodes X
        set X.tag = False
    Visit ( 1st node )
Visit ( X )
    set X.tag = True
    for each successor Y of X
        if (Y.tag = False)
            Visit ( Y )
```

Recursive Graph Traversal

- Can traverse graph using recursive algorithm
  - Recursively visit successors
- Implicit call stack & backtracking
  - Results in depth-first traversal